An Empirical Method for Efficient Spectrum Estimation from LDA Data

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Abstract The paper reports a new methodology for noise suppression of LDA data and accurate spectra estimation. The methodology is based on the empirical mode decomposition method that employs an iterative sifting procedure. The decomposition process leaves a residue which represents the noise and can be subtracted from the data. The advantage of the technique is that it is robust, i.e. it is based entirely on the data and no time reference is required. Hence, it can be applied on both regularly and irregularly-spaced data. The technique is tested against (relatively high resolution) LDA data obtained from three different experiments: a cylinder wake, an axisymmetric jet and grid turbulence. In all cases, the nearest interpolation scheme is employed for the resampling of the data, before or after noise suppression. Estimated spectra are compared with HWA ones as well as spectra obtained using well-established techniques and the effectiveness of the technique is clearly demonstrated.

1. Introduction

Recovery of reliable spectral estimates from LDA data is fraught with difficulties due to the fact that individual velocity realisations are unevenly spaced in time. Historically, spectral estimation is synonymous with fast Fourier techniques which require that the input data are equally spaced in time. In order to overcome these difficulties and improve the accuracy of power spectra (PSD) estimation from LDA data, various techniques have been proposed and further refined over the last 30 years which can be broadly classified into four groups (Broersen et al. 2000): a) Direct transform methods, b) slotting techniques, c) parametric or model-based estimators, and d) time-series reconstruction methods. These techniques have been reviewed by Tropea (1995) and Benedict et al. (2000) and benchmark tests have been carried out using simulated and real data in order to assess their performance and identify strengths and weaknesses (Britz and Antonia 1996; Benedict et al. 1998).

Direct transform methods were introduced by Gaster and Roberts (1977) and make use of the periodogram technique adapted to the randomly sampled data. The random nature of the data introduces additional variability in the estimated spectra which can be reduced by block averaging techniques (Roberts and Gaster 1980). A modification to this method is the Lomb technique which is particularly useful in identifying narrow peaks.

The slotting technique determines the autocorrelation function (ACF) of the velocity fluctuations by dividing the lag time into bins and the spectra are subsequently estimated using conventional Fourier transforms. The technique was firstly introduced by Mayo et al. (1974) but suffers from high statistical variance and is unable to cope with noise and bias. However, its performance has been improved substantially through various refinement schemes proposed over the last 10 years, such as the use of local normalisation (van Maanen and Tummers 1996), local normalisation and

variable lag window (Tummers and Passchier 1996) or the fuzzy slotting technique (Nobach et al. 1998b). Refinements have been shown to reduce the variance inherent in the standard slotting technique and performed well in benchmark tests (Benedict et al. 1998). Further benefits were derived by merging the local normalisation and the fuzzy slotting technique (van Maanen et al. 1999) and also by considering the processor delay time via local time lag estimation in the ACF determination (Nobach 2002). The latter results in more accurate ACF estimates which, nevertheless are irregularly spaced and require further modifications.

Parametric or model based spectral estimators attempt to improve the accuracy of the estimates by fitting a model to either the autocorrelation function or the spectra (van Maanen and Tulleken 1994; van Maanen and Oldenziel 1998; Müller et al. 1998). This requires a priori information about the actual process so that the model adopted can represent it closely. Some of these methods improved the estimation of PSDs even at low data rates.

The fourth group of techniques firstly resamples the LDA data in order to create a time series of equally spaced data which can be subsequently processed by means of conventional FFT techniques. The most common technique is the sample and hold (S+H) or zero-order interpolation technique which has been analysed extensively by Adrian and Yao (1987). This study highlighted the basic limitations of the resampling techniques which are the low pass-filter effect, also called 'particle-rate effect' and added step noise which is white. Other interpolation methods have been implemented such as linear, nearest neighbour and exponential, as well as higher order schemes, Kalman and fractal reconstruction but they were found to alter the basic limitations of reconstruction methods very little (see Benedict et al. 2000).

Attempts have also been made to improve the accuracy of the S+H reconstruction technique by using a digital finite impulse filter (Nobach et al. 1998a). The resulting refined reconstruction method showed the lowest variance among other estimators and scored favourably in the benchmark tests reported in Benedict et al. (1998) together with some of the refined slotting techniques mentioned above. However, it addresses only the problem of particle-rate filter, introduces a bias at low data rates and requires an additional noise-suppression algorithm to improve the PSD accuracy.

A refinement to the S+H approach was recently proposed by Simon and Fitzpatrick (2004). The procedure makes use of a discrete low pass filter which allows the step noise to be determined directly. The results of the technique are comparable to the refined reconstruction of Nobach et al. (1998) but the technique is more straightforward and computationally efficient. It performs well at low data rates but poorly at high data rates where the local time estimation technique of Nobach (2002) appears to be superior. The problem arises at the high frequency end where the correction exhibits a high variability. Nevertheless, their analysis offers a theoretical framework to understand the step noise and filtering errors induced by the resampling process.

