

# **Evaluation of acoustic velocity in mean flow by Laser Doppler Velocimetry**

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## **ABSTRACT**

Non intrusive measurement of a sinusoidal acoustic velocity in a mean movement of fluid is of interest for number of practical application. Although signal processing techniques dedicated to Laser Doppler Velocimetry have been recently developed in order to measure sinusoidal acoustic velocities without flow in a large domain of magnitude and frequency (e.g. Valière et al., 2000), these techniques are not adapted in the presence of a mean flow because of the too short bursts duration and of the presence of turbulence, viewed as an additional high magnitude noise. A new method of signal processing, called 'perio-correlation' is proposed for evaluating sinusoidal acoustic velocity in the presence of mean flow.

This short communication presents the results of an experiment conceived in collaboration between three laboratories. First LDV measurements of acoustic velocity, and microphonic estimate of this velocity are compared.

## 1. INTRODUCTION

Laser Doppler Velocimetry (LDV) is a well-known and mature technique for the experimental study of the dynamic of fluid flows. Its use in the field of acoustics is more recent (Taylor 1976, Sharped 1989, Valeau 1999, Mellet 2000), as this requires specific signal processing and optimised optics to deal with the small orders of magnitude involved in usual acoustic fields. As for an example, the loud traffic noise in a town involves acoustic velocities below 1 cm/s, and nonlinearities are likely to occur when the particle velocity reaches 1 m/s. Conversely, the acoustic phenomenon varies rapidly with time : medium audio frequencies are around 1000 Hz, a much shorter timescale than the one of most natural air flows. All microphones include an equalisation of ambient pressure, which filters out the low frequencies, and so emphasizes the acoustic part of the pressure fluctuations. Existing velocity sensors do not include such a natural filtering, and thus most of their dynamic range is required to cope with non-acoustic components, leaving few opportunities to measure small acoustic velocities. The use of LDV in acoustics has therefore been almost restricted to situations involving very weak flows (e.g. natural convection), for which the validity of the technique has been explored (Valière 2000), and specific signal processing has been developed (Valeau 1999, Mellet 2000).

There are however many situations where a significant flow interacts with the acoustic field, requiring measurements of both pressure and velocity. Among these, the acoustics inside ducts has many applications such as mufflers, air conditioning, wind musical instruments, and plane motors. In this last example, the fan is a strong noise source, which radiates in the outside space through the nacelle, but also blows a huge flow which deeply modifies noise radiation at both ends of the nacelle. Our work is part of a research program about supersonic airplanes, and aims at measuring the effect of flow inside the nacelle on fan noise propagation and radiation. Even if the sound level radiated by a plane motor is quite high, it still involves velocities which are small compared to the flow. It was therefore mandatory to develop new processing techniques to be able to use LDV during this research project. This communication presents such new processing technique, which results are compared to results of acoustic velocity estimates deduced from microphonic measurements.

## 2. EXPERIMENTAL SETUP

An experiment has been set up (see figure 1), designed so that to separately control the flow and the acoustic field, and including acoustic sensors able to give an estimate of the acoustic velocity within the duct. A muffler (zone 3) is set between the fan (zone 1) and the tranquillisation part of the duct (zone 5). The muffler is connected to others parts with convergent duct (zone 2 and 4). The acoustic source (zone 6), is made of 16 loudspeakers. A stabilisation part of the duct (zone 7) is set downstream the acoustic source and just before the measurement area (zone 8, 10 and 11) where LDV measurement and microphonic measurement are performed. Diameter of duct (173 mm) and fan features were chosen in order to reach a maximum flow at the centre of about  $80 \text{ m.s}^{-1}$ , that is a Mach number of about 0.25.

