Spatio-Temporal Correlations for Turbulent Jet Flows using the Point Reference Global Correlation (PRGC) Technique

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Abstract  The present paper examines the sensitivity of the Point-Reference Global Correlation Technique (PRGC) developed by Chatellier & Fitzpatrick (2005) and which combines single and global point measurements such as those obtained from Laser Doppler Velocimetry (LDV) and Particle Image Velocimetry (PIV). The technique allows spatio-temporal correlation functions to be obtained over a 2D area of the flow so that the turbulence statistical characteristics can be extracted together with the inhomogeneous and anisotropic features. Simulated data for typical LDV and PIV measurements are used so that a parametric study can be performed to examine how sensitive the PRGC technique is as a function of sample frequency and the number of samples of both the LDV and PIV data. These parameters are shown to be critical to errors associated with the calculated spatio-temporal correlations as low data rates can lead to significant variations in the estimates. Measurements conducted in a Mach 0.2 jet flow using PIV and LDV systems are reported and the two dimensional spatio-temporal correlation functions as well as the associated length and time scales are shown to be as expected.

1. Introduction

Turbulence statistics such as length and time scales remain of particular importance in fluid mechanics and aeroacoustic applications since these quantities are directly related to the scales at which turbulence motions and conversion mechanisms of the turbulence kinetic energy operate. These properties, which constitute the basis of the statistical approaches to noise source modelling for jets need to be determined as accurately as possible. Inhomogeneous and anisotropic features of the turbulence field due to significant shear stresses and 3D effects are also of importance. For jet flows, these aspects are of some significance as they influence the noise distribution of the noise sources as well as their acoustic contributions. Furthermore, for noise source modelling using a statistical approach based on the acoustic analogy, these features need to be taken into account in the modelling the turbulence velocity correlation tensor which represents both the dynamic and efficiency of the mechanisms of conversion of the kinetic energy into acoustic energy. While significant progress has been made using CFD tools for these modelling space-time properties, it still remains necessary to measure these properties using experiments. Two-point measurements combining intrusive or optical techniques have been extensively used to extract the typical turbulence statistics required for modelling the correlation tensor as reported by, for example, Chu (1956) and Harper-Bourne (2003). However, these measurements allow only one component to be estimated in one direction for each series of measurements. Obtaining a 3D model using such measurements is not realistic and remains a significant challenge.

Although PIV measurements can be used to extract the spatial correlation over a 2D area, because of the relatively low repetition rate, temporal information such as time scales and convection speeds cannot be easily obtained. Thus, the frequency content of the energy containing eddies in high speed flows cannot, at present, be derived from PIV measurements. Although there have been significant advances in the use of PIV for measurement of the space-time correlations, the use of multi-point LDV systems is still the most common measurement approach. More recently, Chatellier & Fitzpatrick (2005) have shown that single and global point measurements techniques such as LDV and PIV can be used together to extract whole-field space time correlations around the LDV reference point in a wide range of flows. Such a combination takes advantage of the global
and high-sampling frequency measurements respectively. Using advanced processing for the correlation estimation such as the slotting technique (Mayo 1978, Nobach et al 1998), the PRGC technique was found to provide good estimates of the space time correlations when compared to the results from two point measurements. One of the main difficulties of the technique is a consequence of the undersampling of the PIV signals compared to those from the LDV. This can give rise to significant errors associated with bias in the correlation estimate. This paper examines the effects of the sampling number and sampling frequency ratios of both the PIV and LDA systems on the quality of the estimation. Simulated turbulence data is used to evaluate the quality of the correlation and turbulence statistical properties estimates using the PRGC technique and its sensitivity to the various parameters is assessed by estimated the errors in the results. Finally, the procedures are applied to a series of measurements using LDV and PIV in a jet flow and the results are shown to be in good agreement with what would be expected.

2. Simulation of space-time statistical turbulence

The approach used to simulate the turbulence velocity field has been inspired by the work reported by Simon & Fitzpatrick (2004) for the development of spectral estimators of irregularly sampled signals based on the sample-and-hold technique and as detailed earlier by Kasdin (1995). Their generating process is here extended to the case of a 2D turbulence field.

