Lagrangian velocity and acceleration measurements, PTVA, in quasi-two-dimensional electromagnetically controlled multi-scale flows

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Abstract A self-adaptive iterative method (Particle Tracking Velocimetry and Accelerometry, PTVA), amenable to accurately extract the Lagrangian velocity and acceleration of a multi-scale flow, has been developed and applied to an electromagnetically controlled, quasi-two-dimensional multi-scale turbulent-like flow. PTVA can be considered as an evolution from previous acceleration measurement methods like PTV, as its parameters only relies on the determination of the smallest flow structures to be measured. Particle positions are obtained from standard PTV and then used to extract trajectories using a polynomial approximation with an adaptive number of tracked positions, depending only on the local flow properties. Velocities and accelerations components along each trajectory are obtained subsequently as the first and second derivative in time of the trajectories. PTVA is here applied to a quasi-two-dimensional multi-scale flow experiment, designed and developed to study and control some aspects of turbulence, stirring and mixing. This flow has a multi-scale (8 in 8) distribution of stagnation points (zero velocity point in Eulerian reference frame) according to the scale of the flow structures which they are connected to. We fully control the topology and the forcing of this flow. The probability distribution function (pdf) of Lagrangian acceleration sustains some of the turbulent-like characteristics of this flow. Measurements of the Eulerian fields (over the whole investigation field) of velocity, acceleration and Navier-Stokes equation’s viscous term are here illustrated, leading to a deeper insight in the physics of the turbulent-like flows we are studying. Particularly, it is confirmed that, as happens in turbulent flows, the number of zero acceleration points is larger than the one of zero velocity points (stagnation points). Moreover, the Eulerian field of Navier-Stokes equation’s viscous term, obtained with a second order spatial derivative, permits to see that the viscous term is weak compared to the acceleration one and concentrated at the small scales (like in turbulent flows) and to obtain an indirect measurement of the pressure gradient over all the area of interest. Finally, to characterize the power input, output and transformation in the flow (useful for efficient mixing purposes), the Eulerian fields of the scalar product $u \cdot a$ and of the vectorial product $u \times a$ are reported.

1. Introduction

A new class of quasi-two-dimensional (Q2D) multi-scale flows is generated and controlled in the laboratory, via electromagnetic forcing, by Rossi et al. (2005 and 2006). The main targets are firstly to model and study some of the multiple-scale characteristics of turbulence, secondly to control stirring and mixing to achieve efficient mixing (same dilution as traditional mixing systems but minimizing the energy input). These flows have a \textit{multiple-scale} or 8-in-8 distribution of hyperbolic stagnation points (zero velocity point in Eulerian reference) according to the flow structures the stagnation points are connected to (see for instance figures 1b and 2). This multiple-scale structure of stagnation points is, according to Davila and Vassilicos (2003), closely related to the flow energy spectra; it has been shown, by Direct Numerical Simulation (DNS) of Rossi et al. (2005) and by experiments of Rossi et al. (2006), that a fractal forcing can impose the flow energy spectrum. Moreover, the energy spectrum of these flows has a power-law shape for more than two decades and presents Richardson-like pair dispersion properties, with the Richardson exponent controlled by the distribution of hyperbolic stagnation points. This electromagnetically-controlled turbulent-like flow is the one we use for our experiments: one advantage of the present flow is that we know and control its topology and time dependency. To study and characterize the properties of transport, stirring and mixing of fluid motion in general and of our flows in particular, Lagrangian
statistics are very important; particularly, the Lagrangian acceleration is at the very base of fluid motion. Despite this importance, only few example of its experimental measurements can be found in literature: for instance, Virant and Dracos (1997), Ott and Mann (2000), La Porta et al. (2001), Voth et al. (2002), Mordant et al. (2001) and Luthi et al. (2005). Among the difficulties to measure and analyze acceleration, it is the fact that the Lagrangian acceleration, $a$, is a Galilean invariant composed of two Eulerian accelerations, local acceleration and convective acceleration (see eq. 3), which are not Galilean invariants: a very high quality of the measurements is then required, to be able to measure both very large and very low values of acceleration. Moreover, as here we focus on multiple-scales flows aspects and consider that local acceleration is important for all scales behavior, we want our acceleration measurements to be accurate at large and small scales.