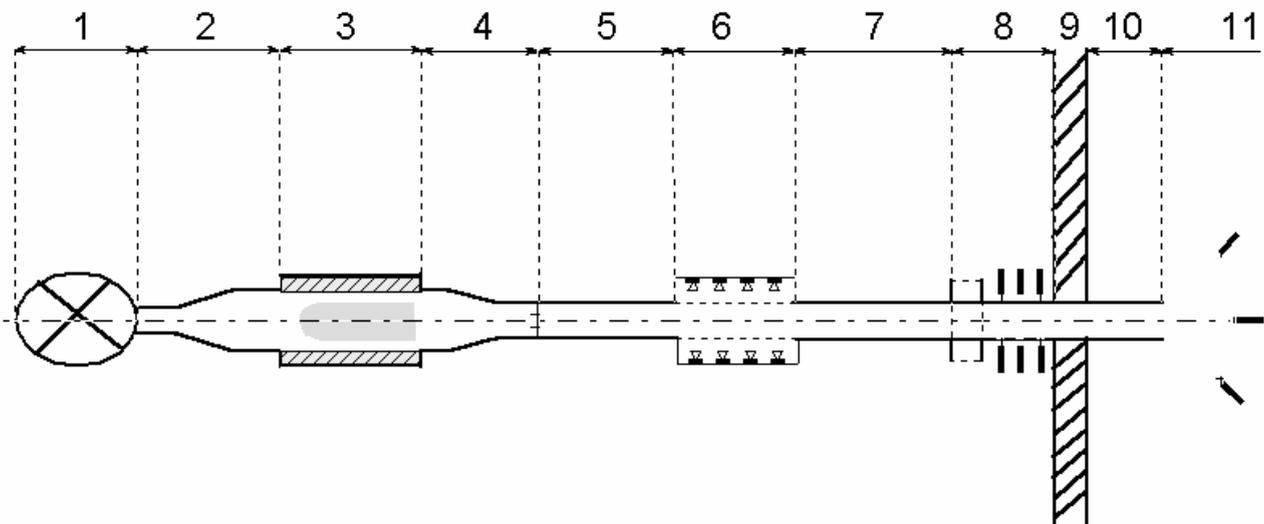


Fig. 1. Schematic drawing of experimental apparatus for acoustic velocity measurement in the presence of mean flow in a circular duct.

This design is the result of several compromises, as we had again to take into account the very different order of magnitudes of Acoustics and Fluid Mechanics quantities. An array of measurement microphones has been used to determine the mean acoustic pressure at two axial positions along the duct. The flow velocity profile was determined by a preliminary scan with Pitot and Hot Wire sensors, and was used as an input of a model that deduces the axial acoustic velocity from pressure measurements. In this model, two plane waves are considered that propagate respectively upstream and downstream, their sound speeds being modified by the measured Mach numbers associated to the flow. Although these simple assumptions may seem questionable, results are found to be consistent with other uncertainties in the measurements. The resulting estimation method has a good dynamic ranges, allowing comparison with our new method as far as the acoustic velocity is not below the LDV processing resolution, i.e. for very high sound levels. A specific sound source has therefore been designed and built, using 16 high performance loudspeakers driven by a real-time processor controlling their relative amplitudes and phases so that they build together a progressive plane wave inside the duct. This allowed to reach high acoustic pressures and velocities while keeping a linear behaviour of the electroacoustic source.

Together, the acoustic source and the microphone antenna allow to control the acoustic velocity in the measurement plane of a LDV system, in the presence of the flow generated by a powerful fan feeding the duct through a silencer. LDV measurements are then collected for increasing flow velocities, and their post-processing is compared to the estimate of the acoustic velocity obtained from the microphones.

