Determination of Turbulent Scales in Subsonic and Supersonic Jets from LDV Measurements

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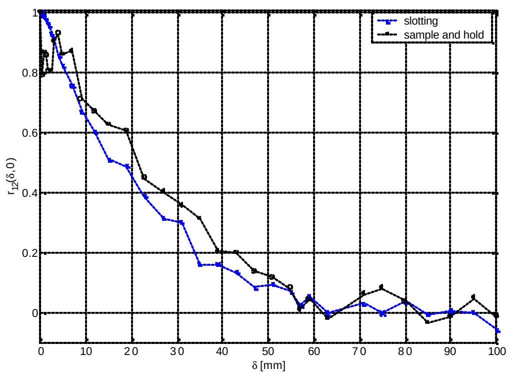
The turbulence statistics of high speed jets provide the basis for the acoustic source mechanisms which are responsible for noise generation. The correlation functions and spectra together with the length and time scales of the turbulence associated with sub and supersonic jets are examined for two point Laser Doppler Velocimetry (LDV) measurements. The paper presents results for two point/single component measurements at Mach numbers of 0.75 and 1.2 for isothermal conditions (i.e. $T_j/T_a = 1.0$). Analysis has been performed to extract the turbulence statistical characteristics using both the slot correlation and sample-and–hold procedures. A technique by which the frequency dependence of the scales can be determined from the auto and cross spectra of the measurements is derived. The results from the two analysis methods are compared and shown, in the main, to be equivalent. The figure below shows the length and time scales determined from the measurements in a subsonic isothermal jet flow at M=0.75.



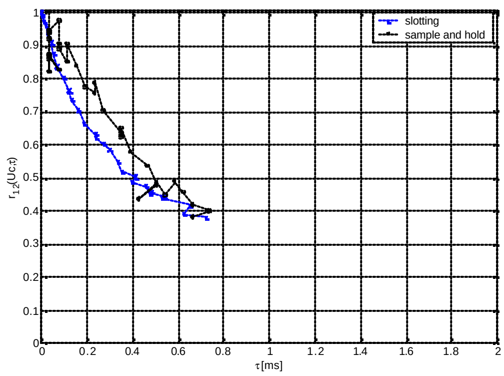


Figure 1: Length and Time Scales for M=0.75

1. Introduction

The statistical characteristics of turbulent structures are used extensively for the prediction of noise generated by jet flows. In order to assess the source mechanisms responsible for the noise generation, reliable data for these characteristics is required as indicated by Jordan & Gervais (2003). In addition, the potential for the use of unsteady CFD methods such as large eddy simulation (LES) to provide the information required for noise prediction methodologies means that reliable data for validation purposes for specific applications is required as noted by Andersen et al. (2003). For single point measurements, the time averaged mean and higher order statistical characteristics (i.e. variance, skewness, flatness), the auto-correlation function and auto-spectrum are important properties. For two point measurements, in addition to these, the cross correlation function and the cross spectrum are used to derive the spatio-temporal correlation functions. At present, there are two widely used techniques to determine turbulence characteristics from LDV data. The first procedure uses the slotting correlation method, which gives the auto or cross correlation function and these are Fourier transformed to obtain the auto or cross spectra (e.g. van Maanen, 1999). For the second, the time domain signal is reconstructed at equal time intervals using a zero order or sample-and-hold interpolation scheme. The resulting time series is then analysed to determine statistical characteristics such as the auto or cross spectra and the cross-correlation function is calculated as the inverse Fourier transform of the latter (e.g. Simon & Fitzpatrick, 2004). The length and time scales are then determined from either the time or frequency domain functions. However, these length scales are usually a function of frequency and the more sophisticated noise prediction models make assumptions concerning this relationship.

In this paper, the results from two point/single component LDV measurements in both a subsonic and a supersonic jet are analysed using both the slot correlation and sample-and-hold procedures. The auto and cross spectra, cross correlation functions and length and time scales thus derived are compared. A technique by which the frequency dependence of the length and time scales can be obtained is given and the results for the two jet conditions are discussed.

2. Experimental Set-Up and Instrumentation

The subsonic tests were conducted at the MARTEL facility of CEAT (Centre d'Etudes Aérodynamiques et Thermiques) in Poitiers, France. Aerodynamic measurements were obtained using two-point one-component LDV arrangements. A traversing system for the LDV measurements was designed and installed at the nozzle exit and two 5-watt argon-ion lasers were used in forward scatter to maximise signal to noise ratios and data rates. A schematic of the layout is shown in figure 1(a). The potential core and shear layer were seeded using Silicon dioxide particles of ~0.4 µm diameter and the signals were processed by a TSI Doppler Signal Analyser. For the subsonic test, a 50 mm diameter nozzle was used with the jet aligned vertically and exhausting into free space. The two-point/one-component measurements were made in a non-coincident mode at the end of the potential core along the lip line for the jet condition at M=0.75. A schematic of the measurement positions is shown in figure 1(b). These measurements allow the correlation functions and spectra of the longitudinal component velocity to be determined and the length and time scales of the flows to be examined.