In addition to the limitation of random sampling in time, another problem often present in LDA measurements is that of inherent noise in the signal. There are various sources of noise in LDA and a comprehensive account of these can be found in van Maanen (1999). As an example of the problem, Fig. 1 shows the particles' velocity crossing a single dual-beam interference volume measured by two individual collection optics (backscatter and sidescatter) and signal processing units (Burst Spectrum Analysers). Regardless of the calibration of the two units, the individual measurements differ by a small random amount. While the signal-to-noise ratio can be optimized so as to avoid erroneous measurements by careful experimental practice, it is hardly the case that, at the same time, a sufficiently high data rate to resolve the highest-frequency scales in the flow can be

achieved. As a result, a compromise often needs to be made between sufficient data rate and an acceptable signal-to-noise ratio which precludes the availability of information at high frequencies (model-based estimators attempt to recover the lost information). On the other hand, if high data rates are desirable then one has to accept the unavoidable error due to measurement noise.

Measurement error inherent in the LDA data and errors due to the resampling process conspire to produce a power spectral density which often does not represent the physics of the flow being measured. In fact, the works cited above in the brief literature review indicate that the most efficient methods rely partially on noise suppression of some kind. Unfortunately, eliminating the noise through filtering techniques is impossible because there is no prescribed frequency content to remove. In this work, a novel methodology is proposed to rectify this problem. The efficacy of the methodology is demonstrated by application on available experimental data and also by comparison of the results obtained from spectral analysis to known results.

2. The empirical decomposition method: denoising

The method is based on the empirical mode decomposition developed by Huang et al. (1998) to treat nonlinear and nonstationary data. Their objective was to compute the Hilbert spectrum of the signal which is meaningful only if the instantaneous frequency can be defined. They proposed a decomposition of the signal into 'intrinsic mode functions' that admit well-behaved Hilbert transforms. Their procedure yields an energy-frequency-time description of the signal. This is particularly useful for amplitude-modulated and transient signals where other processing methods fail, e.g. Fourier analysis, wavelets, etc. The procedure was implicitly developed to treat regularly-sampled (equidistant) data. The objective in the present study is different: the empirical mode decomposition is implemented as a method to eliminate noise from LDA data with a view to computing the unbiased spectrum of turbulence from measurements of sufficiently high resolution in time. Turbulence is by virtue a wide-band process involving a continuous cascade of energy over different scales and, thus, the Hilbert spectrum of such a process is non-definite. Bearing in mind the limitations of the Fourier transformation with amplitude-modulated and transient signals, the present work seeks to extract the spectral characteristics of LDA turbulent signals down to the smallest dissipative scales from measurements with sufficient resolution.

According to Huang et al. (1998) a signal $X(t)$ can be decomposed into empirical modes by using the envelopes defined by the local maxima and local minima. The first step is, therefore, to identify the local extremes in the signal. After the extremes are known, all the maxima are connected by piecewise cubic Hermite polynomials to define the upper envelope and likewise all the minima are connected to define the lower envelope (see Fig. 2). The piecewise cubic Hermite polynomials were employed as an alternative to cubic splines because their performance is more robust, i.e. unrealistic overshoots and/or undershoots in the envelopes can be avoided. As Huang et al. (1998) note, the upper and lower envelopes should enclose all the original data between them (Fig. 2b). The mean of the upper and lower envelopes is designated as m_1 and the difference between the original data and m_1 is the first residual component r_1, i.e.

$$X(t) - m_1 = r_1 .$$
(1)

Clearly, the m_1 and r_1 components contain fluctuations with different time scales. It should be noted, that the exact time scale is irrelevant, i.e. the time scale can be expanded/shrunk without affecting the procedure. Unless for a reason, the fluctuations in both components should represent the physics of the measured signal. However, one may continue the decomposition procedure until the fluctuations contained in the residual component have no physical meaning, i.e. they represent

the noise in the signal. Any real signal is bound to be affected by noise to some extent. At every step of the decomposition (the sifting process) a new residual is produced:

$$r_{i+1} = r_i - m_{i+1}$$ (2)

where the mean in each step is found using the procedure described above. Figure 3 shows the outcome of the repeated siftings. As the number of siftings increases, the residual terms appear more and more like uncorrelated noise. The example shown in Fig. 2 is a measured turbulence signal in a cylinder wake (see section 3.1). In real situations, it is unlikely that the signal is free of noise and the last residual component is ideally the signal noise. However, this is not guaranteed. Furthermore, it might be expected that there is a limit on the number of siftings that can be performed without obliterating the physical meaning of the computed modes. Huang et al. (1998) suggested the following criterion for the sifting process to stop:

$$SD_i = \sum_k \left[\frac{(r_{i-1,k} - r_{i,k})^2}{r_{i-1,k}^2} \right] < 0.2 .$$ (3)

The above criterion essentially suggests that there is little difference between the residuals from successive siftings beyond this point. In the present context, the number of siftings will generally be a function of the amount of noise in the signal; it might be logical to stop the sifting process when the variance of the residual term σ_r^2 matches the noise variance σ_n^2, which has to be estimated from the data. An estimate of the noise can be obtained from the spike in the measured autocorrelation function at zero time-lag (Benedict and Gould 1998) or from model-based techniques (Nobach 2002). In this work, however, the sifting procedure is stopped when the standard deviation falls below 0.1 beyond which further siftings were found to have a weak effect.