### 3. MODEL OF THE SIGNAL

A complete model of burst has been developed by Durst (e.g. Durst et al., 1976) from optic and mechanical features. This model has been used by Sharped and Greated (e.g. Sharped 1989) in acoustic applications. A Doppler burst can be described, following Durst et al. by

$$s(t) = env(t) \times mod(t)$$

where  $env(t)$  is the envelop of the signal and  $mod(t)$  its modulation part. The envelop part can be described by

$$env(t) = A \times e^{-\beta^2 x^2(t)}$$

$x(t)$  being the position of the particle,  $A$  a constant depending on optical features of system, Laser power, etc and  $b$  a characteristic dimension of the measuring volume. The modulation term can be expressed as

$$mod(t) = M + \cos(2\pi[f_D t + Dx(t)] + \psi)$$

with  $f_D$  the Doppler frequency,  $D$  a calibration factor (which is the inverse of the distance between two fringes),  $M$  the pedestal of the signal and  $\psi$  a phase. In our application, an acoustic sine wave propagates in a moving medium supposed to be an uniform flow. Then, a particle going through the measuring volume has a velocity  $v(t)$  given by :

$$v(t) = \bar{v} + V_{ac} \cos(2\pi f_{ac} t + \varphi_{ac}) \quad (1)$$

with  $\bar{v}$  the mean flow velocity,  $V_{ac}$  the amplitude of acoustic velocity,  $f_{ac}$  the acoustic frequency and  $\varphi_{ac}$  the acoustic phase of the signal. Using this expression for the particle velocity, the particle position  $x(t)$  can be deduced by integration yielding:

$$x(t) = x_0 + \bar{v}(t - t_0) + \frac{V_{ac}}{2\pi f_{ac}} \left[ \sin(2\pi f_{ac} t + \varphi_{ac}) - \sin(2\pi f_{ac} t_0 + \varphi_{ac}) \right]$$

where  $x_0$  is a reference initial position (which is taken to be 0 in order to simplify the expression) and  $t_0$  the time corresponding to the particle being exactly at the centre of the measuring volume ( $x_0$  being 0). Introducing this equation in the model of the burst signal leads to the following equations

$$env(t) = A \exp \left[ -\beta^2 \left( \bar{v}(t - t_0) + \frac{V_{ac}}{2\pi f_{ac}} \left[ \sin(2\pi f_{ac}t + \varphi_{ac}) - \sin(2\pi f_{ac}t_0 + \varphi_{ac}) \right] \right)^2 \right]$$

and

$$mod(t) = M + \cos \left( 2\pi f_D t + 2\pi D \left( \bar{v}(t - t_0) + \frac{V_{ac}}{2\pi f_{ac}} [\sin(2\pi f_{ac}t + \varphi_{ac}) - \sin(2\pi f_{ac}t_0 + \varphi_{ac})] \right) + \psi \right)$$

#### 4. SIMPLIFICATION OF THE MODEL.

The model expressed above is uselessly complex for the evaluation of acoustic features of the signal application, because it involves two distinct phenomena with highly different orders of magnitude. The burst occurs during a time inversely proportional to particle velocity, the latter being made up of a convective and an acoustic part. The time duration of the burst is given by the dimension of the measuring volume (about 100 $\mu$ m) divided by the mean velocity of particle. In our experimental setup, an order of magnitude of mean velocity is at least 10 m.s<sup>-1</sup>, that is the burst has a maximum duration time of about 10<sup>-5</sup> s. For frequencies below 10kHz, this duration time is negligible compared to the acoustic period. Then, if the burst occurs during a very short time, the time variation of the acoustic component of the velocity during the burst can be neglected. It follows that an expansion of the acoustic part of the particle velocity (equation (1)) allows to simplify the model as :

$$s(t) = A e^{(-\beta^2 V_m^2 (t-t_0)^2)} \left[ M + \cos \left( 2\pi (f_D + DV_m)t - 2\pi DV_m t_0 + \psi \right) \right]$$

with

$$V_m = \bar{v} + V_{ac} \cos(2\pi f_{ac}t_0 + \varphi_{ac}) \quad .$$

This equation is a model of the signal delivered by a commercial processor (DSA from Aerometrics). This processor records arrival time and velocity for each burst until a defined number has been detected and validated. These data are further processed with different methods in order to evaluate acoustic features of the signal (acoustic velocity  $V_{ac}$ , acoustic frequency  $f_{ac}$  and acoustic phase  $\varphi_{ac}$ ).