The basis for the analysis is a reference signal as proposed by Simon & Fitzpatrick (2004). A regularly sampled signal \( s_{ref}(t) \) is generated using a Kolmogorov type spectrum \( G(f) \) to represent the turbulence spectrum at the reference position. This is using the inverse transform operation defined by

\[
s_{ref}(t) = IFT \left[ G(f)^{1/2} e^{-j \theta(f)} \right] t
\]

where \( \theta(f) \) is a random phase with uniform distribution between 0 and \( 2\pi \). The variance of the simulated data is equal to the sum of the spectral components and the Kurtosis and Skewness factors 3 and 0 respectively, indicating that the data was Gaussian. This is of particular importance as it confirms the use of a Gaussian formulation for the two-point space time correlation functions introduced below.

If \( u_o(t) \) and \( u(t) \) are the fluctuating velocities at location \((x_o,y_o)\) and \((x,y)\) respectively in the flow, for free turbulent flows, \( u(t) \) is correlated with \( u_o(t) \) over time and spatial extent. This is characterized by integral length and time scales. The convection of the turbulence field as the flow develops downstream also requires the convection velocities to be known. The organisation of the turbulence field around a given location is represented by spatio-temporal correlation and the normal form for the 2nd order correlation \( r(x,v,s,v,\tau) \) can be written using the Gaussian formulation of Ribner (1964) as

\[
r(x,v,s,v,\tau) = \text{Exp} \left[ -\frac{\pi \tau^2}{T_x} \right] \cdot \text{Exp} \left[ -\frac{\pi (s_x - U_x \tau)^2}{L_x^2} - \frac{\pi s_y^2}{L_y^2} \right]
\]

where \( x \) denotes the location of the reference point, \( s \) the separation with the second point and \( \tau \) the retarded time with \( L_x, L_y \) the integral length scale in the \( x \) and \( y \) directions respectively, \( T_x \) the integral time scale and \( U_x \) the convection velocity. For this work, the flow filed is considered two dimensional so the two-point space time correlation is used.

The coherence function is even better indicator of how the turbulence field is correlated as it takes account of the frequency content of the turbulence energy. Using Eq. (2), Kerhervé et al. (2006) showed that the coherence and phase between the two velocity measurements can be expressed as
\[ \gamma(x_v, s_v, \omega) = \text{Exp} \left[ -\frac{s_x^2}{2Lx^2} - \frac{s_y^2}{2Ly^2} \right] \cdot \text{Exp} \left[ -j \omega \cdot \frac{s_z}{2L_z} \right] \]  \tag{3}

\[ \Phi(x_v, s_v, \omega) = \omega \frac{s_z}{U_z} \]  \tag{4}

Although the coherence is normally frequency dependent, for this analysis, it is considered to be dependent only on the space coordinates. This has no significant implications for the parametric study reported here so that the coherence and phase of Equations (3) & (4) are used to model the spatial decay of the turbulence as it convects downstream.

Using these relationships together with the reference signal, simulated data for each point in the 2-D flow field are derived using the formulation of Simon & Fitzpatrick (2004) as

\[ u_{piv}(x, y, t) = IFT \left[ \gamma(x_v, s_v, f) G(f)^{1/2} e^{-j\theta(f)-\theta(f)} \right] + IFT \left[ \left( 1 - \gamma(x_v, s_v, f)^2 \right)^{1/2} G(f)^{1/2} e^{-j\theta(f)} \right] \]  \tag{5}

where \( \phi(f) \) is an independent random phase with uniform distribution.

For the PIV data, \( u(t) \) is resampled at a rate appropriate for a typical PIV system acquisition.

To simulate the LDV data, a Poisson process was used to generate random arrival times with a mean sampling frequency of 20kHz. To evaluate the quality of this specific process, the power spectral density of the generated signal was calculated using the slotting technique of Nobach et al (1998) and compared to the target spectrum estimated directly from the regularly sampled signal \( s_{ref}(t) \). As can be seen from figure 1, good agreement is found when compared to the target spectrum, except at the higher frequencies as expected.

### 3. Simulation Results

The overall quality of the correlation estimates depend intrinsically on both the PIV sampled images and the LDV mean sampling frequency. This can be shown analytically as proposed by Chatellier & Fitzpatrick (2005), or directly by the simulations reported here. The main parameters are the number of samples and mean sample frequency for the LDV and the PIV image number and repetition rate. The mean square error between the targeted and simulated PIV/LDV correlations is used to evaluate the accuracy of the results.