Following the procedure described above, the original signal is decomposed into a number n of empirical modes m_i and a residue r_n:

$$X(t) = \sum_{i=1}^{n} m_i + r_n$$ (4)

It should be noted that although the above sifting procedure is similar to that by Huang et al (1998), the resulting modes shown above do not correspond to their 'intrinsic mode functions'. The procedure described above does not require a time reference since it uses only the extremes in the signal. Hence, it can be implemented to treat irregularly-sampled data such as those from LDA. In this case, the residual term can be used to subtract the noise from the signal (denoising), i.e. the noise-free signal is $u(t_i) = X(t_i) - r_n$ where the index i indicates the random arrival time of particles through the measuring volume. A disadvantage of this approach is that the $u(t_i)$ data is also irregular in time and therefore resampling of the data at regular intervals is still required. Even though the resampling process introduces step noise in the data, it has been found that removing the residue from the original data can improve the estimation of the turbulence spectrum, particularly in cases where LDA data is known to be noisy (see section 3.2). On the other hand, denoising can be done after resampling the irregularly-sampled data, in which case the method is likely to remove the step noise due to resampling plus the measurement noise. As shown in the next section, this denoising process proves to be effective and allows unbiased estimation of the turbulence spectrum.

3. Application to experimental data

The denoising method proposed in the previous section is applied on experimental data from three different experiments obtained in different laboratories: a) in a cylinder wake, b) in an axisymmetric jet, and c) in grid turbulence. The mean particle arrival rates are sufficiently high to resolve most of the turbulence scales. Comparisons in terms of the computed spectra using various methods are shown and the results are also compared to spectra from hot-wire anemometry (HWA) in the first two cases while in the last case the mean squared velocity gradient computed from integration of the dissipation spectrum is compared to that directly measured from two-point LDA.

3.1   Cylinder wake

Measurements in the wake of a circular cylinder were obtained by Gjelstrup et al. (2000) using LDA and HWA (data available on the web at http://ldvproc.www.nambis.de). HWA data was sampled at 100 kHz ($\sim$1.3$\times$10^5 samples) and the mean particle arrival rate for the LDA measurements was DR = 75 kHz ($\sim$10^6 samples). Measurements were made for Re = 12000 at a single point located about four diameters downstream of the cylinder. At this location the wake is expected to be fully turbulent with an oscillatory component due to the periodic passage of vortices shed from the cylinder.

Figure 4(a) shows a comparison of the spectrum from HWA to that from LDA using either nearest neighbour (NN) or linear (LN) interpolation to resample the data at equidistant times. The NN spectra follow the reference HWA spectrum up to a frequency of 6 kHz. In this range, the spectral peak is predicted very well. At higher frequencies, the effect of step noise induced by the resampling process is evident and the power is considerably overestimated (Simon and Fitzpatrick 2004). The slightly higher variance of the HWA spectra is due to the smaller number of samples in the data record. Linear interpolation produces a power spectrum which follows the reference one up to $\sim$12 kHz, i.e. there is an improvement compared to NN. However, both methods are incapable of exceeding the limit of DR/2π, a value up to which zero-order resampling is known to produce correct estimates (Adrian and Yao 1987). It should be pointed out that the better performance of the linear interpolation in this case is coincidental. In general, linear interpolation has been reported to be the worst method to resample irregular LDA data (Broersen et al. 2000) and therefore will not be considered any further.

Next, the LDA signal is decomposed using the present empirical denoising method (DN) before (DN+NN) and after resampling (NN+DN). The reference spectrum is compared to that produced in the two cases in Fig. 4(b). When the empirical decomposition is used to remove noise prior to the interpolation, the power at the high-frequency end of the spectrum is still overestimated due to the step noise introduced by the resampling process. Yet, the LDA spectrum follows that from HWA up to 20 kHz which is nearly a quarter of the mean particle arrival rate (cf. to 6 kHz for NN without DN). If, however, noise removal is done after the interpolation, the HWA and LDA spectra agree remarkably well. It might be argued that denoising after resampling removes both the measurement and the resampling-induced noise and hence improves the spectral estimates considerably. However, it should be noted that since measurement and resampling-induced noise are due to independent sources, it is possibly best to apply the denoising method twice: once on the raw data and then on the resampled data as shown further below.

Figure 4(c) shows the spectra computed using the slot correlation technique (SC) with several improvements (Nobach 2002) which is one of the most efficient methods to retrieve spectral information from irregularly-sampled LDA data (code available at http://ldvproc.www.nambis.de).