#### 5. METHODS OF EVALUATION

A main difficulty when processing velocities from the LDV data set is that particles arrival times are random, generally distributed closely to a Poisson law. Specific methods, based on this assumption, have therefore been developed.

In the context of flow measurements, the techniques of post-processing generally used are based on auto-correlation function. Several techniques of estimation of auto-correlation function have been developed such as the 'Slotting technique' which is a burst delay classification technique and the 'Sample and Hold technique' which is a signal reconstruction technique.

In the case of acoustic measurements, the Doppler frequency generally varies with time because particles oscillate in the measuring volume. Then, techniques of estimation of Doppler variation have to be used in order to track acoustic velocity. They generally fail in the case of our application, because they are very sensitive to the mean flow component and because the Doppler frequency varies rapidly as the particle crosses the measuring volume during a short time. However, in the presence of a very low mean flow (about one order of magnitude below acoustic velocity), techniques of quadratic demodulation or synchronous detection have been recently adapted to acoustic measurement (LeDuff 2003).

In the following, a new technique of post-processing is proposed, based on a specific calculus of auto-correlation function, called 'perio-correlation' function. As in the case of synchronous detection, advantage is taken of the fact that we are considering a pure sine wave.

## 6. NEW METHOD OF EVALUATION

An auto-correlation function is calculated that is based on assumptions about the signal to be processed following a synchronous approach. The calculus is optimised by selecting in function of their arrival times. Practically, the 'perio-correlation' function is expressed as :

$$\mathcal{RB}_{t_i}(\tau, T) = \frac{1}{N(\tau)} \sum_{j=0}^{N(\tau)-1} \left[ V_m[t_i + n_\alpha T] - \widehat{v} \right] \times \left[ V_m[t_i + n_\alpha T + n_\beta + \tau] - \widehat{v} \right]$$

where  $V_m[t_i]$  is the velocity  $V_m$  estimated at the arrival time  $t_i$ .  $N(t)$  is the number of burst used for each calculus of values of this function.  $T$  is the analysis period and  $n_a$  and  $n_b$  are two adjustable coefficients. The computation requires an estimate of the mean velocity which is obtained by averaging velocities. Introducing the expressions of the signal model previously given leads to the following theoretical expression for the perio-correlation function :

$$\mathcal{RB}_{t_i}(\tau, T) = \frac{V_{ac}^2}{2} \cos(2\pi f_{ac}\tau) + \frac{V_{ac}^2}{2N(\tau)} \sum_{j=0}^{N(\tau)-1} \cos(2\pi f_{ac}(2t_i + 2n_\alpha T + n_\beta T + \tau) + 2\varphi_{ac})$$

This expression is of interest because its first term, that includes acoustic velocity and frequency, is independent of arrival time and analysis period. The second term of this expression for the perio-correlation function is equal to a residual term that depends on arrival time  $t_i$ . In order to minimise the importance of this term, the computed function is chosen to be :

$$\widetilde{\mathcal{RB}}_{t_i}(\tau, T) = \mathcal{RB}_{t_i}(\tau, T) + \mathcal{RB}_{t_i+T/4}(\tau, T)$$

Indeed, calculating the theoretical expression of this function leads to

$$\widetilde{\mathcal{RB}}_{t_i}(\tau, T) = V_{ac}^2 \cos(2\pi f_{ac}\tau) + \sin(\epsilon) \times \frac{V_{ac}^2}{N(\tau)} \times \sum_{j=0}^{N(\tau)-1} Res(f_{ac}, \varphi_{ac}, t_i, T, n_\alpha, n_\beta, \tau)$$

where  $\epsilon = \pi/2 \times (1 - f_{ac}T)$  converges on zero when the analysis period is chosen close enough to the acoustic period. For a large number of bursts, if arrival times are randomly distributed, the residual term can be neglected. Then this perio-correlation function presents a cosine part which amplitude is the square of acoustic velocity amplitude and which frequency is exactly the acoustic frequency. The residual term is overevaluated by a sine part which converges on zero and a sum of random cosine terms which is also averaged to zero.

