Applications of LDV with conditional seeding in turbulent jets with variable density

by

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ABSTRACT

Pulsed laser Mie scattering and Laser Doppler Velocimetry, both conditioned on the origin of the seed particles, were successively performed in the near development field ($x/d<20$) of turbulent jets with variable density. The instantaneous radial profiles of the mixture fraction derived from Mie scattering are discontinuous and contain sharp peaks and valleys whereas the average profiles are continuous functions like in a laminar flow. The ensemble average Doppler velocities $<V_{LDV}>_1$ obtained when only the jet fluid was seeded are higher than the velocities $<V_{LDV}>_2$ obtained when only the ambient air was seeded, especially for the radial component. Analysis of the marker statistics provides the expressions for these two conditional velocities and shows that their difference is an approximated measure of the turbulent transport of the mixture fraction.

When both channels are seeded the unconditional Doppler velocity $<V_{LDV}>_{1+2}$ depends on the ratio of the seeding rates of channels 1 and 2, and one can ask what should be the seeding ratio for which $<V_{LDV}>_{1+2}$ represents the actual average molecular velocity of the turbulent flow ? In a turbulent flow with strong intermittency where the fluid is not yet well molecularly mixed the mean velocity is defined in terms of ensemble Favre average and this actual mean velocity of the molecules is properly represented by $<V_{LDV}>_{1+2}$ if the ratio of the seed concentrations in the initial channels is set equal to the ratio of the initial fluid densities. Furthermore this actual mean velocity can be restored from the weighted conditional LDV data.

In the course of this investigation it appeared that the form of the relationships between macroscopic molecular properties in a laminar flow with constant molecular weight is the same as the form of the relations between the counterpart Favre average properties in a turbulent mixing flow such as they are portrayed by discrete particles. Provided that specific volumes are used instead of densities, the correspondence above is a homomorphism with ensemble Favre averaging operator ($A \rightarrow <\rho A>/<\rho>$) for any unconditional properties, and conditional Favre averaging ($A_k \rightarrow <\rho_k A_k>/<\rho_k>$ with $k=1$ or $k=2$) for any conditional properties. Then the gradient-diffusion law for a laminar mixing flow of identical molecules, $Z(1-Z)(V_1-V_2)= -D_L\nabla Z$, translates into a counterpart law in turbulent flows where Z is mapped by $<\rho Z>/<\rho>$, V_1-V_2 is mapped by $<V_{LDV}>_1-<V_{LDV}>_2$, and D_L is mapped by D_T. In addition to the turbulent transport of Z, this general form accounts also for the local action of mixing.

1. INTRODUCTION

Turbulent jets are involved in many practical systems because of their ability to provide high mixing rates in simple configurations. Investigations of the scalar and dynamic properties in the near field of turbulent jets are useful as important interacting processes such as mixing, entrainment, recirculations and combustion are initiated and conditioned in the early stages of the jet development. In the near field of turbulent jets where sharp gradients and fluctuations are expected, measurements of the scalar and velocity fields must be performed with high spatial, temporal and dynamic resolutions using non intrusive techniques.

In jets the scalar field is characterized by the mixture fraction; Z(x,y,z,t) is defined (Bilger, 1976) over a local control volume as the ratio of the mass of fluid originating from the injection stream to the total mass of fluid, $Z(x,y,z,t)=\rho_1(x,y,z,t) / \rho(x,y,z,t)$. At first sight Z appears to be a simple local scalar property, but actually the definition above is functional (Pratt, 1976) as conditioning the fluid on its origin involves implicit Lagrangian aspects linked to the history experienced by the jet fluid before it reaches the probe volume. The global state of the mixture over a local probe volume is quantified by Z but locally the action of mixing implicates different velocities of the fluid elements according to their origin. For simple example of this implication, in laminar flows the action of mixing comes out in a diffusion flux

$$J = \rho\ Z\ (1-Z)\ (V_1-V_2) \qquad (1)$$

where V_1 is the local velocity of the fluid originating from the jet, V_2 is that of the fluid issuing from the ambient, and the global (unconditional) flow velocity is defined as the mass weighted composition by

$$V = Z\ V_1 + (1-Z)\ V_2 \qquad (2)$$

In the near development field of turbulent jets where sharp gradients are expected, measurement of the scalar and velocity fields must be performed with high spatial, temporal and dynamic resolutions. The mixture fraction was measured by pulsed laser Mie scattering and the velocity was measured by LDV. In each technique the laser light is scattered by seed particles that are added either to the jet fluid or to the ambient air in a slow coflow. Particular attention must be brought to the seeding conditions and to the statistical differences (Labacci et al, 1988) that may result from unequal density of seed particle in the mixing flow especially for LDV where the sampling is not random as the acquisition of a velocity data is determined by the passage of a single particles in the probe volume. The mean local velocity provided by LDV is an ensemble average velocity, but this ensemble average needs to be specified as the frequency of the Doppler bursts is linked to the local concentration of seed particle originating from the seed channel.

LDV measurements in the near development field of turbulent jets have shown (Dibble et al, 1987) that the ensemble average velocities $\langle V_{LDV}\rangle_1$ obtained when only the jet fluid was seeded are higher than the velocities $\langle V_{LDV}\rangle_2$ obtained when only the ambient air was seeded, especially for the radial components. Expressions for these two conditional velocities can be derived from detailed analysis of the marker statistics (Stepowski and Sautet, 2003). They suggest that the local difference, $\langle V_{LDV}\rangle_1 - \langle V_{LDV}\rangle_2$, in the mean conditional velocities is a measure of the action of turbulent mixing within the probe volume as well as, in laminar flows, molecular diffusion is proportional to $V_1 - V_2$ (Eq. 1).