For the supersonic jet, the tests were conducted using a convergent nozzle of 52mm diameter operating at M=1.2. The nozzle was contained in a wind tunnel with a secondary subsonic flow at Mach 0.15 entrained so the jet cannot be assumed as perfectly free. Nonetheless, the properties of the jet structure and of the mixing noise mechanisms remain unchanged. Details of the experimental facility can be found in Kerhervé et al. (2003). Again, two single component LDV systems were used as shown in figure 1(a) and the signals were processed by a TSI Doppler Signal Analyser. The glass walls of the tunnel allowed optical access up to 10 diameters from the nozzle exit and the measurements reported here were obtained along the lip line of the nozzle at half the potential core length as shown schematically in figure 1(b).

For both series of tests, the data output from the LDV processors were digitally recorded for subsequent off-line analysis.

3. Analysis of LDV Data

For the measurements, the auto and cross spectra, cross correlation function, the length and time scales and the convection speed of the turbulence are of significant interest in determining potential for noise generation in a jet. The temporal irregularity of LDV data means that the standard approaches used for calculation of auto and cross correlation functions and for auto or cross spectra cannot be applied.

Although there are a number of different techniques by which correlation functions and spectra can be derived from LDV data, the two most common approaches are the slot correlation method and sample-and-hold reconstruction.

The slot correlation method used was that proposed by van Maanen (1999), in which the fuzzy slotting technique of Nobach et al. (1998) is combined with a local normalisation approach suggested by van Maanen & Tummers (1996). Spectral estimation via slot correlation involves calculating all possible combinations of cross-product between the data points of two signals, which, plotted as a function of the associated time lags give an estimation of the cross-correlation function (CCF). The cross-products are then accumulated and averaged in equispaced bins (slots) in the correlation domain, giving a regularly discretised estimation of the autocorrelation function which can be subsequently windowed and Fourier transformed to give an estimate of the power spectral density of the signal. The high variance associated with this approach can be mitigated by applying a triangular windowing function to each individual slot (fuzzy slotting), which allows cross-products from adjacent slots to contribute to the local estimate, and to normalise each slot using only those data points which contribute to that estimate (local normalisation).

For the sample-and-hold process, the time domain data is reconstructed by holding the value of each validated data point until the next arrival and re-sampling the data at equal intervals. The resulting data sets are then Fourier transformed and averaged to obtain the auto and cross spectra. It has been shown by both Boyer & Searby (1986) and Adrian & Yao (1987) that the sample-and-hold process of time domain reconstruction of a signal introduces errors that comprise of a step noise, which is white and adds a constant bias to the estimated spectrum, and a low pass filter effect. The errors are functions of the mean sample rate, the maximum frequency to be resolved and the Taylor microscale of the flow. A procedure for the estimation of the auto-spectrum from sample-and-hold data was proposed by Nobach et al. (1998) in which a digital finite impulse response (FIR) filter was used together with an estimate of the noise to yield a corrected PSD. More recently, a procedure was proposed by Simon & Fitzpatrick (2004) in which a discrete filter together with a step noise correction was shown to be a more efficient estimator using sample-and-hold reconstruction. This approach has been extended to the determination of cross spectra from LDV data for both co-incident and non-coincident cases (Fitzpatrick & Simon, 2004). The auto and cross spectra are then inverse Fourier transformed to obtain the respective correlation functions.

In this work, both methods are used to obtain the auto and cross spectra and the cross-correlation functions for the two point measurements. The length and time scales for the measurements are derived from these as detailed in the next section.

4. Length & Time Scales

The spatio-temporal correlations give details of the length and time scales of turbulence in a flow. The definition of this is the cross correlation function given as

$$R_{12}(d,t) = \int_{-\infty}^{\infty} u_1(0,t)u_2(d,t+t)dt \qquad (1)$$

This is generally plotted as a series of curves with d as a parameter or as a space-time iso-contour plot. The length scales are determined at each separation from $R_{12}(0,d)$ as

$$L_x(d) = \frac{R_{12}(d,0)}{[R_1(0).R_2(0)]^{0.5}} \qquad (2)$$

where $R_1(0)$ & $R_2(0)$ are the standard deviations of u_1 & u_2. The integral length scale is determined by direct integration of equation 2. The cross spectrum of turbulence are given as

$$G_{12}(d,f) = \int_{-\infty}^{\infty} R_{12}(d,t)e^{-i2pft} \qquad (3)$$

The real and imaginary parts of this are termed the co and quadrature components of the cross spectrum. It can be shown the $R_{12}(0,d)$ is given by

$$R_{12}(d,0) = \int_{-\infty}^{\infty} \mathrm{Re}[G_{12}(d, f)]df \qquad (4)$$

Thus, the length scales can also be determined from the real part of the cross spectrum and the two auto-spectra as

$$L_x(d) = \frac{\int_{-\infty}^{\infty} \mathrm{Re}[G_{12}(d, f)]df}{\{\int_{-\infty}^{\infty} G_1(f)df . \int_{-\infty}^{\infty} G_2(f)df\}^{0.5}} \qquad (5)$$

Using this formulation, a frequency dependent length scale can be defined as

$$L_x(d, f) = \frac{\mathrm{Re}[G_{12}(d, f)]df}{\{G_1(f)df . G_2(f)df\}^{0.5}} \qquad (6)$$

The convection speed can also be determined as a function of frequency from the phase of the cross spectrum using the relationship

$$f_{12}(d, f) = 2pfh(d, f) = 2pfd / U_c(f) \qquad (7)$$

where ?(d,f)=d/Uc(f) is the time lag for the two measurements as a function of frequency and displacement. Finally, the time scale can be determined as a function of frequency using the relationship

$$L_T(f, h) = \frac{|G_{12}(d, f)|df}{\{G_1(f)df . G_2(f)df\}^{0.5}} \qquad (8)$$

Thus, the frequency dependence of the length and time scales be estimated directly from the auto and cross spectra obtained from two point measurements in a turbulent flow.