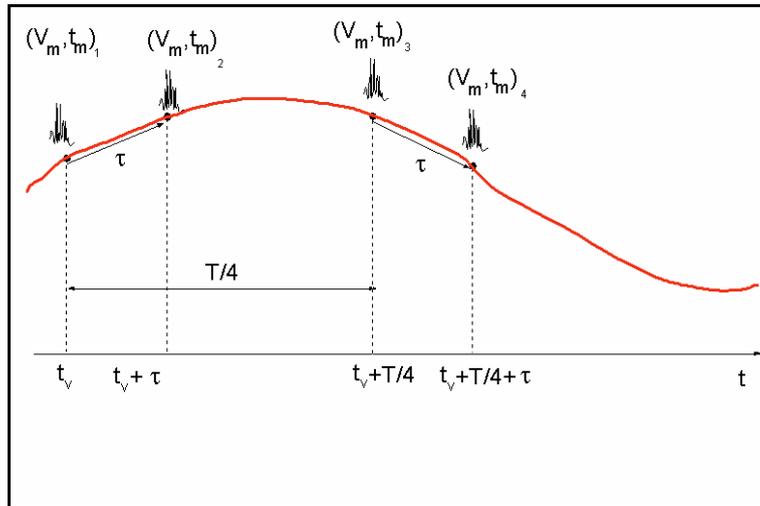


Fig. 2. Schematic diagram of conditions on arrival time of bursts used in perio-correlation.

In practice, as shown by the schematic diagram in figure 2, arrival times are first scanned before processing the velocities. A couple of bursts at arrival time  $t_0$  and  $t_0+t$ , is processed only if there exist bursts at  $t_0+T/4$  and  $t_0+T/4+t$ . These conditions can be somewhat relaxed by adjusting the integer coefficient  $n_a$  and  $n_b$  which allow to shift one or two of those four bursts by an integer multiple of the analysis period.

Acoustic velocity amplitude can then be evaluated by a Fourier transform of the perio-correlation, or by direct synchronous detection of this function.

## 7. FIRST RESULTS

Experiment is made for a frequency (700Hz) below the first cut-off frequency (about 1100Hz) in order to be ensure a uniform acoustic velocity profile. The measurement by the two microphones technique allows the calculation of the pressure and the acoustic velocity at any point of the tube (e.g. Dalmont 2001), using a classical model of propagation of plane waves in a duct, taking into account absorption and convection effects. The flow provided by the fan has a peak velocity (at the core of the duct) of about  $20\text{m}\cdot\text{s}^{-1}$ .

LDV experiments were driven for different measurement points distributed along the whole glass part of the duct. The reference point for velocity measures is about 0.667m away from the microphone planes. The velocity profile of the mean flow has been measured and the mean velocity of the flow is estimated to be  $\bar{M} = 0.76M$  where  $M$  is the peak Mach number. This is the value which is used in the calculation of the wave numbers  $k_{00}^+ = \frac{k_{00}}{1+M}$  and  $k_{00}^- = \frac{k_{00}}{1-M}$  used for the computation of the acoustic velocity from pressure measurements.

Result are presented on figure 3. The estimate by the two microphone technique is plotted in blue line whereas measurement by LDV device and estimate by perio-correlation are shown in red. Results are qualitatively similar considering the assumptions and uncertainties involved.. Values obtained by perio-correlation technique seems to be slightly overestimated. This might be explained by the fact that the absolute accuracy of the pressure measurements is not better than +1dB (that is +12%). On the other hand the uncertainty on the velocity obtained by perio-correlation. ). Further measurements are thus needed to better evaluate the accuracy of the perio-correlation method.

