

Laminar viscoelastic flow through a 1:4 plane sudden expansion

by

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ABSTRACT

The results are reported of an experimental investigation of the laminar flow ($Re_d \approx 130$) of a viscoelastic liquid, a 0.05% aqueous solution of polyacrylamide PAA (Seperan AP 273 E), through a plane sudden expansion, of expansion ratio $R=D/d=4$, immediately preceded by a smooth contraction. As is well known, for Newtonian fluid flow above a critical Reynolds number the flowfield downstream of the expansion is asymmetric. For the viscoelastic PAA solution the asymmetry is greatly reduced (evident from the very similar reattachment lengths for the two recirculation regions) but not eliminated (evident from mean streamwise velocity profiles far downstream). The PAA flow is complex and three-dimensional, especially within the smooth contraction, and becomes increasingly three-dimensional (although always remaining symmetric about the XY-centreplane) with downstream distance from the expansion as flow is forced into the XY-centreplane.

1. INTRODUCTION

For laminar Newtonian fluid flow through a plane sudden expansion it is well known that when the expansion ratio $R=D/d$ exceeds 1.5 the flow field downstream of the expansion becomes asymmetric above a critical Reynolds number. This has been observed both experimentally (e.g. Chedron et al (1978), Fearn et al (1990) and Durst et al (1993)) and numerically (e.g. Drikakis (1997) and Schreck and Schafer (2000)). The critical Reynolds number (Re_{CR}) at which this switch to asymmetric flow is observed is dependent upon both the upstream flow conditions (e.g. fully-developed or uniform velocity profile) and the aspect ratio of the expansion, $A_d=w/d$ or $A_h=w/h$.

In contrast to the situation for Newtonian fluid flows, the laminar flow of non-Newtonian, viscoelastic, liquids through sudden expansions has received scant attention in the literature and is largely restricted to a handful of papers at very low Reynolds numbers, where the flow remains symmetric, and has involved flow visualisation and theoretical modelling but no detailed measurements of the flowfield.

The first to investigate expansive flows of viscoelastic, non-Newtonian liquids were Halmos and Boger (1976). Streak photography was used to visualise the flow while a flash technique was used for point centreline velocity measurements. In earlier work, Halmos et al (1975a and b), these authors presented a numerical solution with experimental verification for the flow of inelastic, power-law fluids through an axisymmetric sudden expansion. Halmos and Boger (1976) concluded that as a viscoelastic fluid flows through a sudden expansion it releases some of its stored energy, resulting in an expansion of the main flow and compression of the secondary cell (recirculation region).

Townsend and Walters (1994) used flow visualisation to observe the flow field downstream of both a two-dimensional and a three-dimensional expansion for a 0.15% aqueous solution of polyacrylamide. The conclusion drawn from the study was that the viscoelasticity of the polymer solution damped out the vortex activity (which is generated by fluid inertia) and caused any recirculating fluid to be pushed into the corners of the expansion. A theoretical model was also developed to simulate the flow field numerically. The results were in good physical agreement with the flow visualisation but only qualitative in nature due to the lack of quantitative rheological and velocity data.

The numerical works of Darwish et al (1992) and later Missirlis et al (1998) are very similar. Both use a finite-volume technique to simulate the flow of a viscoelastic liquid through a two-dimensional 1:4 plane sudden expansion. Neither validate their work by comparison with experimental data but infer verification by using grid refinement to obtain grid-independent results. Missirlis et al show that the suppression of vortex activity is related to the Deborah number ($De = t / T$ where T is a characteristic time of the deformation process being observed and t is a characteristic time of the fluid). They show that as De is increased beyond a critical value of 3.0 the recirculation zone is completely eliminated.

The experiments of Townsend and Walters (1994) were also used as the basis for comparison in the numerical simulation work of Baloch et al (1996). Expansion flows were modelled in two and three dimensions using a class of constitutive models due to Phan-Thien and Tanner (1977). Once again good qualitative agreement is seen with the experimental visualisations and the conclusion again drawn that viscoelasticity suppresses vortex activity and that this suppression is linked to the phenomenon of die swell.