When used with the raw LDA data, the SC yields a spectrum which agrees with the reference spectrum (HWA) up to ~10 kHz. Beyond this frequency, the spectrum exhibits high variability which is a known shortcoming of the technique; in fact, negative values are possible (Boersen et al 2000). On the other hand, the spectral estimate is consistent with the reference spectrum up to ~20 kHz when the SC is used after denoising the raw LDA data. The result is very similar as that with NN interpolation. The above might indicate that the raw LDA data are noisy, which is not surprising. Finally, the spectrum from the present empirical methodology is compared to the reference spectrum and that from the slot correlation technique in Fig. 4(d). Clearly, the performance of the present method is excellent and the result is comparable to that from HWA.

3.2   Axisymmetric jet

Turbulent velocity fluctuations near the edge of an axisymmetric jet were measured with HWA and LDA by Nobach et al. (1996) in order to test the efficacy of refined reconstruction methods for spectral estimation from LDA data. The measurements employed here are for Re = 4×10^4; HWA: ~4.1×10^5 samples @ 20 kHz; LDA: ~10^5 samples @ ~7 kHz (low noise level) or @ 8.2 kHz (high noise level). Figures 5 and 6 show the corresponding power spectra from LDA compared to that from HWA.

The spectra computed from the low-noise level LDA data resampled at 10 kHz using the NN interpolation without de-noising (Fig. 5a) agree with the reference spectrum up to 1 kHz, as expected (~DR/2π). Beyond this point the filter and step noise errors due to resampling impose a a roll-off with a constant slope of –2 (DANTEC 2000). This effect is seen more clearly in Fig. 5(b) where the –2 slope in the spectrum continues up to the Nyquist frequency of 10 kHz (NN@20 kHz). Denoising the resampled data in Fig. 5(a) brings about a considerable attenuation of the power spectra at high frequencies which is not unexpected given the low level of noise in the raw signal. The numbers in parenthesis indicate the number of siftings employed. In this case, the empirical decomposition does not perform well in removing noise in the data simply because the level of noise is too low and the sifting procedure cannot cope with it. If, however, the resampling rate is increased to 20 kHz (Fig. 5b), then denoising the resampled data produces a spectrum which is comparable to that from HWA over the entire range of frequencies. It is interesting to note that the change in slope at ~2kHz is correctly captured by the LDA spectra in this case (NN+DN).

With a high level of noise, the spectral distribution from the LDA data resampled at 10 kHz without denoising (NN@10kHz in Fig. 6a) deviates from the reference spectrum even at lower frequencies (~0.3 kHz) than the low-noise level case (cf. Fig. 5a). Denoising the resampled data appears to recuperate some of the spectral characteristics but the level of noise in the raw data causes a hump in the power spectra at ~0.9 kHz. To recover the situation, denoising can be applied to the raw LDA data before resampling and the corresponding spectra are shown in Fig. 6(b). Resampling the noise-free data and computing the power spectrum yields a close agreement between LDA and HWA up to ~4 kHz (nearly half the data rate). Only a short tail is introduced at the high-frequency end due to the resampling. Attempting to remove this tail by denoising the resampled data for a second time attenuates considerably the power in the spectra [DN+NN+DN in Fig. 6(b)]. As in the low-noise level case, it is difficult to remove step noise introduced due to resampling if the resampling rate is not considerably higher than the mean data rate. However, by increasing the resampling rate to 20 kHz a good agreement between LDA and HWA is obtained as shown in Fig. 6(c). It is worth noting that the best spectra from LDA data for both low and high levels of noise are very close together and indeed close to the reference spectrum from HWA.

3.3   Grid turbulence

This dataset is from homogeneous isotropic turbulence downstream of a grid (Ducci et al. 2002). Two-point point LDA measurements were obtained in a water test facility at ECLAT in order to compute the mean squared velocity gradients and the dissipation rate of turbulence kinetic energy. Here, the data from a single probe are used to estimate the power spectral density. Data was sampled for approximately 40 s with a mean data rate of ~0.8 kHz.

Figure 7(a) shows the power spectrum computed with three different methods: 1) by resampling the LDA data at 1 kHz using nearest neighbour interpolation (NN), 2) by denoising the resampled data using 4 siftings (DN), and 3) by using the refined slot correlation technique (SC) of Nobach (2002). The NN spectrum might be expected to be correct up to 0.1 kHz, i.e. $\sim DR/2\pi$ but beyond this point the filtering and step noise effects impose a roll-off with a constant slope of -2. The SC technique implements model-based noise suppression and its effect is evident at frequencies above 0.1 kHz. The DN spectrum also exhibits a gradual roll-off which appears more natural than in the SC spectrum while the present denoising method produces a spectrum with less power at high frequencies.