Now when both the jet fluid and the ambient air are seeded with particles, what are the seeding conditions for which the ensemble average laser Doppler velocity $\langle V_{LDV}\rangle_{1+2}$ represents the actual average molecular velocity of the turbulent flow? and is it possible to restore this mean flow velocity from the conditional Doppler velocities $\langle V_{LDV}\rangle_1$ and $\langle V_{LDV}\rangle_2$ that were measured separately ? To answer these questions it is necessary to clarify the definition of the local mean flow velocity in a turbulent flow with strong intermittency where the fluid is not yet well molecularly mixed.

2. EXPERIMENTAL ARRANGEMENT

2.1 Turbulent jets and seeding

The experimental setup schematized in Fig. 1 has been described in previous papers (Sautet and Stepowski 1994, 1995). It consists of a vertical pipe (d = 10 mm, L = 1 m) fed with various mixtures of hydrogen and nitrogen (bulk density $\rho^{(1)}$) flowing in a regime of fully developed turbulence. The jet exit velocity $V_x^{(1)}$ is fixed for all experiments at 45 ms^{-1} with a turbulence level of 4.5% . Several jet mixtures were used so as to vary the initial density ratio $R_\rho = \rho^{(1)}/\rho^{(2)}$ relative to the ambient from 0.07 up to 1 while the Reynolds number varied from 4100 up to 27000.

The jets are discharging into a slow coflow air stream ($V_x^{(2)}$ = 4.5 ms^{-1}) which is properly conditioned by a honeycomb section. The coflow Reynolds number is 27000 and its turbulence level is 2%. The jet and the coflow can be separately seeded at large upstream distance with nebulized silicon oil droplets (diameter ≈ 1 µm, ρ = 1.1 $g.cm^{-3}$, ν = 500 $mm^2\ s^{-1}$). Based on their Stokes number these particles are able to track accurately the fluctuations of the fluid motion up to 0.2 MHz. This seeding system is used for laser Mie scattering as well as for LDV with the same kind of particle but the seeding rate is reduced to a very low level for LDV.

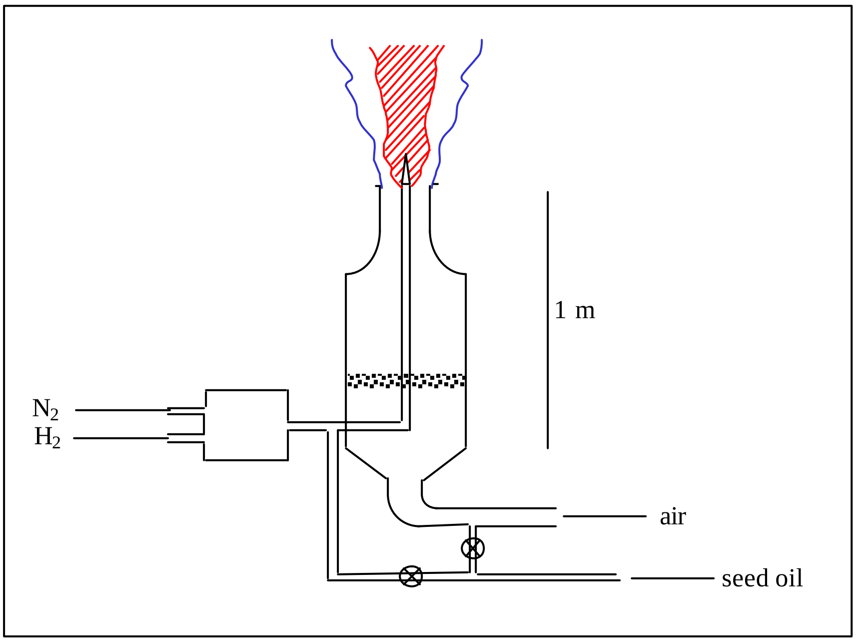

Fig. 1. Experimental setup

2.2 Laser Mie scattering

Radial profiles of the mixture fraction and density at various axial station in the turbulent jets were obtained by laser Mie scattering with conditional seeding. A pulsed YAG laser (λ = 532 nm, F = 10 Hz, Δt = 8 ns) is horizontally focused at a given station along a diameter of the jet with a beam waist of 0.2 mm. At each laser shot, the laser light scattered by the seed particles is collected at 90° and focused onto a 1 D gated intensified diode array consisting of 1024 pixels (25 µm wide each).
Part of the incident laser beam is split and focused into the potential core of the jet to induce a reference scattering signal. At each laser firing this reference is used to correct for drifts in the jet seeding rate and shot to shot fluctuations of the laser energy. For jet seeding operations it is assumed that the seeding rate does not vary during the short delay taken by the fluid to reach the measurement station. The seeding rate was adjusted to have about 10^5 particle in the probe volume (mass fraction lower than 0.5%). These conditions insure that the particles do not modify the fluid properties while keeping a low marker shot noise of about 1%.
For each station in the jet development, 700 instantaneous profiles are registered at 10 Hz. The mean profile over these realizations is an ensemble average because the medium stays frozen during each laser pulse and the repetition rate of the sampling is much lower than the turbulence frequency (random sampling).