5. Experimental Results

The flow properties recorded at the end of the potential core for the subsonic case were mean velocities of 250m/s & 160m/s at the core edge and on the shear layer axis respectively with turbulence intensities of 9.5% and 17.5%. For the supersonic case, the properties at the mid length of the potential core were 400m/s, 375m/s & 180m/s for the jet axis, core edge and shear layer axis respectively with turbulence intensities of 2.5%, 7.5% & 30%. Data rates for the LDV acquisitions varied from 5–20kHz for the subsonic case and from 15- 45kHz for the supersonic tests.

5.1 Auto-Spectra, Cross Spectra & Correlation Functions

The auto spectra for the reference position, the cross spectrum (for a separation of 1.1D for subsonic and 0.5D for supersonic) and the cross correlation functions for series of separations are shown in figure 2 for both cases. The auto-spectra are on the shear layer axis at the end of the potential core for the isothermal Mach 0.75 jet and along the shear layer axis at half the potential core for the isothermal Mach 1.2 jet. Along the shear layer axis, the standard turbulent shape is well reproduced up to the Nyquist frequency of the corresponding data, with sample-and-hold achieving a slightly higher frequency resolution than the slot correlation. The cross spectra for a separation of 1.1 and 0.5D are shown in figure 2(b). From these, it can be seen that the magnitude of the estimates for the subsonic jet using both techniques are equivalent. For the supersonic case, sample-and-hold doesn't achieve as high a frequency resolution as the estimate obtained using slot correlation. However, on examination of the phase estimates of the corresponding data, both techniques are efficient at low frequencies with sample-and-hold achieving a higher frequency resolution for the supersonic case.

The cross correlation functions for a number of normalised separations are presented in figure 2(c). These show the classical form of the spatio-temporal correlations. Some discrepancies between the two techniques are observed. These are due to the different approaches used in both techniques to estimate the cross-correlation as previously detailed. The spatial-temporal correlation function $R_{12}(d,t)$ can be separated into its spatial and temporal decrease denoted by $R_{12}(d,0)$ and $R_{12}(Uc,t)$ respectively. The values for these obtained from the results of figure 2(c) are presented in figure 3. It can again be seen that the estimates for both the length and time scales for both the jets are in very good agreement, with the exception of the temporal estimate using sample-and-hold for the supersonic case. The integral length and time scales obtained by integrating these functions over the separation distance and time respectively are reported in table 2. The sample-and-hold results give higher estimates of length and time scales for both jet configurations compared to the slot correlation, whereas lower estimates of convection velocity are noted. Overall differences of less than 15% are globally found between both techniques.

5.2 Frequency Dependence of Turbulent Scales

The frequency dependence of the length & time scales and the convection velocity were then investigated using the relationships given by equations 6, 7 and 8. For the length scales, the frequency dependence is shown in figure 4 for both subsonic and supersonic jets. From this, it can be seen that the length scales are decreasing as the frequency increases as would be expected with oscillatory behaviour evident at higher frequencies. The agreement between the results obtained from slot correlation and sample-and-hold is seen to be good, for the illustrated frequencies. The frequency dependence of the integral length scale is obtained from integration of each of the graphs in figures 4 (a) and (b), these are shown for both jet configurations in figure 5 (a) and (b). Sample-and-hold estimates a lower frequency dependent length scale compared to the slot correlation for both the subsonic and supersonic jets. For the convection velocity, the frequency dependence is shown in figure 6. Both techniques reveal comparable estimates. For the subsonic case, a definite increase in velocity with increasing frequency is evident up to approximately 2kHz. The bulk convection velocities estimated using the space-time iso-contour plot from sample-and-hold and slot correlation were estimated at 142 m/s & 145 m/s respectively for the subsonic jet and at 205 m/s and 217m/s respectively for the supersonic case corresponding to frequencies of approximately 800 and 1100Hz. For the supersonic jet, the convection velocity appears to fluctuate, with some increase with frequency, about the iso-contour estimates for both techniques. Again the estimates are evident up to a frequency resolution of approximately 2.5kHz. The time scales in a moving frame of reference are shown in figure 7 for the subsonic case only as the instability of the convection velocity for the supersonic case rendered it impossible to extract the information from the results. Again, it can be seen that these are clearly a function of frequency and the integral time scales determined from these are shown in figure 8.

6. Conclusions

The turbulence statistics which provide valuable information for the understanding of noise generation in jets have been examined. The data from two-point LDV measurements have been analysed using both sample-and-hold and slot correlation to obtain the length and time scales and convection velocities. From the results, the following conclusions can be drawn.