A rigorous test for the performance of the different methods used is to estimate the mean squared velocity gradient by integrating the dissipation spectrum (see Hinze 1972):

$$\overline{\left(\frac{\partial u_1}{\partial x}\right)^2} = \int_0^\infty k_1^2 F_{11}(k_1)\mathrm{d}k \tag{5}$$

and compare the result to that directly measured from two-point LDA (Ducci et al. 2002). In Eq. (5), k_1 is the wavenumber and F_{11} is the power spectral density. The wavenumber is obtained by $k_1 = 2\pi f/\overline{u}_1$. The result from the present method with denoising is shown in Fig. 7(b). The agreement of the result to that from two-point LDA is very good, especially if one considers the uncertainties in the determination of the reported quantities (see also Ducci and Yianneskis 2005). It should be noted that the values of the mean gradients computed from the SC spectrum are nearly three times higher than those shown in Fig. 7(b) while those from the NN spectrum are an order of magnitude higher. The above example demonstrates that the proposed method allows unbiased estimation of the turbulence spectrum from single-point LDA data.

4. Summary and conclusions

A methodology was developed to suppress noise from LDA data and improve the accuracy of spectral estimates. The methodology is based on the empirical mode decomposition which involves an iterative sifting procedure. The residual component of the decomposition determines the uncorrelated noise in the data. The methodology was rigorously tested against LDA measurements of velocity fluctuations from different experiments. Power spectral densities were estimated using the new methodology in conjunction with the nearest interpolation scheme and were compared with HWA spectra and spectra obtained using other estimation techniques. The focus of the present study was on LDA data with a sufficiently high data rate (mean particle arrival rate) to resolve most of the flow frequencies which are typically affected by noise (but not always). The main conclusions drawn from the results presented are summarized below:
- Resampling of the LDA data using zero-order interpolation, i.e. sample+hold or nearest neighbour, introduces step noise and filtering errors which affect mainly the power spectral

density at the high-frequency end. These errors are relatively well understood (Simon and Fitzpatrick 2004) but it has been difficult to remove their effects effectively.

- Zero and first order interpolation techniques, e.g. the linear interpolation briefly examined here, might be expected to give good spectral estimates as long as the data rate is sufficiently high, i.e. above 2π times the highest flow frequency to be resolved, but they tend to distort the flow physics and produce a roll-off at the high-frequency end of the spectrum with a constant slope of –2 which can easily be confused with the inertial range of turbulence. The performance of the interpolation methods is severely affected by measurement noise.

- All the examples considered in this paper demonstrate that the estimation of the power spectra from LDA data is severely affected by both measurement noise and step noise plus filtering effects due to resampling.

- The empirical mode decomposition is a robust technique, i.e. it is intuitive, direct, a posteriori, and adaptive, with the basis of the decomposition being derived from the data. Because it uses no time scale it is also suited for processing of LDA data which are irregular in time.

- The denoising method proposed in this work was found to be efficient in suppressing measurement noise inherent in the LDA data. In the cases considered, denoising the irregularly-sampled LDA data and subsequently resampling the signal at regular times resulted in a two-fold increase in the region of the spectrum which was consistent to that from HWA. That alone represents a significant advantage of the proposed methodology.

- The denoising method was also found to be effective in removing the noise introduced by the resampling process. Power spectra computed from LDA agreed well over the entire range of frequencies to those from HWA. Problems arise when the level of noise in the data (raw or resampled) is low in which case the power of the 'denoised' signal is considerably underestimated.

- The mean squared velocity gradients in grid-generated turbulence were accurately estimated from the dissipation spectrum computed from single-point LDA and using the present methodology to suppress noise. This result indicates that the methodology developed allows unbiased estimation of the turbulence spectrum down to the smallest dissipative scales.

The paper introduced a new and powerful alternative technique to suppressing noise from LDA data. Although the technique produces irregularly spaced data, it has been shown that together with the nearest interpolation technique it can result in accurate estimation of the power spectra. Nevertheless, more testing is required to fully verify its performance, possibly under more controlled conditions, e.g. using simulated turbulence data.

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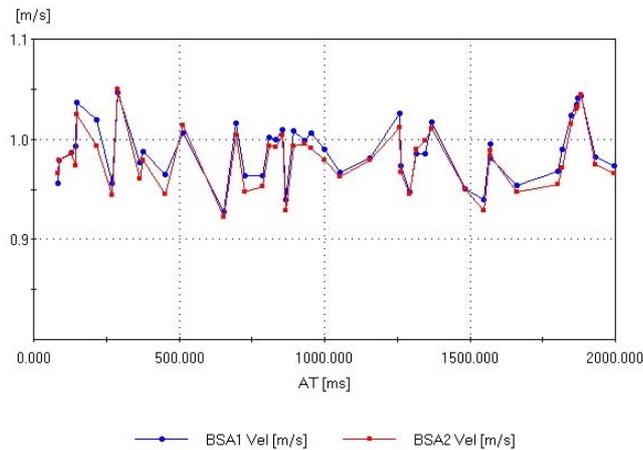