Typical results for the correlation function for different separation distances between the point-LDA and a point-PIV simulation are shown in figure 2 for PIV repetition rates of 4Hz, 8Hz and 2kHz respectively, the last being effectively time-resolved PIV. While the peak of correlation is not affected, significant discrepancies, similar to background noise with threshold increasing when decreasing the PIV scan rate, are manifest. While the correlation peak remains higher than the noise level, turbulence statistics can be thus expected to be extracted either using the raw correlation estimates or indirectly using a fitting process without any significant lost in the accuracy. The influence of the LDV scan rate and the number of PIV fields on the mean square error is shown in figure 3 for the 8Hz repetition rate. Significant decrease in the error with both parameters is manifest with a general trend confirming the results obtained by Chatellier & Fitzpatrick (2004) using analytical developments. For the 8Hz PIV repetition rate, which is used as this is the same as that for the experimental results presented, errors of less than 20% are obtained for a combination of LDV data rate of 25kHz and 350 PIV velocity fields. Figure 4 shows a comparison between the cross spectrum estimated from the zero separation correlation functions and the target cross spectrum. It can be seen that even at 8Hz PIV rate, the cross spectrum is well estimated up to 1 kHz.

The spatial and temporal organization of the turbulence field can be represented by mapping the
space time correlations around a given point of the flow at different time delays. At least two representations can be used depending on which features are to be investigated, viz. either space-time or 2D-space correlations. The first gives information on the convection speed and the rate at which the turbulence field is regenerated while the second leads to spatial properties such as the length scales, which give a measure of the region over which the turbulent field can be considered correlated. Both of these are representative of the equivalent local noise source dimensions. For the second representation, the combination of single and global point measurements is unnecessary as the latter contains all the required spatial information. However, how the 2D spatial turbulence velocity correlations change with time is of particular interest for modelling noise source mechanisms. Space-time turbulence velocity correlations of stationary turbulence are often estimated assuming that the space and time variables are separated but the interaction between the different spectral components, or scales, of the turbulence field mean that the temporal part of the space-time correlation is scale dependent. It has been suggested by Kerhervé et al (2006) that these interactions can be accounted for using frequency dependent length and time scales instead of the common integral ones. However, this requires turbulence velocity cross and auto-spectra to be known locally at all points.

Since both time-space and 2D-space representations are required for modelling the turbulence velocity correlation, the effectiveness of the PRGC technique in estimating these is now examined. Results obtained for the simulated PIV/LDV and targeted correlations using the 2D-spatial representation are shown figure 5. The figure presents iso-contours of correlations and it can be seen that, with increasing time delay, the maximum of correlation decreases and moves away from the reference point in the x direction due to convection of the turbulence, the Gaussian-shape gradually diminishing in the downstream direction. For these simulations, effects such as mean-shear flow or variable frequency dissipation have not been accounted for. While differences between simulated and target results can be observed, the simulated LDV/PIV correlations show that the technique gives a good estimate of the temporal evolution of the spatial correlation. The same results are plotted in figure 6 as a space-time representation. These can be seen to be iso-contours of correlations which are of particular interest when the convection velocity is concerned since the slope of these curves is linearly related to this specific property. Here also, while both samples number and sampling frequency of the PIV signals are low compared to that of LDV, the general trend of the iso-contours are well reproduced, leading to a good estimation of the convection velocity.

The corresponding spatial correlation functions for the x & y directions are given in figure 7 for both simulated LDV/PIV and target results. The lines corresponds to a curve fitting process using the model proposed by Kerhervé et al. (2004) for modelling the space-time correlation functions for jet flows. Sufficiently good agreement between the results can be observed except for larger spatial separations where discrepancies can be observed when using PIV signals. Using this result, a given component of the correlation tensor can be modelled in detail over a 2D area, and anisotropic effects can be accurately accounted for.