2.3 Laser Doppler Velocimetry

LDV was performed with a dual color dual beam system. Axial and radial velocity components are measured using the green line and the blue line of an argon-ion laser. Directional ambiguity in the radial component is eliminated by frequency shifting (40 MHz on each beam pair). The four beams are focused using a 480 mm focal length lens. The spatial resolution is 500 µm (radial) by 90 µm (axial and azimuthal).
Particular attention was paid to the seeding conditions : Alternatively one of the two channels was seeded (solely the jet stream then solely the coflow) at low rate with great care to avoid residual particles in the other channel. The seeding rate was reduced to have a validation ratio better than 90% and to make sure that further reduction of the seeding rate did not improved this validation ratio. The probability of having more than one particle in the probe volume was then negligible. The ensemble average velocity data were obtained over 1500 samples.

3. SCALAR FIELD

3.1 Principle of the measurement

The laser Mie scattering technique has been intensively used (Becker, 1977 ; Ebrahimi and Kleine, 1977 ;Long et al, 1881 ; Stepowski and Cabot, 1988)) to measure density and mixture fraction in turbulent flows. As the seed particles are assumed to track the turbulent fluid motion accurately without perturbing it ($Y_p \ll 1$), with the assumption of a marker continuum (marker shot noise < 1%), the mass fraction of particle is a conserved property that can be linearly expressed as a function of the mixture fraction,

$$Y_P = Z\, Y_P^{(1)} + [1-Z]\, Y_P^{(2)} \qquad (3)$$

where $Y_P^{(i)}$ is the mass fraction of particles in pure stream i and (index 1 refers to the jet stream, index 2 refers to the coflow). The intensity of the scattered light is proportional to the local number of particles (ie to ρY_P), to the constant scattering cross section of the particles and to the laser irradiance.
When only the jet is seeded ($Y_P^{(2)} = 0$), the local signal normalized by the signal at the nozzle (where Z=1) is

$$s_1(t) = \rho(t)Z(t)/\rho^{(1)} \qquad (4)$$

When only the coflow is seeded ($Y_P^{(1)} = 0$), the local signal normalized by the outward signal (where Z=0) is

$$s_2(t) = \rho(t)[1 - Z(t)]/\rho^{(2)} \qquad (5)$$

As the two complementary conditional measurements cannot be performed simultaneously, the instantaneous profiles of ρ and Z are derived by using $1/\rho = Z/\rho^{(1)} + [1 - Z]/\rho^{(2)}$ for isothermal and isobaric flow

$$\rho(t) = [\rho^{(2)} - \rho^{(1)}]s_2(t) + \rho^{(1)} \qquad (6) \qquad \text{and} \qquad Z(t) = 1 - \frac{\rho^{(2)}}{\rho(t)} s_2(t) \qquad (7)$$

For mean values it is better to use the two mean conditional data (Equ. 4 and 5) to derive the mean density because our results show that $\langle s_1 \rangle + \langle s_2 \rangle = 1$ within 2%, which validates the assumption that on average drifts in seeding rate are negligible during the short delay taken by the jet fluid to reach the measurement station. Then,

$$\langle \rho \rangle = \rho^{(1)}\langle s_1 \rangle + \rho^{(2)}\langle s_2 \rangle \qquad (8) \qquad \text{and} \qquad \frac{\langle \rho Z \rangle}{\langle \rho \rangle} = 1 - \frac{\rho^{(2)}}{\langle \rho \rangle}\langle s_2 \rangle \qquad (9)$$

3.2 Results and discussion

Examples of instantaneous and average radial profiles of the mixture fraction at two axial stations (x/d = 0.5 and 10) are shown in Fig. 2 for a pure hydrogen jet ($\rho^{(1)}/\rho^{(2)} = 0.07$), and in Fig. 3 for a pure nitrogen jet. In both cases the radial profiles across the exit section looks like a top hat function. However in the early stages of the jet development the instantaneous profiles are discontinuous and contain sharp peaks and valleys with different widths. Since the profiles at the neck of the injector appear as a homogeneous top hat function, the strong inhomogeneities observed in the near field of the mixing flow cannot be attributed to marker shot noise nor to spatial resolution problems. The separation between the scale of these scalar inhomogeneities and the scale of the mean flow is not large, and this is generally considered (Tennekes and Lumley, 1972) as the main limitation of gradient-diffusion modeling of $\overline{Z'V'}$ in turbulent jets. These instantaneous profiles (see Fig. 2, 3) show that various mixed fluid compositions can be found in structures with different extents and that unmixed ambient fluid can be sometimes entrained deep in the jet. As also observed and commented by Dahm and Dimotakis (1987) the strong inhomogeneities in the instantaneous profiles – whereas the average profiles are continuous functions as in a laminar flow – point out an essential difference between molecularly mixed fluid at a given composition and an average over mixed and unmixed fluids yielding the same mean composition. The basic difference could be due to the dynamic effects induced by the coherent structures of a turbulent flow when mixing proceeds.