Two recent works have, for the first time, investigated non-Newtonian fluid flow through a plane sudden expansion ($R=3$) at high enough Reynolds numbers (ca 100) for asymmetric flow to occur. Neofytou and Drikakis (2003) investigated three non-Newtonian models used to simulate blood: the Casson, power-law and Quemada models. All three models are inelastic and only represent the shear-thinning behaviour of the shear viscosity. Viscoelastic effects (such as extensional viscosity variations or the appearance of normal-stress differences) are not modelled. They found that the asymmetry also occurred for these shear-thinning models and that the critical Reynolds number was dependent on the parameters in each model's constitutive equation. In contrast the work of Oliveira (2003) does attempt to model a real non-Newtonian fluid and to incorporate viscoelastic effects by employing a modified FENE-CR constitutive equation. Here again, asymmetric flow occurs above a critical Reynolds number. The effect of viscoelasticity on the critical Reynolds number is dependent on the concentration and extensionability parameters chosen for the model but the trend is always for Re_{CR} to increase beyond the Newtonian value i.e. the effect of viscoelasticity is a stabilizing one. Both of these studies are purely numerical in nature and have to rely on grid independency or agreement with past Newtonian simulations for validation, a course of action that is clearly not ideal and something the current work aims to address.

The prediction of viscoelastic flow in complex geometries such as sudden expansions has both scientific interest and industrial relevance. The development of theoretical models to predict these flows has, as we have discussed, been investigated by a number of different researchers. The verification of these models has always been at best qualitative in nature and it is one of the purposes of the present work to address this deficit by providing detailed rheological and velocity data to enable quantitative validation of these models.

2. EXPERIMENTAL RIG AND INSTRUMENTATION

The flow loop used for the present experiments was a modified version of that used by Escudier and Smith (2001) for their investigation of fully developed turbulent flow through a square-duct. The square duct consisted of ten stainless steel modules each of length 1.2m with an internal cross section of side length $w=80$ mm. The plane sudden expansion, for which the key dimensions are given in **Figure 1**, replaced one of the existing modules 9.6 m from the inlet connection. This arrangement provides a length of 120 hydraulic diameters ($D_H = w = 80$ mm) for the flow to become fully developed. The duct width w throughout is 80 mm, the inlet height d is 10 mm and the step height h is 15 mm. The downstream duct height D is 40 mm. The expansion is preceded by a short (53.5 mm in length), smooth contraction (40 mm radius followed by 20 mm radius). The side-walls of the expansion were made of borosilicate glass to permit velocity measurements using a laser Doppler anemometer. Distributions of mean streamwise velocity (U) were obtained along the XY-centreplane from transverse traverses (i.e. in the y -direction) at nine streamwise locations corresponding to x/h values of 0,1,2,3,4,5,6,8 and 12. Distributions of mean streamwise velocity were also obtained across the duct along the XZ-centreplane from spanwise traverses (i.e. in the z -direction and at $y/D=0.5$) both within the smooth contraction ($x/h=-3.33, -1.67, -1$ and 0) and downstream of the expansion ($x/h = 1,2,3,4,5,6,8$ and 12).