Fig. 1 Measurement of the particle velocity crossing a single control volume using two individual LDA systems (receiving optics and signal analysers). One photomultiplier operates in back scatter and the other in side scatter.
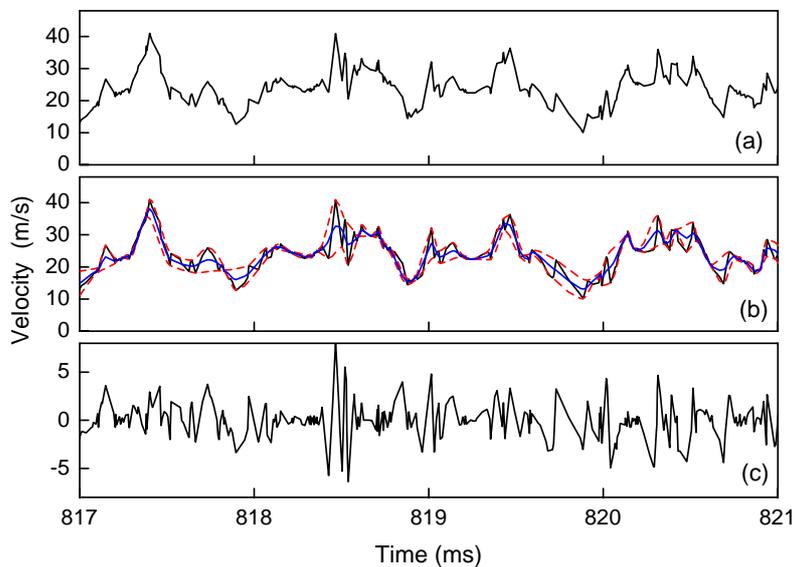

Fig. 2 Illustration of the empirical mode decomposition: (a) original data; (b) the upper and lower envelopes enclosing the original data (dashed lines) and their mean; (c) the difference between the original data and the mean of the envelopes.
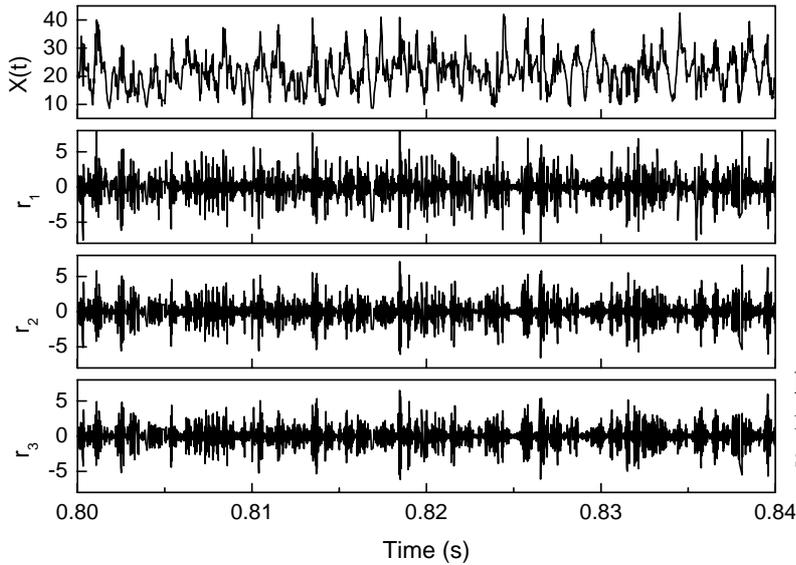
Fig. 3 The original signal and the residual components after three successive siftings.
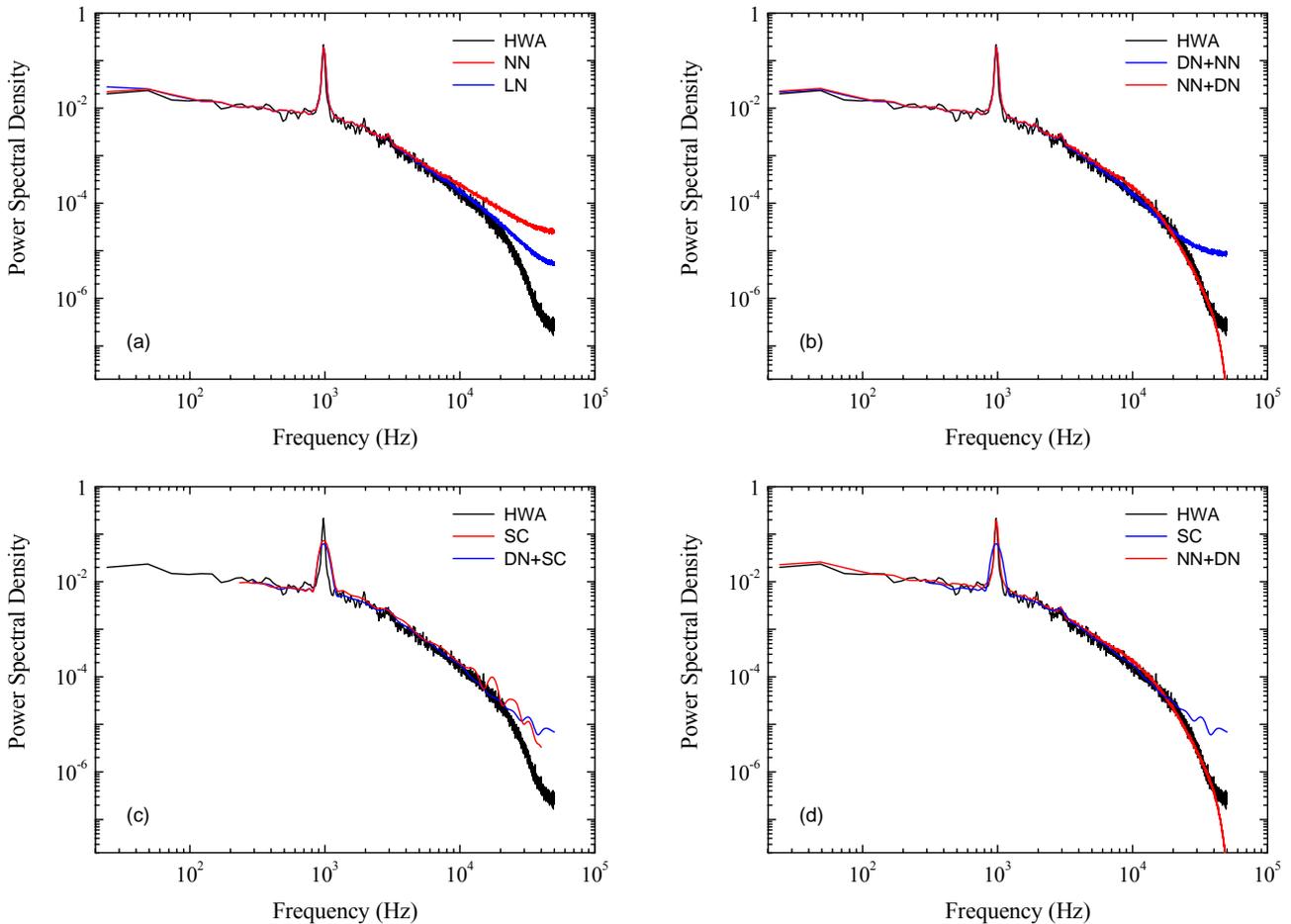