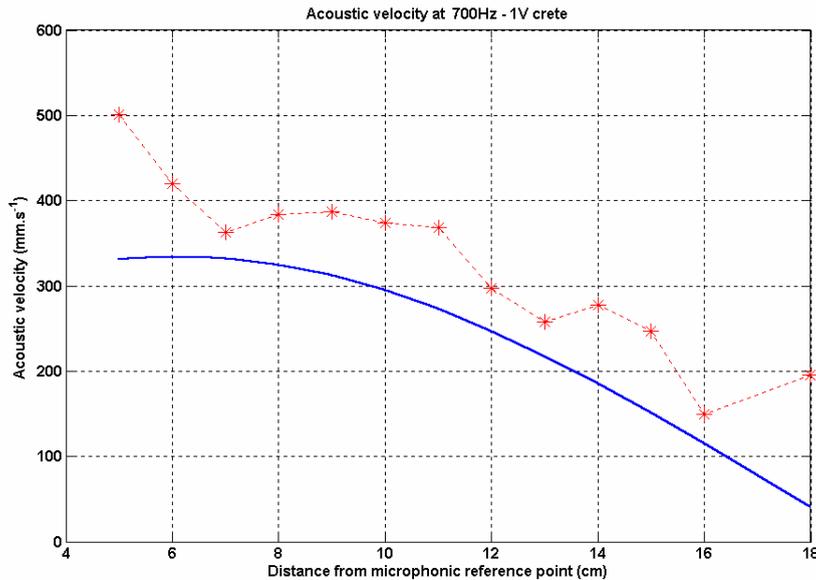


Fig. 3. Comparison between LDV measurement of acoustic velocity (red) and estimation by the two microphone technique (blue).

## 8. CONCLUSION

A new technique has been proposed for post-processing LDV signals in the case of acoustic velocity measurement in the presence of a strong mean flow, based on the calculus of a 'perio-correlation' function. This function uses an estimate of auto-correlation function with conditions on arrival times of each burst in order to optimize the calculus, using an analysis frequency which must be chosen close to the actual acoustic frequency. A first comparison between microphonic and LDV measurements is presented, showing a good potential for this technique. Further analysis is in progress in order to determine its strong and weak points, and quantify its performances (bias, standard deviation, etc...). Comparison of results obtained with this new technique and those given by other techniques is in progress, and should highlight the respective advantages of each technique.

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## REFERENCES

- Dalmont, J.-P. (2001). "Acoustic impedance measurements Part I: a review", *Journal of Sound and Vibration*, 243(3), pp 427-439.
- Le Duff, A. (2003). "Contribution à l'estimation paramétrique de signaux à variation sinusoïdale de la fréquence instantanée et à amplitude variable : application à l'anémométrie Laser Doppler pour l'acoustique", PhD Thesis. Université du Maine.
- Mellet, C. (2000). "Estimation paramétrique de signaux à phase sinusoïdale : application à la vélocimétrie Laser à effet Doppler pour l'acoustique", PhD Thesis. Université du Maine.
- Minotti, A., Simon, F., Piet, J.-F. and Millan P. (2003). "In-Flow Acoustic Power and Intensity Fields Measurements with a 2D LDV System", AIAA/CEAS Aeroacoustic Conference AIAA 2003-3262, Hilton Head, 12-14 May.
- Sharp, J.P. and Greated, C.A. (1989). "A stochastic model for photon correlation measurements in sound field.", *J. Phys. D: Appl. Phys.*, vol 22, pp 1429-1433.
- Taylor, K.J. (1976). "Absolute measurement of acoustic particle velocity", *Journal of Acoustical Society of America*, 59, pp.91-694.
- Valeau, V. (1999). "Mesure de la vitesse particulaire acoustique par anémométrie Laser Doppler : estimation de fréquence instantanée à variation sinusoïdale, validation de la mesure", PhD Thesis. Université du Maine.
- Valière, J.-C., Herzog Ph., Valeau V. and Tournois G (2000). "Acoustic velocity measurement in the air by means of laser Doppler Velocimetry : dynamic and frequency range limitations and signal processing improvements", *Journal of Sound and Vibration*, 229(3), pp. 607-626.