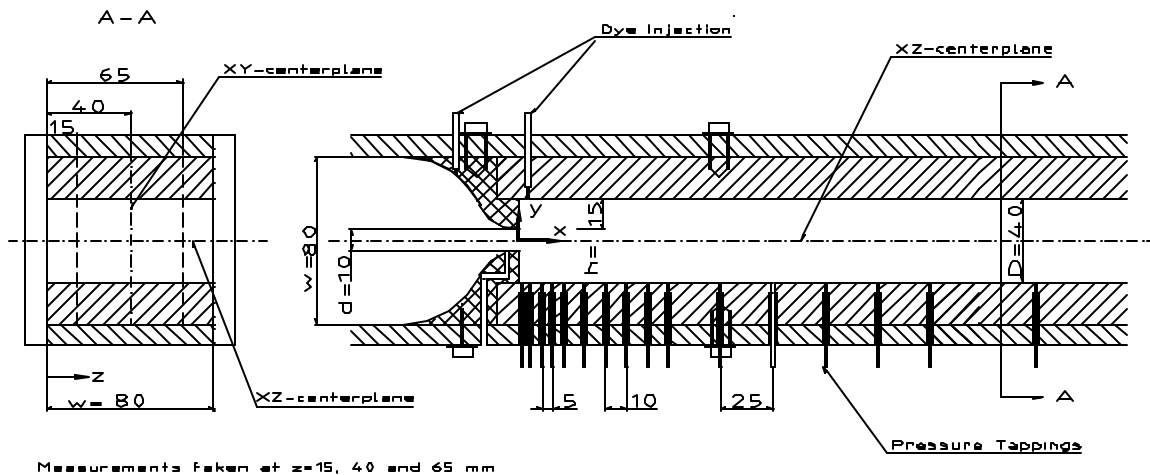


Figure 1: Plane sudden expansion geometry, dimensions in mm

A Dantec Fibreflow laser Doppler anemometer system was used for the measurements and comprised a Dantec 60x10 probe and a Dantec 55x12 beam expander in conjunction with a Dantec Burst Spectrum Analyzer signal processor (model 57N10). The beam separation at the front lens was 51.5 mm and the lens focal length 160 mm which produces a measurement volume in water and the working fluid with principal axis of length 0.21 mm and diameter 0.02 mm. Ensemble averages were formed from not less than 9500 velocity samples.

Flow rates were measured using a Fischer and Porter electromagnetic flowmeter (model 10D1) incorporated in the flow loop upstream of the sudden expansion with the flowmeter output signal recorded via an Amplicon PS 30AT A/D converter.

All rheological measurements were carried out using a TA Instruments Rheolyst AR 1000N controlled-stress rheometer. A temperature of 20 °C was maintained for all the rheological measurements, which was also the average temperature of the fluid for the duration of the experimental runs. Temperature control of the TA rheometer is achieved via a plate that uses the Peltier effect to maintain the temperature of the sample to within ± 0.1 °C. The rheological characterisation included measurements of shear viscosity μ , the storage modulus G' and the loss modulus G'' .

3. WORKING FLUID CHARACTERISTICS

The working fluid used in this investigation was an aqueous solution of 0.05% w/w polyacrylamide (PAA); Separan AP273 E supplied by Floerger. The solvent used was filtered tap water with 100 ppm of 40% formaldehyde solution added to retard bacterial degradation. Approximately 0.25 gm of Timiron seeding particles were added to the fluid (total volume of fluid 575 litres) to improve the LDA signal quality. PAA was chosen as it is highly viscoelastic, is optically transparent (thereby permitting LDA measurements) and has been used extensively in previous investigations in the same laboratory (see e.g. Escudier et al (1999) and Escudier and Smith (2001)).

PAA is generally regarded (see e.g. Walters et al (1990)) as being ‘very flexible’ in its molecular structure and it is believed this flexibility gives rise to its increased elastic properties compared to other water-soluble polymers such as xanthan gum and carboxymethylcellulose. The average molecular weight for the PAA used in this study, ascertained using gel phase chromatography, was determined to be 1.94×10^6 g/mol with a polydispersity of 1.05. The measured viscosity versus shear rate data for PAA is shown in **Figure 2** together with the Carreau-Yasuda model fit:

$$\mu_{CY} = \mu_{\infty} + \frac{\mu_0 - \mu_{\infty}}{\left(1 + (I_{CY}\dot{\gamma})^a\right)^{n/a}}$$

$\mu_0 = 0.614$ Pa.s being the zero-shear-rate viscosity, $\mu_{\infty} = 0.0028$ Pa.s the infinite-shear-rate viscosity, $I_{CY} = 25.7$ s a time constant, $n = 0.58$ a power-law index and $a = 0.99$ a parameter introduced by Yasuda et al (1981). The parameters were determined using the least-squares fitting procedure outlined in Escudier et al (2001).