To be accurate in measurements of velocity and acceleration, a great care is accorded to the trajectory extraction: indeed, we need to accurately measure trajectories at all the scales, with a critical insight given to close vicinity of the hyperbolic stagnation points where the velocities are small but with strong curvature of the streamlines. After that particle positions are extracted using Particle Tracking Velocimetry (PTV), trajectories are obtained via a polynomial approximation that uses an adaptive number of those tracked positions: this is needed because achieving the required accuracy with a fixed number of positions can be very tricky. The adaptive number of positions depends only on the local flow properties. We thus use the trajectories extracted with this adaptive measurement to obtain velocities and accelerations via temporal derivatives.

To completely characterize our flows (which, being multiple-scale laminar flows are repeatable from the Eulerian point of view), we need not only the multiple-scale distribution of $u$ and $a$ but also of $\nu \nabla^2 u$; as this needs a double spatial derivative of $u$, also the measure of $u$ has to be very precise. Moreover, for the reduction of power input required by efficient mixing purposes, the scalar product $u \cdot a$ and the vectorial product $u \times a$, characterizing indeed the power input, output and transformation in the flow, must be also measured, according to the flow topology and to its fractal forcing. Finally, this study is completed by the probability density functions (PDF) of modulus and components of the Lagrangian velocities and accelerations.

2. Experimental set-up

The experimental set-up (figure 1) is basically made up of a tank containing a shallow layer of brine electromagnetically forced: the electromagnetic body forcing ($j \times B$) is produced by the use of permanent magnets ($B$), placed under the horizontal bottom of the tank, and of electrical currents ($j$), generated by platinum electrodes on each side of the tank. Each one of these electrodes is made of 16 platinum wires, each one 40 mm long and with a diameter of 11.5 µm. Two opposite tank’s sides (West and East) are covered with 43 electrodes, with a typical spacing of about 4 cm between two electrodes of same polarity: the electrical field has been checked as uniform above the active part of the wall. The electrical current used in the experiments presented in this paper is $I = 0.53 \, A$, leading to a two-dimensional Reynolds number of 5900.

To design the stagnation point distribution of figure 1b, EM forcing is used over many scales simultaneously: the three scales of forcing are associated with a size of the square magnets of 160 mm, 40 mm and 10 mm. The size of the tank is of 1700 x 1700 mm$^2$, so it is large compared to the size of the magnets and the EM forcing area represents only 2.8 % of the area of the bottom wall, which is small if compared with previous works (e.g., Cardoso et al., 1994, Williams et al., 1997). Each forcing scale is made of a pair of North and South magnets, whose spacing is about the typical length of the associated magnets; each couple of magnets is placed on an iron plate of sufficient thickness to close the magnetic field. Figure 1c shows a distribution of Lorentz body forces in the plane $(x, y)$ where the forces work.
Due to the particular kind of EM forcing, we generate wall-jets, due to the North-South magnet pairs, and so control the hyperbolic stagnation points connected with each scale of forcing, generating the multi-scale flow illustrated in figure 2 by some of the particle trajectories measured. In this experimental set-up, the dissipation due to the bottom friction is compensated by the sustained forcing; once the viscous damping is overcome, Lorentz forces provide a good way to control the intensity of the flow, as shown in Thibault and Rossi (2003). The thickness of the shallow brine layer (salt water, 158 g/l NaCl) is about \( H = 5 \) mm (\( H_{\text{MEAN}} = 5.023 \text{ mm} \pm 0.140 \text{ mm} \)). The quasi-two-dimensionality of the flow has been checked and verified (for more details, see Rossi et al., 2006).

![Fig. 1](image1.png)

**Fig. 1** (a) Sketch of a the tank’s vertical section with a schematic for the electromagnetic forcing of the shallow brine layer (5 mm thickness); (b) plan view of a schematic of the a fractal flow of “8 in 8” and associated permanent magnets; (c) electromagnetic forcing distribution, computed with \( I = 1 \text{ A}, B = 1 \text{ T} \); \( f_y \) in N/m³.