- The auto and cross spectra estimated by both slot correlation and sample-and-hold are well estimated up to the Nyquist frequency associated with the mean data rates. Some differences have been observed but these are not considered significant. From the results presented, either method is suitable for the determination of the spectra.
- There is very good agreement for the cross correlation functions and hence the length and time scales for the subsonic jet. The agreement of the cross correlation functions for the supersonic jet is poor although the magnitude and phase of the cross spectral values are in reasonable agreement. This has resulted in poorly estimated time scales for the supersonic case.
- Despite this, differences of less than 15% are globally found between both techniques for estimations of the integral length and time scales.
- The phase of the cross spectrum has been used to determined the effect of frequency on the convection velocity. The results show an almost linear increase in the eddy convection velocity with frequency.

- A method by which the frequency dependence of the length and time scales can be determined has been outlined. When applied to the results, it has shown clearly that both the scales are a function of the frequency with smaller scales associated with the higher frequencies as might physically be expected.

7. References

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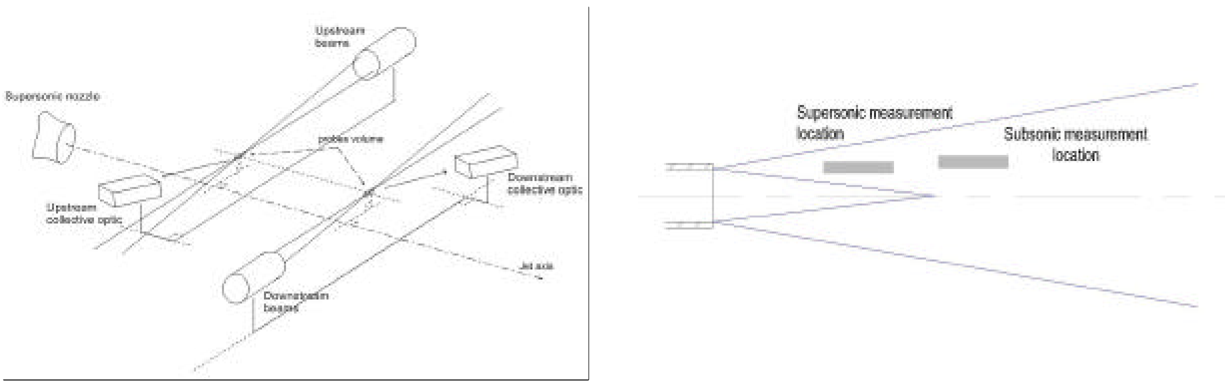
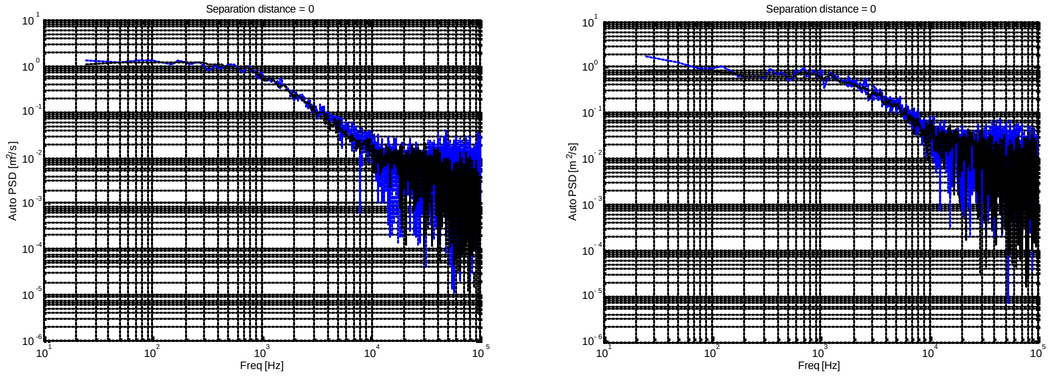
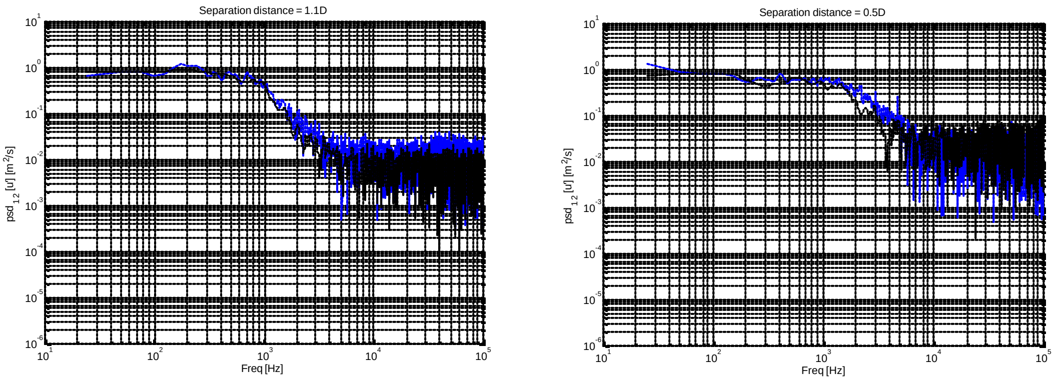


Figure 1 (a) schematic view of the optical 2-point measurements using LDV (b) Location of the measurement points into the flow in the cases of subsonic and supersonic jets