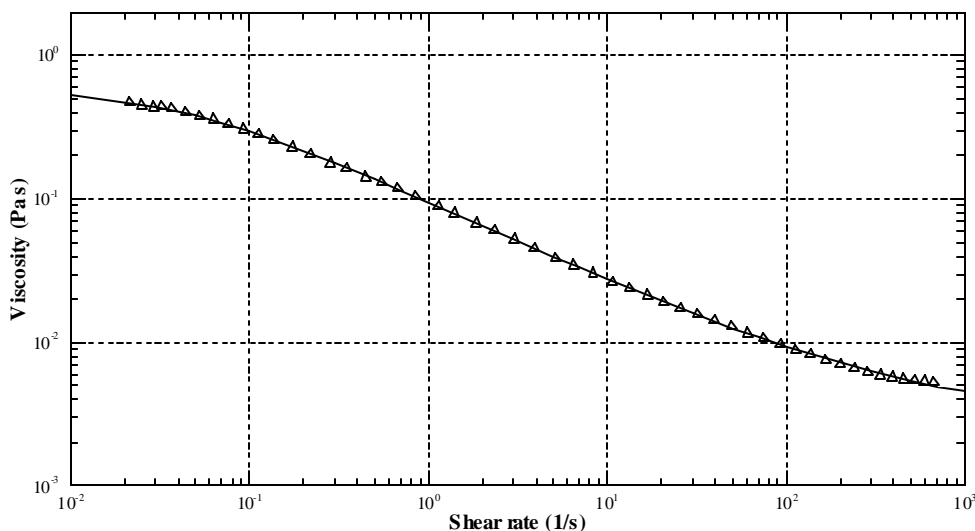


Figure 2: Shear viscosity versus shear rate for 0.05% polyacrylamide (including Carreau-Yasuda fit)

The viscoelastic properties of the PAA used were investigated firstly in shear but the N_1 values produced were below the sensitivity of the rheometer even at the highest shear stresses. The oscillatory results can be seen in **Figure 3** in terms of the storage modulus, G' , and the loss modulus, G'' , against angular frequency. A linearity check was conducted to determine the linear viscoelastic region prior to the frequency sweep.

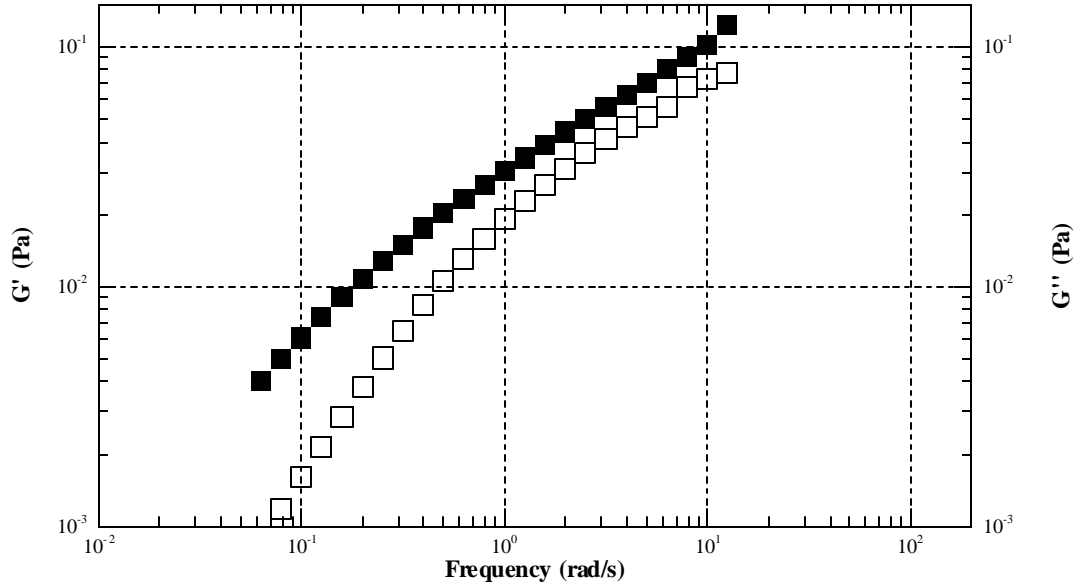


Figure 3: Storage (G' open symbols) and loss (G'' closed symbols) moduli versus angular frequency for 0.05% polyacrylamide

The frequency sweep shown in **Figure 3** was performed at a shear stress of 0.5 Pa, a value well within the linear regime. Comparison with frequency-sweep data at a higher shear stress, again within the linear regime, confirmed that the viscoelastic properties observed were independent of the shear-stress value. The oscillation data reveal a degree of elasticity within the fluid as G' is non-zero. The response of an inelastic liquid would be $G'=0$ and $G''=\mu\omega$ where μ =the shear viscosity of the liquid at the applied (oscillatory) shear stress. Elasticity increases with frequency, the ratio G'/G'' reaching a maximum value of 0.73 at a frequency of about 0.5 Hz.