We use Image Analysis techniques to perform measurements on this flow, so the experimental set-up is completed by a 2 ADC high definition camera (2048x2048 pixel² for a maximum acquisition frequency of 14 Hz and a 14 bit depth), placed orthogonally to the measurement plane and by two 500 W lamps that uniformly enlighten the investigation field. The flow is seeded with particles of Chemigum. Data used in the present paper were filmed with an acquisition frequency of 10 Hz and

![Fig. 2](image2.png)

**Fig. 2** Examples of tracked trajectories: on the left hand side, the entire flow, on the right hand side a zoom on its South-West quarter; 1 pixel is 0.411 mm.
consist of a first group 23 measurements; other 110 runs with a larger investigation frame are currently under analysis and will be used to increase the quality of the statistics in Ferrari and Rossi (2006).

3. PTVA (Particle Tracking Velocimetry and Accelerometry): aim, functioning and comparison with some previous method

Since the first PTV measurements, where velocity was computed simply as the distance between two consecutive positions of the centroid of a particle divided by the time steps, to try increasing the accuracy of measurement, moving polynomial approximation of various order has been used to extract particle trajectories: e.g., La Porta et al. (2001), Voth et al. (2002), Luthi et al. (2005). One issue of the polynomial approximation of the trajectories is the number of position used to fit the polynomial, that is usually chosen according to the specific set of data but without any physical link to the flow: this may lead to an excessive smoothing of the trajectories when this number is too large or, on the other side, to keep a lot of noise together with the measures. One of the aim of PTVA is so to use an adaptive number of positions, optimized according to the physics of the flow. We use standard Particle Tracking Velocimetry softwares (DigiFlow, Dalziel 1992, and GPTV, Querzoli 1996) only to extract particle positions and then an adaptive number of tracked positions, depending on the local flow intensity, to approximate the trajectories as polynomials of order \( n \) in time.

A first-step approximation is done to obtain a preliminary raw velocity measurement which is used, in a second-step, to compute the local and adaptive mean vector velocity:

\[
\bar{u} = \frac{1}{N} \sum_{i=1}^{N} \bar{u}_i
\]

(1)

This mean velocity is used to compute \( \Delta s \), the bulk parameter of PTVA:

\[
\Delta s = \sum_{i=1}^{N} \left| (\bar{x}_{i+1} - \bar{x}_i) - \bar{u} \Delta t \right|
\]

(2)

A value of \( \Delta s \) close to zero is typical of particles traveling along a straight trajectory, a high value of a particle trapped in a vortex. The target for the value of \( \Delta s \) is the perimeter of the smallest eddy one want (or can, because of technological limitations) to measure, so the number of positions used for the approximation \( Np \) is increased or reduced according to the value of \( \Delta s \). The only adjustable parameter is then the diameter \( D \) of the smallest vortex to measure that we choose as a target for \( \Delta s \).

For each measured position, the value of the noise (measured as the root mean square of the distances between the approximated positions and the particle coordinates from PTV) is compared to the value of \( \Delta s \) used: if the ratio of noise over \( \Delta s \) is too high, the position is not considered in the measured data set. Once the polynomial equations of the trajectories’ x and y components are obtained, velocity’s and acceleration’s components are extracted as the derivatives in time of the trajectories. It is worth to highlight that changes on \( D \) do not affect the root mean square of the velocity and acceleration.

The order of the polynomials used for trajectory’s extrapolation, \( n \), is important because determines a maximum of changes in particles direction: an order of interpolation higher than two allows larger maximum of displacement whilst correctly tracking the small eddies and decreases the size of the minimum eddy’s trajectory correctly interpolated (with a same level of noise intensity). Moreover, with higher order of \( n \), the displacement can be increased while still resolving the flow’s small
scales: this gives many advantages to the method, e.g. the ability to work with “lower frequencies” cameras (compare, for instance, to La Porta et al. 2001 and Luthi et al. 2005) and the decrease of the ratio noise versus displacement between two frames and/or of the ratio noise versus total displacement of extrapolated trajectories. So, the smallest turn-over time and the size of the smallest scale eddies which are to be measured are only used to characterize the trajectory approximation. Moreover, \( n \) higher than two allows to measure also strongly time dependent flows.