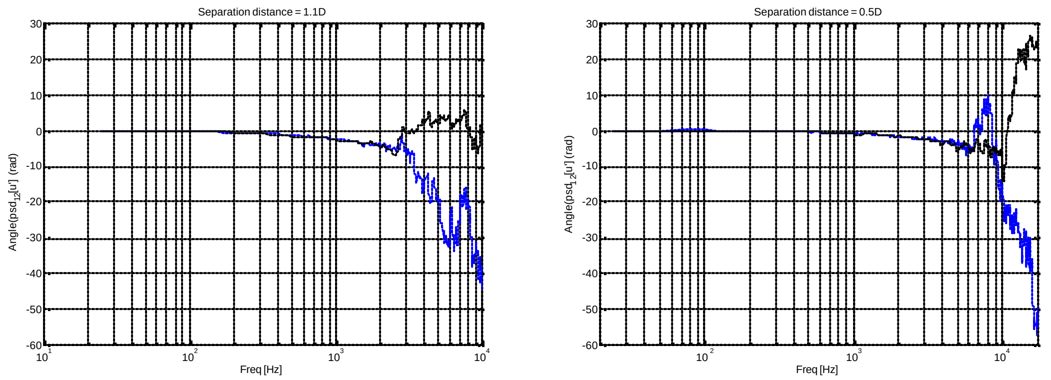
subsonic ***supersonic***



(a) Auto Spectra

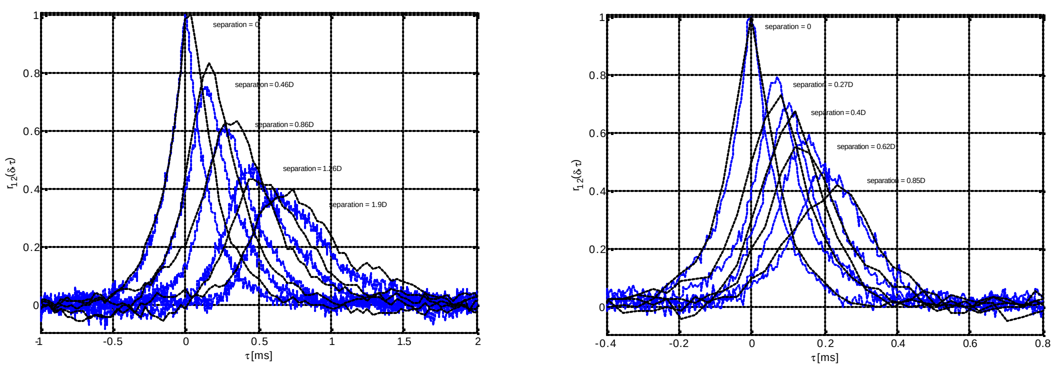


(i) magnitude



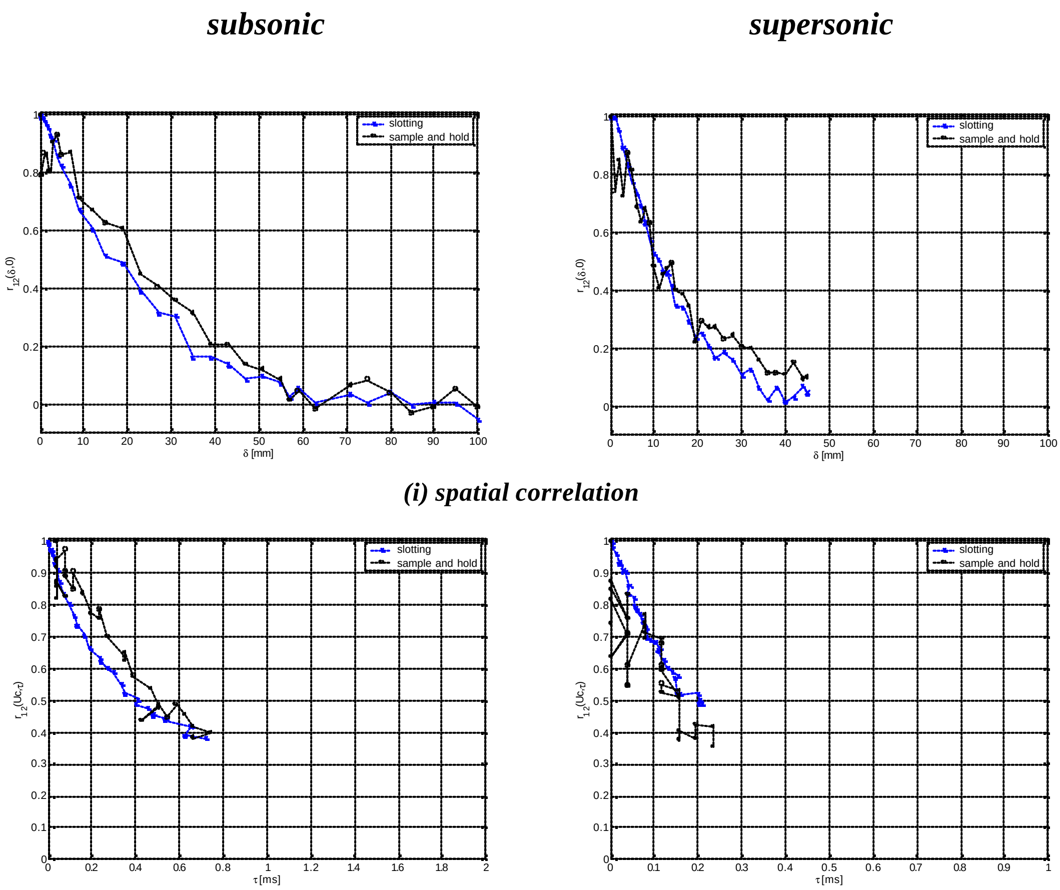
(ii) phase

(b) Cross Spectra: magnitude and phase



(c) Cross Correlation

Figure 2: ***[blue – slotting, black – Sample & Hold]***

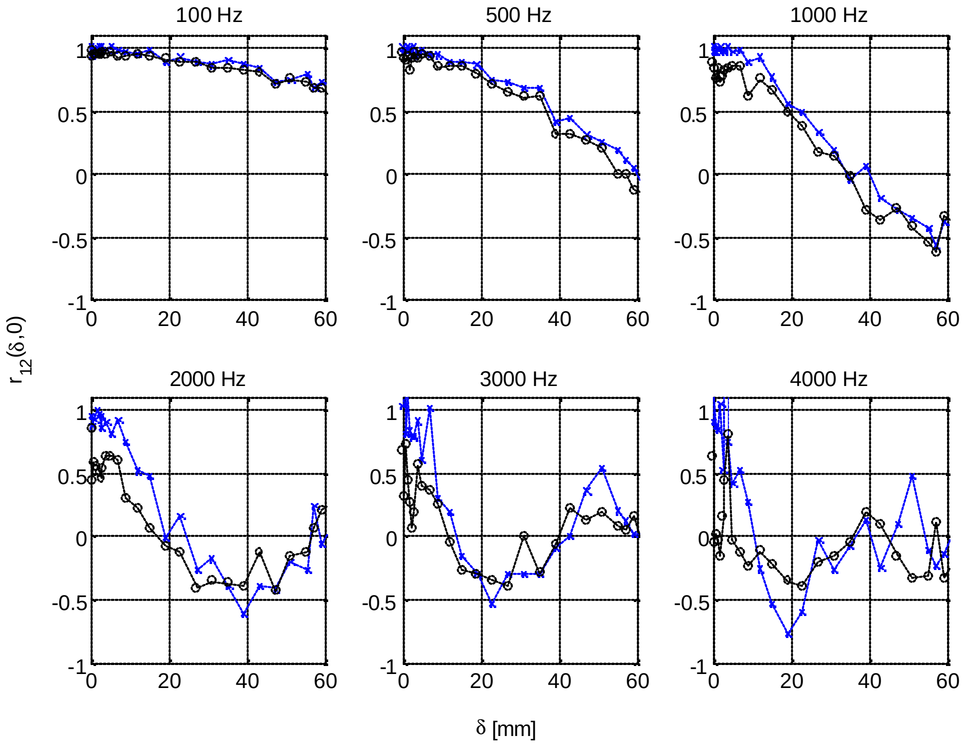