4. ESTIMATION OF REYNOLDS NUMBER

Many workers (see for example Townsend and Walters (1994)) do not attempt to define a Reynolds number for shear-thinning liquids because no single value for the viscosity completely characterises the fluid. In spite of this difficulty we feel that it is useful to quantify the relative importance of inertial to viscous forces. The classic definition of a Reynolds number is

$$Re = \frac{\rho UL}{\mu}$$

where U and L are characteristic velocity and length scales, respectively, and ρ is the fluid density. In the present work the bulk velocity determined from the flowmeter ($U_B=0.270$ m/s = Q/A where A = inlet area at the expansion plane = $w d$ and Q is the volumetric flowrate) was used as the characteristic velocity. Either the inlet height ($d=0.01$ m) or the step height ($h=0.015$ m) can be taken as a length scale. The density of the fluid ρ was constant and essentially that of the solvent, water. There exist a number of possible ways to estimate a characteristic viscosity μ_c all of which involve estimating a characteristic shear rate $\dot{\gamma}_c$ and then using the Carreau-Yasuda model to calculate the corresponding viscosity. The various methods of estimating μ_c and hence Re are shown in **Table 1**.

Table 1: Reynolds number estimates for various characteristic shear rates.

\dot{g}_c (1/s)	η_c (Pa.s)	$Re_d = \frac{rU_B d}{\eta_c}$	$Re_h = \frac{rU_B h}{\eta_c}$
$\rightarrow 0$	0.614	4.4	6.5
$\dot{g}_c = \frac{U_B}{h} \approx 18$	0.0204	131	196
$\dot{g}_c = \frac{U_B}{d} \approx 27$	0.0167	159	240
$\dot{g}_c _{Inlet} = \frac{dU}{dy} _{Wall} \approx 400$	0.0058	460	690
$\rightarrow \infty$	0.0028	953	1429

The difficulties in selecting an appropriate Reynolds number are highlighted by **Table 1** where it can be seen that the Reynolds number varies between about 4 and 1400 (i.e. more than two orders of magnitude) depending upon the choice of \dot{g}_c . The minimum and maximum possible values of the Reynolds numbers are based on the shear rate tending to either zero or infinity and are best used as a guide. Determining the shear rate at inlet has a direct physical interpretation but requires a priori knowledge of the flow field to determine the value and cannot easily be used to compare to numerical simulations because, unless the measured velocity profile at inlet agrees with the simulation, the Reynolds numbers will differ. In view of the foregoing we favour the use of a more simplistic ‘characteristic’ Reynolds number Re_d based on a viscosity corresponding to a ‘characteristic’ shear rate $=U_B/h$.

5. DISCUSSION

5.1 Flow within the smooth contraction

The mean streamwise velocity profile measured along the XY-centreplane at the expansion plane (i.e. $x/h=0$) is shown in **Figure 4(a)**. The remaining downstream profiles are discussed in the following section. The shape of the mean velocity profile at $x/h=0$ is a consequence of the smooth contraction that immediately precedes the expansion. The effect of a smooth contraction is well known for Newtonian fluid flows. The anticipated effect of the smooth contraction (for high Reynolds number flows at least) is to produce a uniform ‘top-hat’ velocity profile. A uniform profile is expected because the smooth contraction converts upstream pressure energy (which must be essentially uniform) into downstream kinetic energy. The effect of viscosity (combined with the no-slip condition at the walls) is to produce a negligibly thin boundary layer near the solid surface. In contrast the PAA solution displays a velocity overshoot ($\sim 1.1U_B$) near the wall and a slight dip in the velocity on the centreline (i.e. the XZ-centreplane $y/D=0.5$) to $1.05U_B$. In an attempt to further investigate this effect, and the inlet profile in more detail, we measured the spanwise variation of the mean streamwise velocity profile along the XZ-centreplane ($y/D=0.5$) both at the expansion plane (i.e. $x/h=0$) and upstream, within the contraction ($x/h=-3.33, -1.67$ and -1). The results can be seen in **Figure 4(b)**. To provide a visual guide of the inherent symmetry within the flow, mirror-image points (about $z/w=0.5$) are included as filled symbols. Within the central core of the flow ($0.25 < z/w < 0.75$) the effect of the smooth contraction is as one would expect for a Newtonian fluid flow: as the area decreases, the velocity increases, and at inlet the velocity in this section of the duct is practically uniform and two-dimensional. Nearer to the wall however, the shape of the velocity profiles is unexpected. At the furthest upstream location ($x/h = -3.33$) a small velocity ‘overshoot’ is seen near the wall. As the flow progresses downstream this ‘overshoot’ is amplified (at $x/h = -1.67$ for example it is almost double the velocity on the centreline) and the width of this high-velocity ‘overshoot’ increases. The transition between the central ‘two-dimensional’ part of the flow and the overshoot near the wall is sudden and the velocity gradient is high ($\approx 100 \text{ s}^{-1}$ which is about the same order of magnitude as the maximum shear rate when the shear layer separates from the step into the expansion **Figure 4(a)**). The mechanism for this behaviour is unclear but it may well be related to the normal-stress differences produced which increase with increasing shear rate. The highest shear rate in the