Fig. 4 Power spectra in a cylinder wake. The black line in all plots shows the reference spectrum from HWA; NN = nearest neighbor interpolation, LN = linear interpolation, DN = denoising, SC = slot correlation with improvements (Nobach, 2002). All spectra were computed using the same FFT technique employing a Hanning window with 4096 points, 25% overlapping and no zero-padding, except for the SC technique where the spectrum is logarithmically sampled.
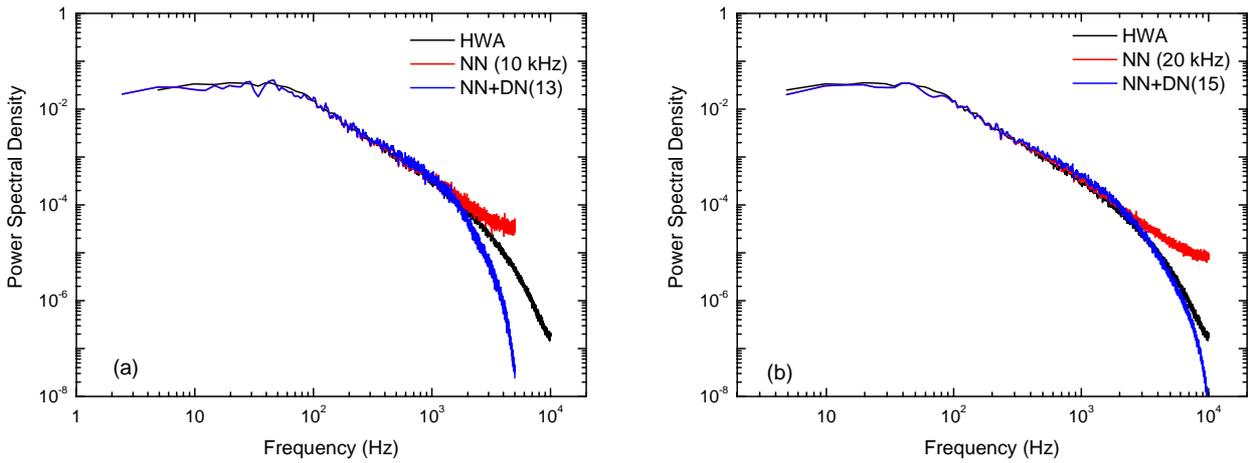
Fig. 5 Power spectra in an axisymmetric jet (LDA data with low level of noise). The black line in all plots shows the reference spectrum from HWA; NN = nearest neighbor interpolation, DN = denoising. All spectra were computed using the same FFT technique employing a Hanning window with 4096 points, 25% overlapping and no zero-padding.
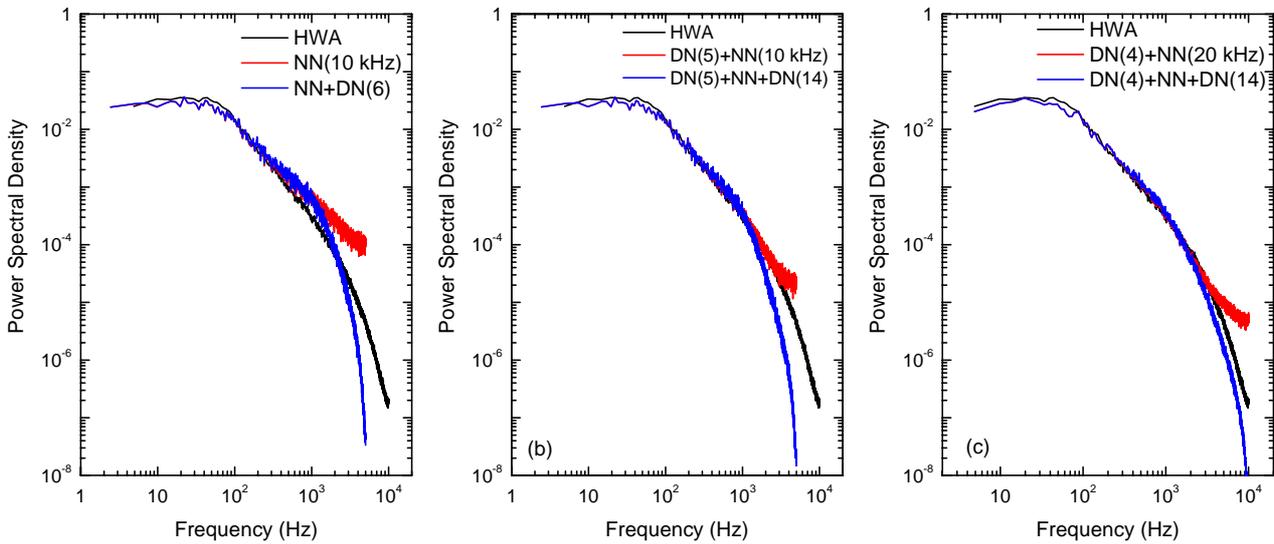