In figure 3 a comparison between the x-component of the acceleration in time measured, along some trajectories (on the first plot on the left), with a moving polynomial approximation of order 4 and a fixed number (21) of positions (plot in the middle) and with PTVA with \( D = 8 \) pixel (plot on the right) is reported: it is clear that PTVA is able to give an accurate measure of the acceleration without deleting small fluctuations of it (see the last part of the acceleration on the black trajectory). We are currently performing a geometrical validation of PTVA on artificial trajectories, with a known parametrical equation in time, with different kind and intensity of noise added and on trajectories generated by a DNS simulation of the flow. These validations along with more complete and concise details on PTVA are included in Ferrari and Rossi (2006).

4. Results and discussion

An illustration of the multi-scale flow generated in our experiment is given in figure 2, where some samples of the extracted trajectories are plotted; the different colors on the picture have the only aim of allow a better visualization of the single trajectories. On the left hand side, the whole flow picture shows the central stagnation point, connected to the large scales of the flow and generated by the two biggest magnets. The streaks, related to this large scales’ stagnation point, contain the streaks connected to the medium scales. The right hand side of figure 2 shows the small scales of the flow. We are so able to extract trajectories at all the scale of the flow that are very different in shape, scale and behavior, from straight lines crossing the biggest magnets, to very small vortices turning many times around an elliptical stagnation point, to trajectories described by particles traveling between two different scales. A very high density of trajectories tracked for a long time is essential to give quality to both Lagrangian statistics and, particularly, Eulerian fields. Moreover, the reduction of the noise (typical of traditional PTV measurements) we match with the trajectory approximation is at the very base of the velocity and, especially, acceleration measurement showed here.

In figure 4, the pdf of the Lagrangian velocity is normalized with the root mean square velocity \( u_{RMS} \) of the flow (22.89 pixel/s corresponding to 9.38 mm/s), assumed as the typical velocity of the flow. On the left hand side, the shape of the pdf of the velocity intensity shows that the
measurements cover a broad range of velocity intensities. The cut-off corresponding to 3 times the \( u_{\text{RMS}} \) suggests the bounded conditions for the velocity we have in this flow: the possible values of the velocity go from zero (over stagnation points) up to a maximum value, reached by the fluid elements when they pass over the big magnets, that, for the present case of \( I = 0.53 \) A, is about 25 mm/s (see also figure 6). On the right hand side of figure 4, the pdf of the two component of the velocity in the x and y-direction are plotted, highlighting the signature of the multi-scale forcing.

![Fig. 4](image1.png)

Fig. 4 Probability density functions of the Lagrangian velocity: on the left hand side, the velocity intensity, on the right hand side the velocity x component (red circles) and the y one (black spots); on both plots, velocity is normalized by \( u_{\text{RMS}} = 22.89 \) px/s that is 9.38 mm/s.

![Fig. 5](image2.png)

Fig. 5 Probability density functions of the Lagrangian acceleration: on the left hand side, the acceleration intensity, on the right hand side the acceleration x component (red circles) and the y one (black spots); the blue line is the best-fit Gaussian; on both plots, acceleration is normalized by \( a_{\text{RMS}} = 8.69 \) px/s\(^2\) that is 3.56 mm/s\(^2\).

The pdf of acceleration intensity, normalized with the root mean square acceleration \( a_{\text{RMS}} \) of the flow (8.69 pixel/s\(^2\) corresponding to 3.56 mm/s\(^2\)), is plotted in the left hand side of figure 5. Also in this case we have a large tail but, differently from the velocity intensity, there is not a sharp cut-off. Regarding the pdf of the acceleration x and y components, they are showed on the right hand side of figure 5: the x-component is plotted with red circles while the y component with black dots; the Gaussian curve that best-fit the data in a least mean square sense is drown with a blue dotted line. Both these components are Gaussian for more one decade of probability density, as they are in the acceleration components reported in La Porta et al., 2001. After this Gaussian behavior, the two
components’ pdfs show broader than Gaussian tails: the shape of these tails is the one usually encountered in turbulence, confirming some turbulent-like natures of this flow.