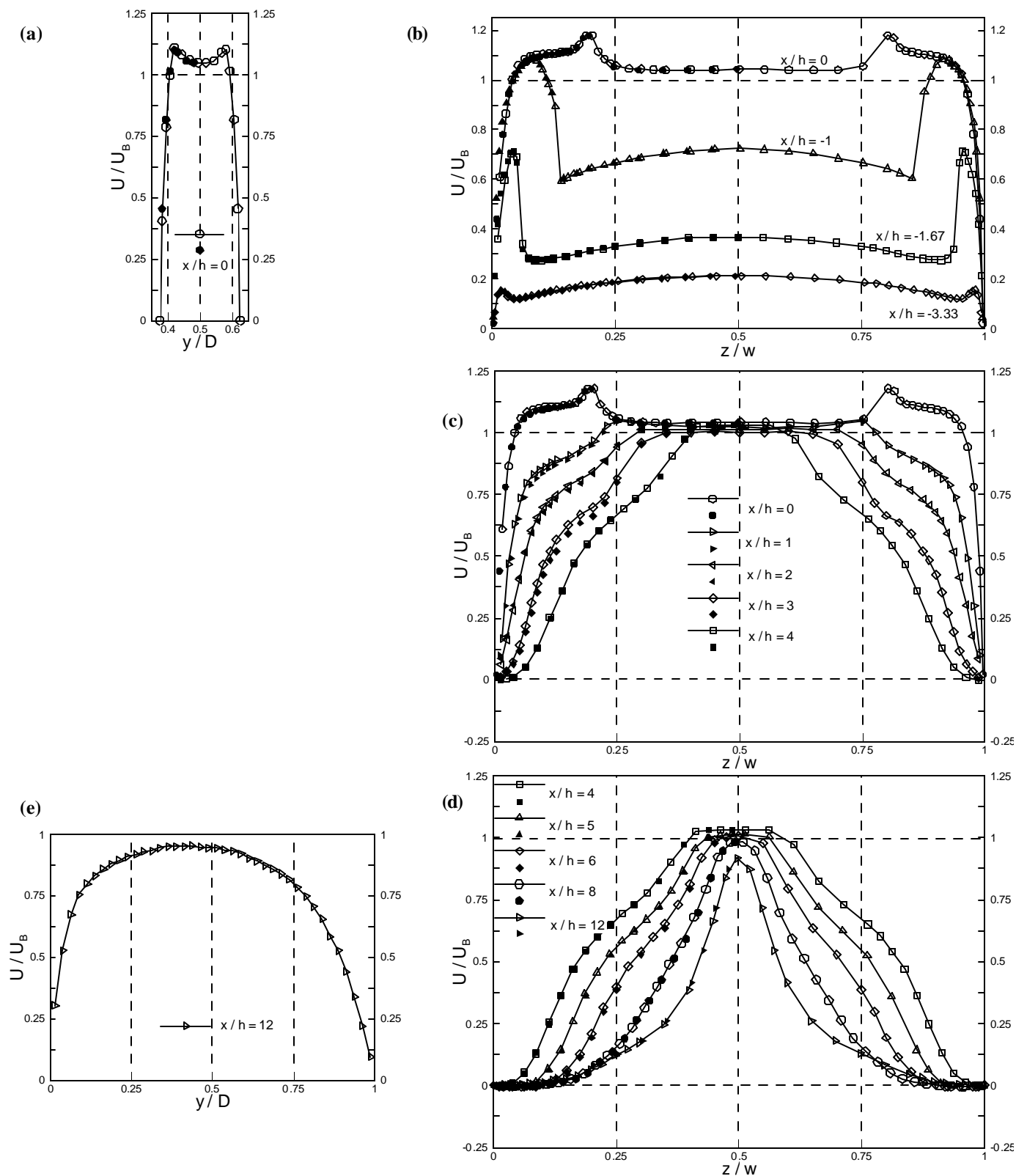


Figure 4: (a) and (e) Transverse variation of streamwise (U/U_B) velocity profiles along XY-centreplane (i.e. $z/w=0.5$)
 (b), (c) and (d) Spanwise variation of streamwise (U/U_B) velocity profiles along XZ-centreplane (i.e. $y/D=0.5$)
 *(a) and (b) are within contraction ($-3.33 \leq x/h \leq 0$), (c), (d) and (e) downstream of expansion ($0 \leq x/h \leq 4, 5 \leq x/h \leq 12$)