(ii) temporal correlation

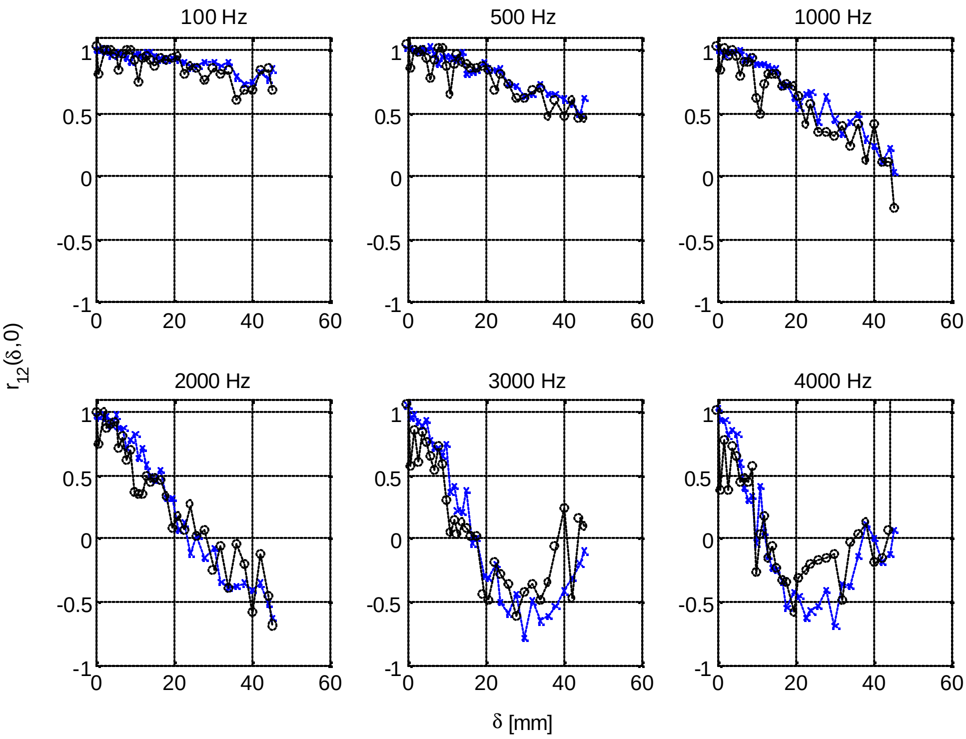
Figure 3: Spatial and temporal correlation functions

	Technique	Length scale (mm)	Temporal scale (ms)	Convection velocity (m/s)
Isothermal Mach 0.75 jet (end of the core)	S & H	56	0.77	142
	Slotting	45	0.69	145
Isothermal Mach 1.2 jet (half of the core)	S &H	38	0.31	205
	Slotting	30	0.28	217