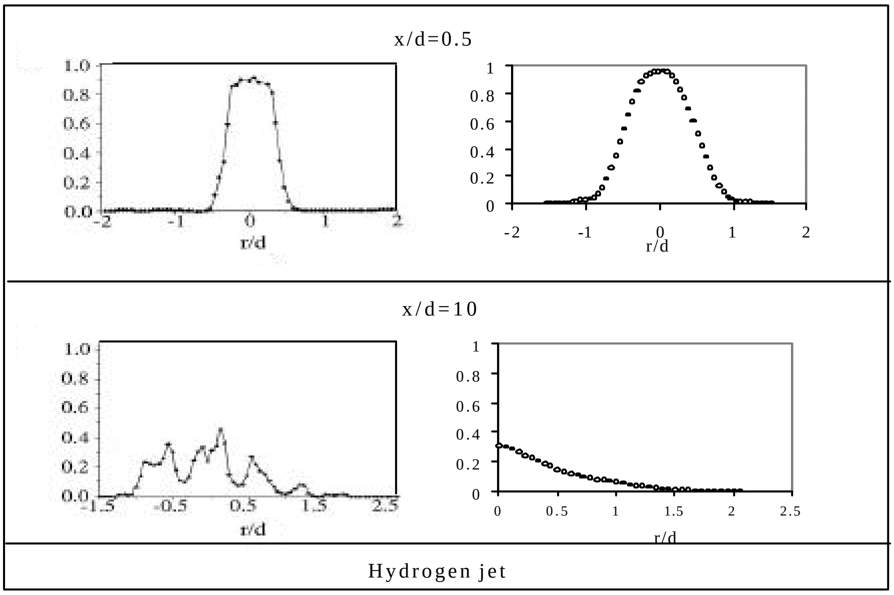

Fig. 2 .Instantaneous (left) and average (right) radial profiles of the mixture fraction for a pure hydrogen jet.

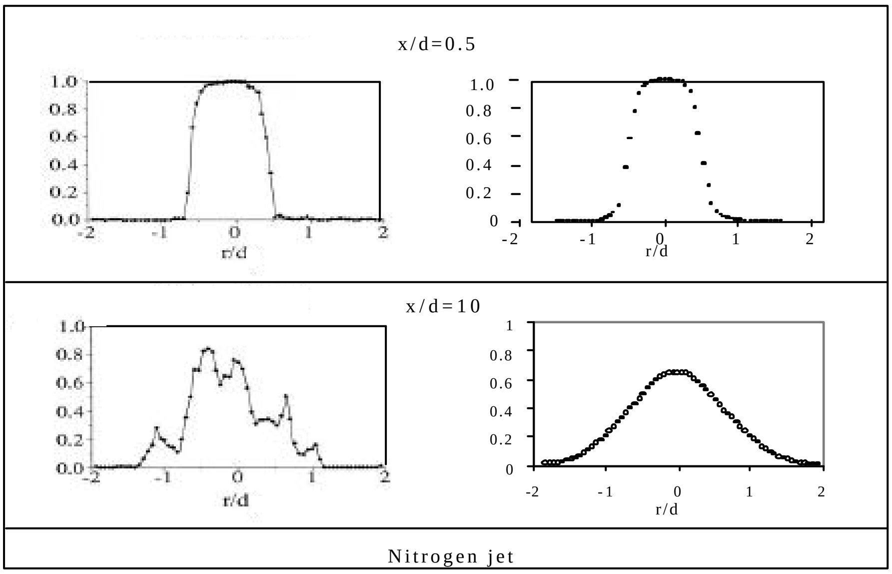

Fig. 3. Instantaneous (left) and average (right) radial profiles of the mixture fraction for a nitrogen jet.

4. LDV WITH CONDITIONAL SEEDING

LDV applies only when a single particle crosses the probe volume and provides the ensemble average velocity over these single events irrespective of the mass or the residence time of each particle in the probe volume. However this ensemble average needs to be specified as the frequency of the sampling is linked to the local concentration of seed particles that have been transported by the turbulent flow

4.1 Marker statistics in conditional LDV

On average the particles are assumed to track the turbulent motion of the fluid without perturbing it. When only the jet fluid is seeded with identical particles P_1, the local ensemble average mass of particles in the probe volume relative to the average mass of particle in the jet stream is equal to the local average mass of fluid relative to the average mass of fluid in the jet stream, then

$$\langle N_{P1}\rangle = \langle N_{P1}^{(1)}\rangle \langle \rho Z\rangle / \rho^{(1)} \qquad \text{with} \quad \langle N_{P2}^{(2)}\rangle = 0 \qquad (10)$$

This relation is valid provided that the averages are performed over large numbers of samples such that ergodicity makes it possible to consider a continuum for the ensemble average properties even if the number of particle in each sample is one or zero (Stepowski and Sautet, 2003).

When only the coflow is seeded with particles P_2, the local average number of particles is given by

$$\langle N_{P2}\rangle = \langle N_{P2}^{(2)}\rangle \langle \rho(1-Z)\rangle / \rho^{(2)} \qquad \text{with} \quad \langle N_{P1}^{(1)}\rangle = 0 \qquad (11)$$

In order to yield expression for the ensemble average velocity provided by the LDV system in each seeding case, the statistical analysis must consider probabilities for having a single particle in the probe volume. Assume first that the fluid density an motion could be continuously portrayed – apart from molecular agitation – by a high number N_{ID} of ideal particles. In the local probe volume the number of seed particles can be considered as the number N_{ID}^* of ideal particles that are marked. N_{ID}^* is then binomially distributed with P^* being the probability that an ideal particle is marked by a seed one. Then $\langle N_P\rangle = \langle N_{ID}^*\rangle = \langle N_{ID}\rangle \times P^*$ with $P^* \ll 1$. Under these conditions the statistical distribution of N_P is locally given by the Poisson law

$$P_{(N_P=n)} = \frac{\langle N_P\rangle^n}{n!} e^{-\langle N_P\rangle} \qquad (12)$$

where $P_{(N_P=n)}$ is the probability of having n particle in the probe volume, and $\langle N_P\rangle$ is given by Eq. 10 or Eq. 11 according to the seeding conditions. The next step of the analysis will consider the case of jet seeding (for co-flow seeding $N_P^{(1)}$ is replaced by $N_P^{(2)}$, $\rho^{(1)}$ is replaced by $\rho^{(2)}$ and Z is replaced by [1-Z]).