4. Application to a subsonic jet flow

4.1 Experimental set-up

A series of 2D-PIV and 2D-LDV measurements have been conducted simultaneously in a 50mm, Mach 0.24 subsonic round jet. The LDV set up was based on a 500mW Argon-ion laser with a Dantec optical system and a burst spectrum analyser type BSA F50. The PIV system consisted of a double-pulsed Nd:Yag laser (New-Wave Solo II, 15 mJ, 15 Hz, 532 nm), a double-exposure camera (LaVision Flowmaster 3, 12bit, 1,280x1,024 px) and the LaVision DaVis software. For seeding, an Antari Z300 fog generator which generated particles of ~1µm diameter was used. A sketch of the experimental set up is given in figure 8 with the LDV beams pointed directly towards the PIV
camera, the measurement volume being located along the shear layer axis and 3D downstream the exit nozzle in the PIV measurement plan. Since both PIV and green LDV wavelengths are close together, two narrow-band filters (532nm 10nm) were used to limit reflections and glare effects on the PIV camera from the LDV beams. A direct consequence of this was a significant reduction in the intensity of light reaching the CCD sensor, but its high sensitivity enabled good quality images to be recorded as reported in Chatellier & Fizpatrick (2005).

The flow area investigated by the PIV was 1.4D x 0.28D the center of which was the location of the LDV point measurements. The PIV timing reference was recorded by the LDV processor and stored with LDV data. Five series of 500 PIV images were recorded simultaneously with the LDA and, to reduce noise in the space-time correlation estimates, results from these five series have been averaged and the final result is presented.

4.2 Experimental results
Iso-contours of the turbulence longitudinal velocity 2-D spatial correlations obtained in the free single-jet for different retarded time steps are given in figure 9. Coordinates x and y refer to the distance from the LDV reference point which was located along the shear layer axis and 3-D downstream the exit nozzle. At zero retarded time, the results correspond to the usual 2-D spatial correlation coefficient which can be obtained when correlating PIV velocity fields only. From this, the typical turbulence integral length scales associated with the velocity component in both x & y directions can be estimated so that local anisotropy in the spatial characteristics can be established. From figure 9, it can be seen that, as the turbulence is convected downstream, the distortion of the original 2-D spatial characteristics due to different turbulence mixing mechanisms is clear as previously detailed in the simulations. The effect of the shear stress can be also noticed as the 2-D spatial correlation pattern exhibits a significant slope.

5. Conclusion
Simulations of the turbulence velocity field in a free shear flow have been used to examine the sensitivity of the Point Reference Global Correlation (PRGC) technique in deriving space time correlations from whole-field measurements using PIV together with LDV point measurements. The effect of parameters such as the mean sample frequency and number of samples for the LDV and number of images and repetition rate for the PIV have been examined using a series of simulations. It has been shown that the accuracy of the method depends on both and that low repetition rates for PIV can be compensated by higher data rates for LDV and vice versa. The results show that the method has good potential for establishing the space time correlations for turbulent in an efficient manner with a minimum of experimental effort and analysis. Thus, although low-speed PIV measurements cannot capture the temporal dynamic of the turbulence velocity, has been shown that when combined with a higher frequency measurement technique such as LDV, statistical properties which show how the turbulence is organised in a domain around a reference point can be extracted.

A direct application of the technique in a Mach 0.24 jet at using PIV and LDV has shown that the turbulence length and time scales can be extracted in 2-D and how the scale dependent nature of the temporal dynamic of turbulence the space-time correlations can be captured. This last point can have significant implications in noise prediction where more realistic noise source models are still necessary.
References


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Fig. 1: Target spectrum and spectrum from simulated LDA data

Fig. 2: LDV-PIV correlations for different separation distances and PIV repetition rates
Fig. 3: Normalised mean square error as a function of LDV data rate and number of PIV velocity fields (repetition rate rate - 8Hz)

(a) 4 Hz

(b) 8 Hz

(c) 32 Hz

(d) 2 kHz

Fig. 4: Simulated (black) and target (red) LDV-PIV cross-power spectrum at zero separation
Fig. 5: Iso-contours of correlations at arbitrary time steps
Fig. 6: Iso-contour of correlation results

(a) Simulations

(b) Target

Fig. 7: 1D spatial correlation in x and y directions
Fig. 8: Schematic of experimental LDV/PIV setup and trigger configuration.
Fig. 9: Iso-contours of correlations at different normalised time steps $\tau T_x$.
From top to bottom: 0, 0.7, 1.4, 2, 2.7
(LDV point reference located at x/D=0 and y/D=0. Jet develops from left to right)