Table 2



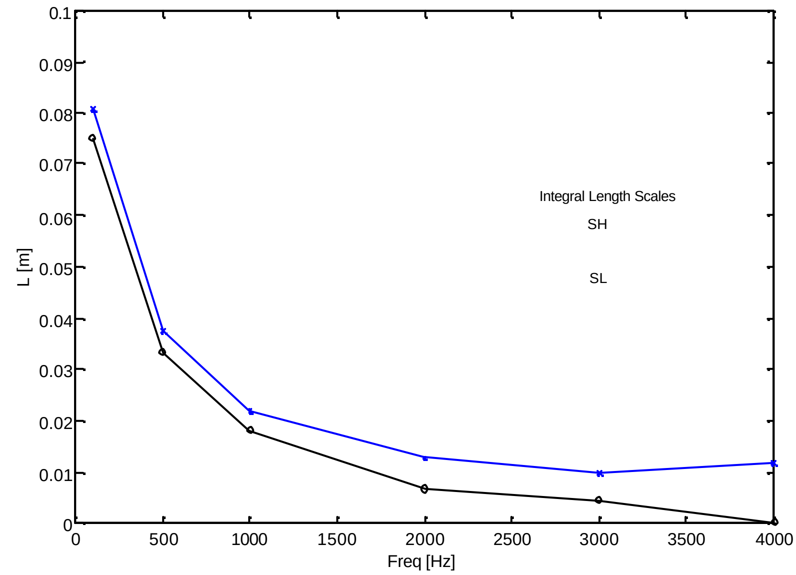
(a) subsonic



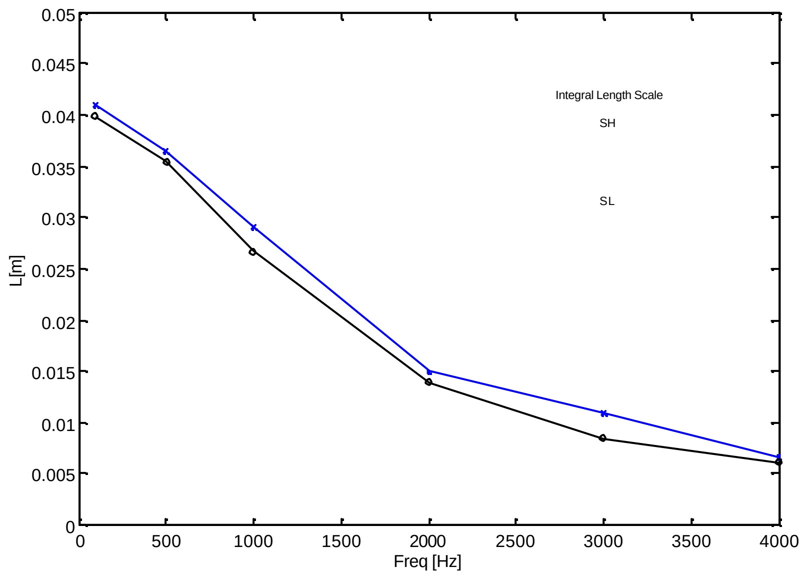
(b) supersonic

Figure 4: Frequency dependence of the length scales

[blue – slotting, black – sample and hold]