The Eulerian repeatability of our experiments, combined with the large number of trajectories tracked for a long time, allows us to extract the Eulerian information all over the investigation field from the Lagrangian measurements. This permits us not only to extract the velocity field but also, more significantly, to decompose and analyze the single components of the Navier-Stokes equation

\[ \ddot{\mathbf{u}} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla P/\rho + \nu \nabla^2 \mathbf{u} + \mathbf{f} \]  

(3)

where P is the pressure, \( \rho \) the fluid density, \( \nu \) the fluid kinematic viscosity and \( \mathbf{f} \) the forcing. The left hand side of figure 6 gives a raw velocity field extracted from PTVA. This clearly illustrates the multi-scale flow generated by the forcing given in figure 1: the highest velocities (colors close to the red) are found above the two big magnets; the stagnation point connected to the large scales of the flow, is clearly noticeable as the spot with zero velocity values in the in the center of the field. Similar considerations are possible from the analysis of the other scales (see, for example, the hyperbolic stagnation point, highlighted with a white square, and the elliptical one, marked out by a white circle, connected to the medium scales on the left hand side of figure 8).

![Fig. 6 Eulerian fields: on the left hand side, an illustration of the intensity of the velocity field, ||u|| in pixel/s, 1 arrow every 64 is plotted; on the right hand side, a zoom at the small scales of the flow in the South-West quarter of the field; 1 pixel is 0.411 mm.](image)

![Fig. 7 Eulerian fields: illustration of the intensity of the acceleration field, ||a|| in pixel/s², along with some acceleration lines; on the right hand side, illustration of the intensity of the Navier-Stokes equation’s viscous term, ||\nu \nabla^2 \mathbf{u}|| in pixel/s²; 1 pixel is 0.411 mm.](image)
The flow is well defined at all the scales, as it is shown on the right hand side of figure 6, where three elliptical stagnation points generate three vortices, two belonging to the small scales and one to the medium scales; two hyperbolic stagnation points are visible too.

The Eulerian acceleration field, along with some acceleration lines, is shown on the left hand side of figure 7. For a steady flow, the critical elliptical and hyperbolic points in the velocity field are also zero acceleration points. The acceleration field shows also some zero acceleration points which are not velocity stagnation points (e.g. above the two biggest magnets). In fact, in the case of zero acceleration points arising from velocity stagnation points, the acceleration lines going toward them have all the same direction: to be more precise, they go toward the elliptical stagnation points and out from the hyperbolic ones. The zero acceleration points belonging only to the acceleration field display both acceleration lines going toward the point and away from it. Regarding the spatial distribution of acceleration intensities, the highest acceleration values arise where both strain and velocity are high: this happens in the vicinity of the hyperbolic stagnation point of the medium scales.

On the right hand side of figure 7, the measured field of Navier-Stokes equation’s viscous term is shown: as it comes out from the Laplacian of the velocity field and so implies a second order spatial derivative, it is very sensible to the noise. Despite that, the measured velocity field is good enough to allow the measurement of the velocity Laplacian. The rms of the viscous term, which is about 0.23 px/s², is about 40 times smaller than the acceleration one: this two terms are of the same magnitude only at the small scales, as it happens in turbulent flows, confirming another of the turbulent-like aspects of this class of flows.

To highlight the coherence of the computed viscous term, local zooms are plotted in figure 8 and figure 9 with some pointed areas: the white circle points to the center of an eddy (elliptical stagnation point), the white square to a velocity hyperbolic stagnation point and the white triangle to a region of shear flow. Where the velocity vectors and the viscous term vectors have opposite direction, the viscosity is subtracting energy from the flow and so pulling it, where they have the same direction the viscosity is pushing the flow. As the vectors of velocity and viscous term around the elliptical stagnation point have opposite direction, it is clear that the viscosity is acting against the eddy to rotate. On the left of the triangle, a tongue of faster fluid is squeezing in a zone of slower fluid: so the viscosity is acting a lot to slow it down. On the tongue over the triangle the viscous term is acting in the same direction of the velocity as the small scale low speed area bridges to higher speed areas.

As the forcing is known (and also of weak percentage area), the pressure gradient field, which is difficult to measure experimentally, can be extracted from the difference between acceleration and viscous terms in Navier Stokes equation, see equation (3).