When only the jet fluid is seeded with particles P_1, for a given local value of ρZ, $\rho Z=a$, the probability of having one particle in the probe volume (Eq. 10 with $\rho Z=a$, and Eq. 12 with n=1) is given by

$$P_{(N_{P1}=1|\rho Z=a)} = a\langle N_{P1}^{(1)}\rangle / \rho^{(1)} \exp\left(-a\langle N_{P1}^{(1)}\rangle / \rho^{(1)}\right) \qquad (13)$$

Application of the total probability theorem (Bayes) over all possible local values a taken by ρZ, with weighting by the probability distribution function P(a) for the local density of jet fluid, provides

$$\left\langle V_{LDV}\right\rangle_1 = \frac{\int V_{P1}(a) P_{(N_{P_1}=1|\rho Z=a)} P(a)da}{\int P_{(N_{P_1}=1|\rho Z=a)} P(a)da} = \frac{\left\langle \rho_i Z_i V_{P_1,i} \exp\left(-\left\langle N^{(1)}_{P1}\right\rangle \rho_i Z_i / \rho^{(1)}\right)\right\rangle}{\left\langle \rho_i Z_i \exp\left(-\left\langle N^{(1)}_{P1}\right\rangle \rho_i Z_i / \rho^{(1)}\right)\right\rangle} \qquad (14)$$

where ρ_i, Z_i and V_{P1i} are the density, the mixture fraction and the particle velocity respectively in each sample i. Thus, for jet seeding, LDV may under-estimate the contribution of events i with high values of $\rho_i Z_i$ in as much as $< N^{(1)}_{P_1} >$ is high. In the worst case of high seeding rate ($< N^{(1)}_{P_1} > \;>>1$), LDV with jet seeding would give the mean velocity associated with intermittent engulfment of unmixed ambient air in the probe volume.
On the other hand, for low seeding rate this bias becomes negligible, and Eq. 14 reduces to

$$\left\langle V_{LDV}\right\rangle_1 = \frac{\left\langle \rho_i Z_i V_{P_1 i}\right\rangle}{\left\langle \rho_i Z_i\right\rangle} \qquad \text{if } < N^{(1)}_{P_1} > \;<<1 \qquad (15)$$

In the complementary experiment when only the ambient air is seeded at low rate with particles P_2, LDV gives

$$\left\langle V_{LDV}\right\rangle_2 = \frac{\left\langle \rho_i (1-Z_i) V_{P_2 i}\right\rangle}{\left\langle \rho_i (1-Z_i)\right\rangle} \qquad \text{if } \left\langle N^{(2)}_{P_2}\right\rangle <<1 \qquad (16)$$

In the next it will be assumed that seeding is always performed at low rate, such that only one particle can be found in the probe volume. Beyond its requirement for unbiased measurement of conditional velocities, this seeding condition will be essential for measurement and definition of unconditional mean velocity.
In expressions (15) and (16) the sampled velocities of the marked flow are still specified by the origin of the marker. However, notice that even if it was assumed that $V_{P1,i}$ =V and $V_{P2,i}$ =V, the two conditional mean velocities would still have different expressions.
This basic result (Expression 15 for instance) merely yields that, for jet seeding, each sampled velocity of the marked flow is weighted by the corresponding density of jet fluid $\rho_i Z_i$ as well as the mean local number of markers is proportional to the mean local density of fluid originating from the jet stream, $<\rho Z>$. In short hand, this operation can be called "ensemble conditional Favre averaging".

4.2 Results and discussion

Figure 4 shows examples of radial profiles of the ensemble average conditional LDV data for the axial component at stations x = 1.5d, x = 5d, x = 15d in the developments of nitrogen and hydrogen jets. The solid lines indicate data collected when seed particles were added only to the jet fluid (subscript 1), the dotted lines indicate data collected when seed particles were added only to the ambient air (subscript 2). On the centerline the difference between the conditional axial velocities is negligible as was also observed by Dibble et al (1987). At a given station x, with increasing radius $<V_{x,P}>_1 - <V_{x,P}>_2$ increases significantly in the hydrogen jet and reaches a maximum at radius $r_M(x)$, whereas it keeps a weak value in the nitrogen jet. With increasing distances above the nozzle in the hydrogen jet the maximum difference between the conditional velocities decays from ΔV_x=12m/s (at $r_M \approx$0.25d) at station x=5d down to ΔV_x=1.4m/s (at $r_M \approx$1.7d) at station x=15d. Notice that different velocity scales were used to plot the profiles in the two jets because the centerline decay of the axial velocity is faster in the hydrogen jet. The profile at 5d in the hydrogen jet looks like the profile at 15d in the nitrogen jet but the difference in conditional velocities is still lower in the nitrogen jet.