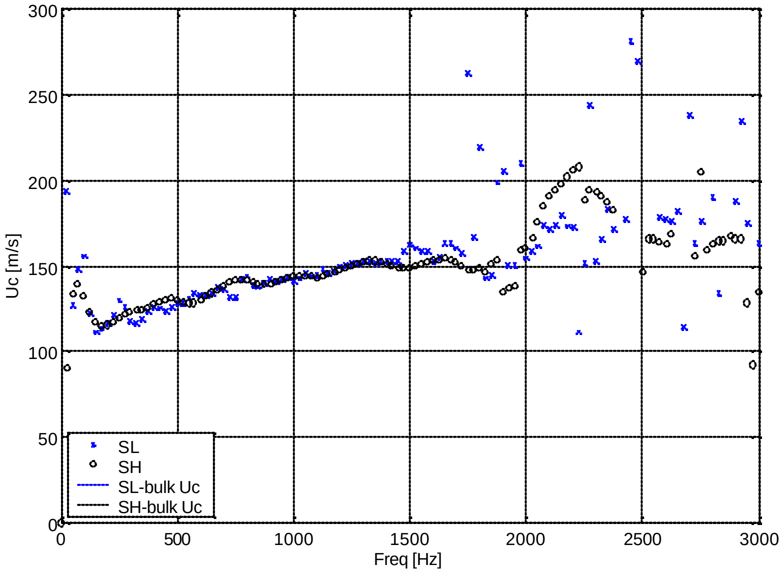


(a) subsonic

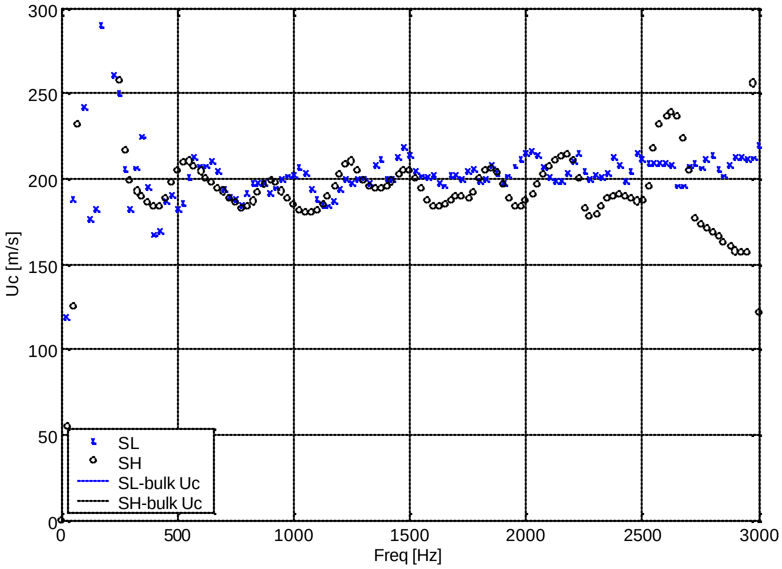


(b) supersonic

Figure 5: frequency dependence of the integral length scale
[blue – slotting, black – sample and hold]



(a) subsonic



(b) supersonic

Figure 6: Frequency dependence of convection velocity

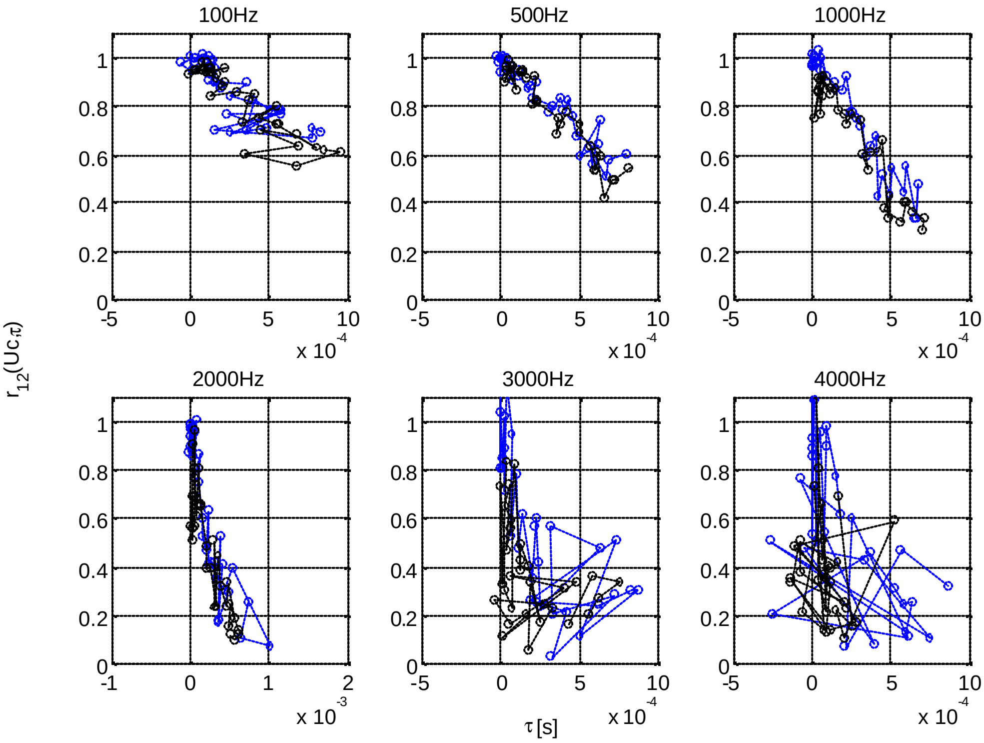


Figure 7: Time scales in a moving frame of reference for the subsonic case
[blue – slotting, black – sample and hold]

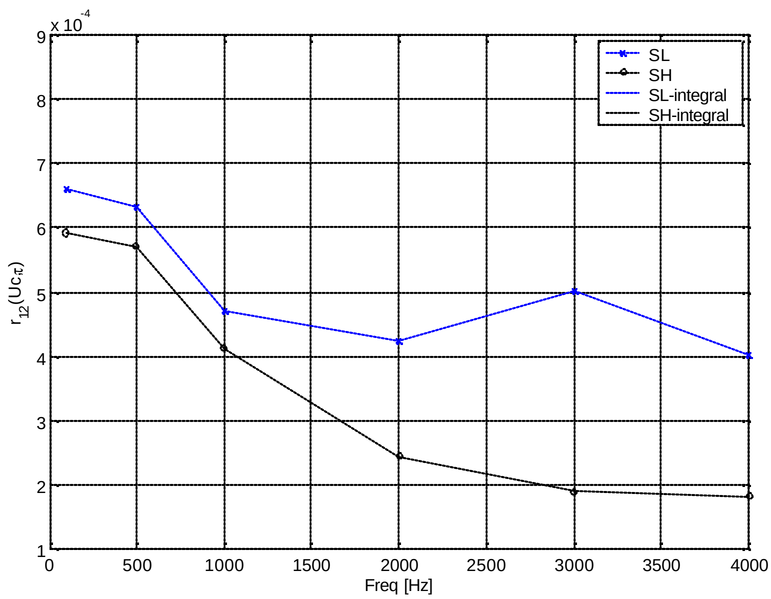


Figure 8: Frequency dependence of time scales