flow will be immediately adjacent to the wall so the largest first normal-stress difference will occur at this location. This normal-stress difference will act to produce a force normal to wall and therefore the wall will exert an equal and opposite force on the fluid near the wall. The incompressibility of the fluid, in conjunction with this force, causes the fluid near the wall to form a jet-like stream of high-velocity fluid i.e. the fluid near the wall is being ‘squeezed’. As the flow progresses through the contraction (and the cross-sectional area decreases) continuity requires the bulk velocity of the flow to increase, which in turn increases the shear rate at the wall thereby amplifying this effect. Presumably the spreading of this high-velocity ‘jet’ is then a viscous effect.

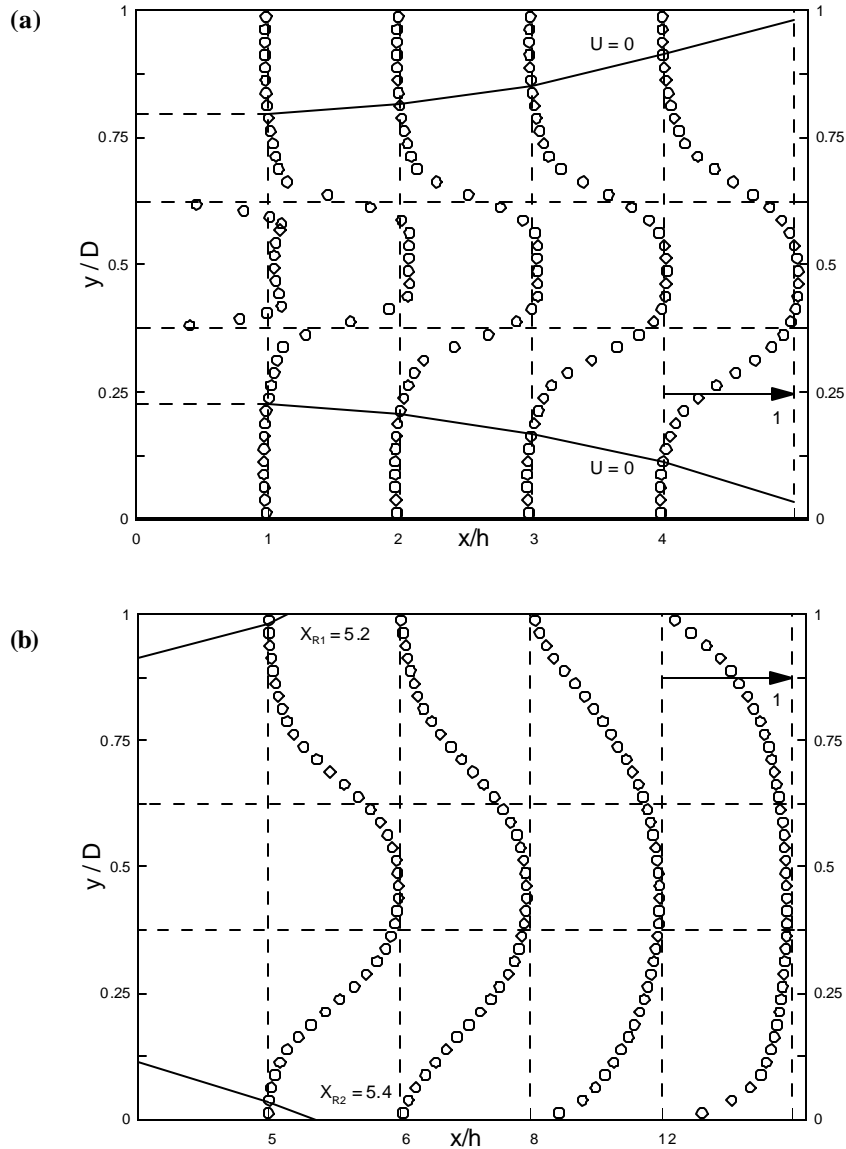


Figure 5: Transverse variation of mean streamwise (U/U_B) velocity profiles in XY-centreplane (i.e. $z/w=0.5$) downstream of expansion (a) $0 \leq x/h \leq 4$ (b) $5 \leq x/h \leq 12$