To analyze the acceleration components, as well as the power input and output in the flow, on the left hand side of figure 10 the scalar product $u \cdot a$ is shown, while the vectorial product $u \times a$ is plotted on the right hand side. $u \cdot a$ is proportional to the tangential acceleration so it shows where fluid particles increase their velocity (velocity and acceleration in the same direction) or decrease their velocity (acceleration and velocity in opposite directions). This scalar product also gives the power input and output in the flow per unit mass: as in this flow the power input comes from the magnets, it is physically correct that $u \cdot a$ identifies the magnets’ positions.

The vectorial product $u \times a$ is linked to transformation of power input to force the rotation in the flow and to allows the formation of eddies in it. Moreover, it is proportional to the transversal component of the acceleration: for instance, the tilting of the shear flow already highlighted on the left hand side of figure 8 generates in a zone of the flow where the tangential acceleration is small compared to the perpendicular one.
Fig. 8 Eulerian fields: on the left hand side, an illustration of a zoom on the intensity of the velocity field, $|\mathbf{u}|$ in pixel/s; on the right hand side, an illustration of the same zoom on the intensity of acceleration field, $|\mathbf{a}|$ in pixel/s$^2$; a white circle is superposed to highlight an elliptical stagnation point, a white square to highlight a hyperbolic stagnation point and a white triangle to highlight a shear flow region; 1 pixel is about 0.411 mm; 1/4 arrows are plotted.

Fig. 9 Zoom on the Eulerian field of intensity of the Navier-Stokes equation’s viscous term, $|\nu \nabla^2 \mathbf{u}|$ in pixel/s$^2$; a white circle is superposed to highlight an elliptical stagnation point and a white triangle to highlight a shear flow region; 1 pixel is about 0.5 mm; 1/4 arrows are plotted.

Fig. 10 Eulerian fields: on the left hand side, an illustration of the field of scalar product $\mathbf{u} \cdot \mathbf{a}$ in pixel$^2$/s$^3$; on the right hand side, an illustration of the field of vectorial product $\mathbf{u} \times \mathbf{a}$ in pixel$^2$/s$^3$; 1 pixel is 0.411 mm.
5. Conclusions, current and future works

The self-adaptive iterative Particle Tracking Velocimetry and Accelerometry PTVA, performed on a multi-scale turbulent-like experiment, provides good measurements of the velocity and acceleration field. The velocity field is precise enough to allow a second order spatial derivative on it and so to give the measurement (slightly noisy but meaningful) of the viscous term of the Navier-Stokes equation. The PTVA allow us to experimentally measure each of the Navier-Stokes equation’s components, as well as the power input-output-transformation in the whole investigation field, fundamental for minimization of power input needed for efficient mixing purpose. It is to notice that the measurements shown here were filmed at only 10 Hz, so the use of higher frequency cameras could allow the application of the PTVA on faster turbulent flows. It is also meaningful that we use the smallest scale of the smallest structure to be measured as the only adjustable parameter of our method, without artificial smoothing that can change the measured values depending on the filter applied. A complementary set of data (110 realizations of a similar experiment with same forcing but a larger investigation field) is currently processed to reinforce Ferrari and Rossi (2006).

The pdf of Lagrangian acceleration components has highlighted some of the turbulent-like characteristics of this flow. From the comparison between the Eulerian fields of velocity and acceleration, it is found that the number of zero acceleration points is larger than the one of zero velocity points (stagnation points), as it happens in turbulent flows. Moreover, we are able to measure the Eulerian field of Navier-Stokes equation’s viscous term over the whole investigation field: this permits to see that the viscous term is weak compared to the acceleration one and concentrated at the small scales (like in turbulent flows) and to obtain an indirect measurement of the pressure gradient over all the area of interest. Finally, the possibility of measuring the power input, output and transformation in the flow according to its topology may be useful to better understand how it is related to mixing.

We are currently applying PTVA measurements on an electromagnetically-controlled multi-scale turbulent-like flow with various time dependent forcing. The time dependent forcing allows a larger interchange of fluid between the scales and is one more step in the direction of fully controlled turbulent-like flows.

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References