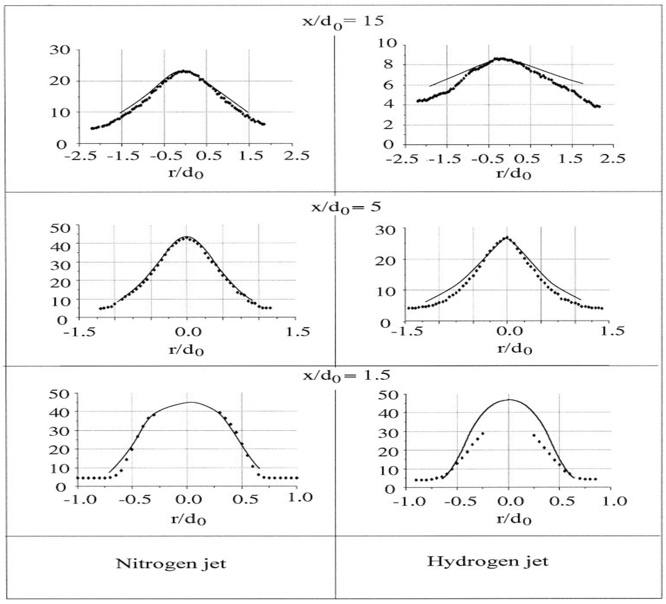

Fig. 4. Radial profiles of the axial component of LDV (in m/s). Solid lines for jet seeding, dots for coflow seeding

Figure 5 shows the profiles of conditional radial velocities in the same jets. Noticeable differences between the conditional velocities are observed in both jets although the difference is still lower (factor 0.5) in the nitrogen jet. The absolute magnitude of the difference in conditional radial velocities is rather lower than for the axial component, but observation of the relative radial profiles reveals striking features : In the earlier stages of the turbulent mixing layer for both jets $<V_{r,P}>_1$ is centrifugal whereas $<V_{r,P}>_2$ is centripetal ! At downstream stations both conditional velocities are centrifugal but $<V_{r,P}>_1$ is still higher than $<V_{r,P}>_2$. The maximum difference in conditional radial velocities decays with increasing distances from the nozzle. In the hydrogen jet at station x=1.5d the maximum velocity difference ΔV_r is 5 m/s (at $r_M \approx 0.25d$), at station 5d ΔV_r =1.4 m/s (at $r_M \approx$ 0.5d), at station 15d ΔV_r=0.5 m/s (at $r_M \approx$ 1.5d). Thus for the hydrogen jet the relative axial decay of the maximum difference between the conditional radial velocities is practically the same as for the axial components, and the radius of maximum difference increases with increasing axial stations as the jet develops.

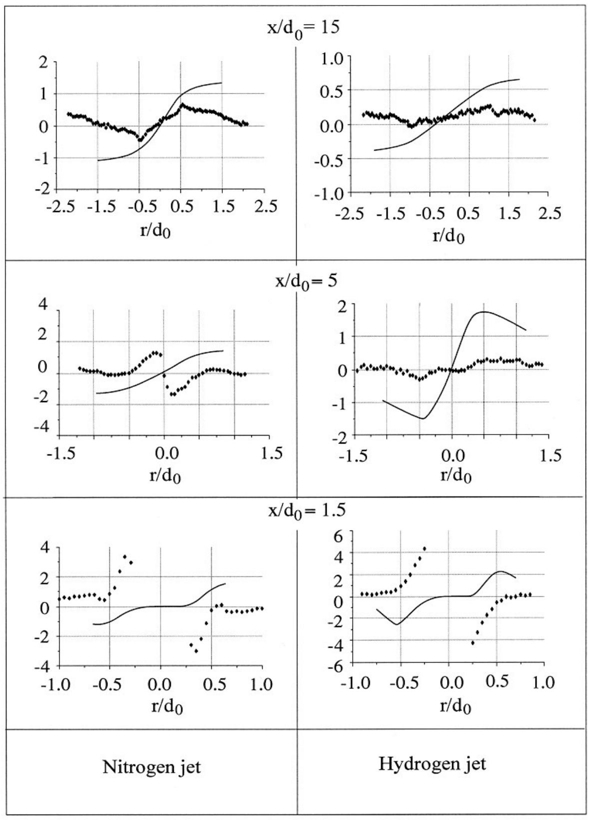

Fig.5. Radial profiles of the radial component of LDV(in m/s). Solid lines for jet seeding, dots for coflow seeding

These observations suggest that the local difference between the velocity data conditioned on the origin of the marker contains information on the dynamic of turbulent mixing within the probe volume:
With the quasi steady state approximation where it is assumed that on average at each sampling the marker velocity has reached the velocity of the local molecular mixture ($V_{p1i}=V_i$ in Eq. 15 and $V_{p2i}=V_i$ in Eq. 16), the local differences in conditional LDV data reduces to

$$\langle V_{LDV}\rangle_1 - \langle V_{LDV}\rangle_2 \approx \overline{\overline{Z''V''}} \Big/ \overline{\overline{Z}}(1-\overline{\overline{Z}}) \qquad \text{with} \quad \overline{\overline{A}} = \overline{rA}\big/\overline{r} \quad \text{and} \quad A'' = A - \overline{\overline{A}} \qquad (17)$$

The correlation term called turbulent transport of the mixture fraction is usually modeled by gradient-diffusion

$$\overline{\overline{Z'V''}} \approx -D_{TT}\nabla\overline{\overline{Z}} \qquad (18)$$

where it is expected that Favre averaging will properly account for density fluctuations.

5. UNCONDITIONAL VELOCITIES

As the ensemble average velocities provided by LDV when only the jet fluid was seeded are different from those obtained when only the ambient air was seeded it can be thought that both channels must be seeded to obtain the relevant – unconditional – average velocity of the mixing flow.