Fig. 6 Power spectra in an axisymmetric jet (high level of noise). Caption as in Fig. 5.
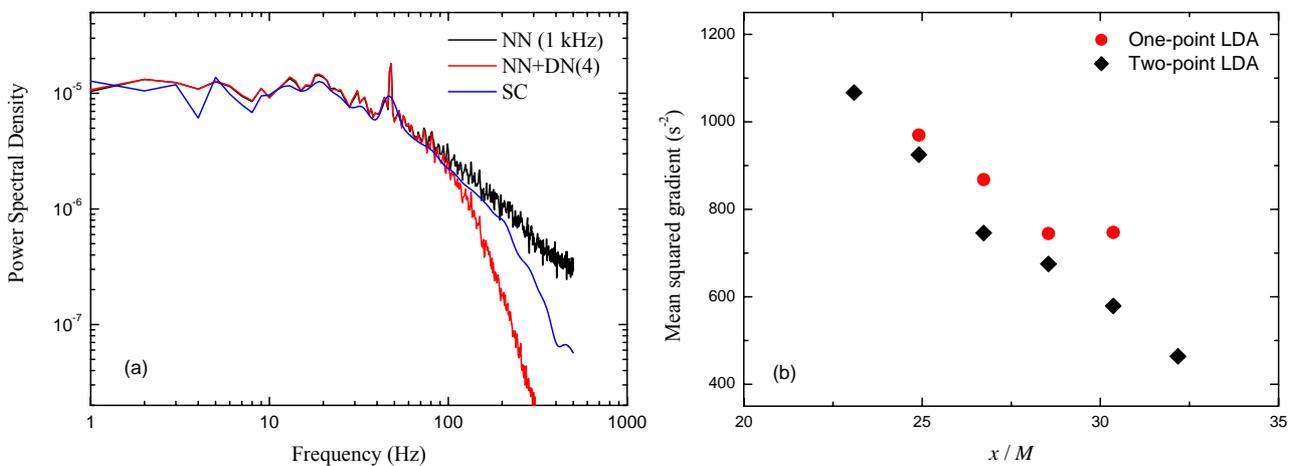

Fig. 7 a Power spectra in grid turbulence. **b** Comparison of the mean squared velocity gradient computed via spectral analysis from single-point LDA data and via averaging two-point LDA data.