5.2 Mean streamwise velocity profiles downstream of sudden expansion

The transverse variation of the mean streamwise velocity profiles along the XY-centreplane downstream of the expansion can be seen in **Figure 5(a)** and **(b)**. Also shown are the loci of zero mean velocity to provide a visual guide to the length of both recirculation regions and the essentially symmetric nature (at least upstream of reattachment) of the flow field. Streamlines would, of course, show this much more satisfactorily but cannot be constructed due to the three-dimensionality of the flowfield. **Table 2** shows integrated “flowrates” (determined numerically assuming 2D flow) at each streamwise

location and demonstrates that the flow is far from two-dimensional. The increase of the apparent flowrate with downstream distance combined with an inspection of the streamwise profiles (Figure 4(a) and (b)) show that fluid is being forced into the measurement plane (i.e. the XZ-centreplane) as it progresses downstream.

Table 2: Apparent flow rates at various axial locations in the XY centreplane

	Flowmeter	x/h=0	x/h=1	x/h=2	x/h=3	x/h=4	x/h=5	x/h=6	x/h=8	x/h=12
Q (m ³ /h)	0.778	0.718	0.817	0.931	1.050	1.233	1.418	1.631	2.087	2.358

An expansion of the high-velocity core is evident immediately downstream of the expansion ($0 < x/h < 1$) as has been observed previously at much lower Reynolds numbers (Townsend and Walters (1994)), and must be related to the release of elastic energy stored in the polymer molecules after the expansion. The reattachment lengths for the two recirculation regions differ slightly, $X_{R1} = 5.2$ step heights compared to $X_{R2} = 5.4$ step heights, and show that the asymmetry, although greatly reduced compared to a Newtonian fluid flow at the same Reynolds number (Drikakis determined the critical Reynolds number for a Newtonian fluid flow through a ‘two-dimensional’ 1:4 plane sudden expansion at about 53), is not eliminated. This is confirmed by the profiles at $x/h=8$ and 12 (**Figure 5 (b)**) which are noticeably asymmetric.

To investigate the three-dimensionality of the flowfield and to understand the mechanism responsible for the increase in apparent flowrate along the XY-centreplane with downstream distance, spanwise profiles were taken along the XZ-centreplane at discrete streamwise locations ($x/h=1,2,3,4,5,6,8$ and 12) and can be seen in **Figure 4(c)** and **(d)**. These profiles show that the increase in apparent flowrate observed in the XY-centreplane with downstream distance (see **Table 2**) is due to fluid being ‘fed’ from flow near the sidewalls. Filled symbols again demonstrate the symmetry of the flow about the XY-centreplane. What is remarkable is that by $x/h=12$ the majority of the flow is contained within a narrow vertical wedge of fluid centred around the XY-centreplane with almost stagnant fluid on either side. This is demonstrated in **Figure 4 (d) and (e)** which show profiles along the XZ-centreplane (**4(d)**) and the XY-centreplane (**4(e)**) for $x/h=12$.

6. CONCLUSIONS

The results have been reported of an experimental investigation of the laminar flow ($Re_d \approx 130$) of a viscoelastic liquid, a 0.05% aqueous solution of polyacrylamide (Seperan AP 273 E), through a plane sudden expansion, of expansion ratio $R=D/d=4$, immediately preceded by a smooth contraction.

Downstream of the expansion the asymmetry seen in a Newtonian fluid flow at a comparable Reynolds number is largely absent. Slight differences in the length of the two recirculation-regions and more significantly, the transverse variation of the mean velocity profiles, show that asymmetry is not entirely eliminated. Integration of the transverse variations of the mean streamwise velocity profiles in the XY-centreplane reveals significant departures from two-dimensionality with the apparent flowrate along the XY-centreplane increasing with downstream distance from the expansion. Spanwise profiles show that this increase is supplied from the flow nearer the sidewalls although the flow always remains symmetric about the XY-centreplane.

The flow through the smooth contraction that precedes the expansion is spectacularly different from that expected of a Newtonian fluid flow. As the flow progresses through the contraction a velocity overshoot develops next to the wall that is amplified in magnitude as the area decreases through the contraction. It is speculated that the cause of the three-dimensionality, both within the smooth contraction and downstream of the expansion, is the normal-stress differences produced as a consequence of the viscoelasticity of the liquid.

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