5.1 LDV with seeding of the two channels

Measurement of the unconditional flow velocity can be performed by seeding the jet fluid with particles P_1 and seeding the ambient air with particle P_2 . It is essential that both the jet fluid and the co-flow air be seeded at low rate. Then the LDV data accounted for in the ensemble average provided by the system correspond either to the passage of a single particle P_1 issuing from the jet fluid or to the passage of a single particle P_2 issuing from the ambient. Owing to this simple alternative, the ensemble average velocity provided by LDV is

$$\langle V_{LDV}\rangle_{1+2} = P\langle V_{LDV}\rangle_1 + (1-P)\langle V_{LDV}\rangle_2 \qquad (19)$$

where P is the local probability that the probed particle comes from the injection channel 1. By using expressions (10) and (11) for the average numbers of particles, this probability can be written

$$P = \langle N_{P1}\rangle \big/ \left[\langle N_{P1}\rangle + \langle N_{P2}\rangle\right] = \left[\langle r Z\rangle / \langle r\rangle\right] \big/ \left[a + \left(\langle r Z\rangle / \langle r\rangle\right)(1-a)\right] \qquad (20) \qquad \text{where } a = \langle N_{P2}^{(2)}\rangle r^{(1)} \big/ \langle N_{P1}^{(1)}\rangle r^{(2)} .$$

Thus $\langle V_{LDV}\rangle_{1+2}$ depends on the ratio of the seeding rates in the two channels relative to $R_\rho=\rho^{(1)}/\rho^{(2)}$.

What should be the seeding ratio for which $\langle V_{LDV}\rangle_{1+2}$ *represents the unconditional ensemble average molecular velocity of the turbulent flow ?*

This question raises the problem of defining the mean flow velocity in a the early stages of a turbulent jet with strong intermittency where the fluid is not yet molecularly mixed.

5.2 Definition of the unconditional velocity in mixing flows

In flows mixing proceeds by mass and momentum transfers with global conservation of these transferable properties during the interaction. Momentum must be considered as a fundamental variable and velocity can be defined only as a derived variable by the ratio of the momentum to the involved mass. The definition of conditional flow velocities are then derived from application of the local continuity equation to the conservation of the mass of the fluid elements originating from the specified channel whereas the unconditional velocity is derived from application of the continuity equation for the global mass of fluid.

5.2.1 Laminar flows

In laminar flows of perfect gas where the fluid elements are single molecules, the macroscopic variables are defined by ensemble averaging of the mass and momentum of the involved elements over a huge number of samples. This can be achieved either by averaging over the huge number of molecules present in a local macroscopic control volume at a given time, or by averaging over a macroscopic temporal window the huge number of instantaneous samples for which each time only one or zero molecule is present in an infinitely small local control volume. Both approaches gives the same result (ergodicity) but the second one is similar to the averaging performed by a LDV system. Owing to the huge number of molecules involved in the averaging, the macroscopic variables are always defined and continuous, allowing for application of the continuity equation.
The local continuity equation for the mass of jet fluid is

$$\frac{\partial r Z}{\partial t} + \nabla r Z V_1 = 0 \qquad (21)$$

The continuity equation for the mass of fluid originating from the ambient is

$$\frac{\partial r(1-Z)}{\partial t} + \nabla r(1-Z)V_2 = 0 \qquad (22)$$

The continuity equation for the global mass of fluid is

$$\partial r / \partial t + \nabla r V = 0 \qquad (23)$$

Identification of (20) + (21) with (22) gives the relation between unconditional and conditional velocities

$$V = ZV_1 + (1\text{-}Z)\ V_2 \qquad (2)$$

Developing the first term in Eq. 20 and using (22) and (2), we obtain

$$r\frac{\partial Z}{\partial t} + rV\nabla Z + \nabla r Z(1-Z)(V_1 - V_2) = 0 \qquad (24)$$

5.2.2 Turbulent flows

Turbulent flows involve the motion and collisions of groups of molecules rather than only those of individual molecules as in laminar flows. The tenuous nature of this grouping is still a matter of discussions: The Lagrangian concept of lumps of fluid over which pressure, velocity and composition would be practically uniform is somewhat ambiguous as the spatial extent of such laminar elements can be defined only on average with respect to the local dissipation scale. According to Pope (2000), the consistency between the random nature of turbulent flows and the deterministic nature of the Navier-Stokes equations is due to the extreme sensitivity of these equations to the slightest variation in the initial conditions; beyond a given spatio-temporal domain, the flow properties can no longer be predicted and the molecules have to be regrouped over such domains where their macroscopic scalar and vector properties are continuously predictable and defined. Then, at a given instant the continuity equations would be valid only over a small spatial domain whereas at a given location they would be valid only during a short delay. Nevertheless local average can be performed over a random series of samples where during each brief sampling the probed fluid satisfies the local continuity equations. Random sampling and ensemble averaging provides the unconditional mean density

$$\left\langle r \right\rangle = \left\langle r_1 \right\rangle + \left\langle r_2 \right\rangle \qquad (25)$$

Rather than average Eq.2, it is meaningful to average ρV=ρ$_1$V$_1$+ρ$_2$V$_2$ to obtain the Favre average global velocity

$$\frac{\left\langle r\mathrm{V} \right\rangle}{\left\langle r \right\rangle} = \frac{\left\langle r\mathrm{Z} \right\rangle}{\left\langle r \right\rangle} \times \frac{\left\langle r\mathrm{ZV}_1 \right\rangle}{\left\langle r\mathrm{Z} \right\rangle} + \left[1 - \frac{\left\langle r\mathrm{Z} \right\rangle}{\left\langle r \right\rangle}\right] \times \frac{\left\langle r(1-\mathrm{Z})\mathrm{V}_2 \right\rangle}{\left\langle r(1-\mathrm{Z}) \right\rangle} \qquad (26)$$

This definition is to be compared with the ensemble average velocity (Eq. 19) for unconditional LDV

$$\left\langle \mathrm{V}_{\mathrm{LDV}} \right\rangle_{1+2} = \mathrm{P}\ \frac{\left\langle r\mathrm{ZV}_{\mathrm{P1}} \right\rangle}{\left\langle r\mathrm{Z} \right\rangle} + [1-\mathrm{P}] \frac{\left\langle r(1-\mathrm{Z})\mathrm{V}_{\mathrm{P2}} \right\rangle}{\left\langle r(1-\mathrm{Z}) \right\rangle} \qquad (27)$$

5.3 Seeding conditions for equitable marking

We can now answer our question:

In a turbulent jet the ensemble average velocity of the marked flow provided by LDV when both channels are seeded represents the unconditional ensemble average molecular velocity if the local probability P that the probed particle comes from the injection stream is equal to $\left\langle r\mathrm{Z} \right\rangle / \left\langle r \right\rangle$; thus, according to expression (20) for P, the ratio of the seed concentrations in the initial channels must be equal to the ratio of the initial fluid densities,

$$a = 1 \qquad \Leftrightarrow \qquad \left\langle \mathrm{N}_{\mathrm{P1}}^{(1)} \right\rangle / \left\langle \mathrm{N}_{\mathrm{P2}}^{(2)} \right\rangle = r^{(1)} / r^{(2)} \qquad (28)$$

Furthermore, when condition (28) is satisfied $\left\langle \mathrm{N_P} \right\rangle = \mathrm{k}\left\langle r \right\rangle$, $\left\langle \mathrm{N_{P1}} \right\rangle = \mathrm{k}\left\langle r_1 \right\rangle$ and $\left\langle \mathrm{N_{P2}} \right\rangle = \mathrm{k}\left\langle r_2 \right\rangle$.

It is worth noticing that this condition for equitable seeding is independent of the masses w_{P1} and w_{P2} of the particles P_1 and P_2 as far as on average they track accurately the turbulent motion of the fluid without perturbing it. Owing to the absence of collision between the passive markers, the mean conditional macroscopic properties of the turbulent mixing flow (<ρ$_1$>, <ρ$_2$>, <ρ$_1$V$_1$>/<ρ$_1$>, <ρ$_2$V$_2$>/<ρ$_2$>) can be obtained by seeding only one or the other channel. The mean flow velocity can be restored from <V$_{LDV}$>$_1$ and <V$_{LDV}$>$_2$ that were measured separately by seeding only the jet fluid and then only the ambient air using relation (19) with P=<ρZ>/<ρ>.

6. HOMOMORPHISM

The form of the relations between unconditional and conditional velocities is the same in terms of ensemble Favre average values in a turbulent mixing flow as in terms of simple values in a laminar mixing flow, and the same analogy is obtained with random sampling and ensemble averaging of the local continuity equations (21-24). In this correspondence the operator is simple averaging for the densities, it is Favre averaging for the mixture fraction and the global velocity, and it is conditional Favre averaging for the conditional velocities. To have a homogeneous correspondence with ensemble Favre averaging (A→ f(A)=<ρA>/<ρ>) as operator for any unconditional property, and conditional Favre averaging (A$_k$ → f$_k$(A$_k$)=<ρ$_k$A$_k$>/<ρ$_k$> with k=1 or k=2) as operator for any conditional property, the scalar variables to be transformed must be expressed only in terms of specific volumes (u=1/ρ and u$_k$=1/ρ$_k$) and the mixture fraction must be considered only as a transfer function such that u=Zu$_1$. With this convention the correspondence between the macroscopic properties of a laminar mixing flow and their counterpart in term of ensemble Favre averages in a turbulent mixing flow is a homomorphism. A homomorphism is a mapping of a group or vector space *L* into another *T* in such a way that the result obtained by applying an operation to variables of *L* is mapped onto the result obtained by applying the same operation to their images in *T* . The basic role played by the specific volume has been underlined by several authors (Bilger, 1979; Pope, 2000; Rey, 2000). Pope gave a useful interpretation of the continuity equation expressed in terms of specific volume u , material derivative and dilatation as $\mathrm{D}\ln \mathrm{u}/\mathrm{Dt} = \nabla\mathrm{V}$. In this form the continuity equation can be viewed as a consistency condition between the change of the specific volume following a fluid particle, and the change in the volume of an infinitesimal material volume element.

The formal analogy between macroscopic properties in a laminar flow and their counterpart averages in a turbulent mixing flow takes an additional physical dimension if one remembers that local macroscopic properties of a laminar flow can be defined as ensemble averages over instantaneous samples where one or zero molecule is present in an infinitely small control volume, while in a turbulent mixing flow the corresponding mean properties can be accessible by the ensemble average properties of seed particles over samples where one or zero particle is present in a local probe volume. To concretize this physical supplement in the analogy it is necessary to verify that the homomorphism is also satisfied for the involved features of the constituents (i.e. actual elements or representatives) of the two kinds flows. In a turbulent mixing flow with equitable seeding, the local probability that a particle comes from the jet fluid is P=<ρZ>/<ρ>=Map(Z), whereas in a laminar mixing flow the local probability that a molecule comes from the jet fluid is *P*=N$_1$/N. To have a complete homomorphism it is necessary that the molecules of the laminar flow have the same elemental mass. Then any relationship between properties in a laminar flow with constant molecular weight can be translated into the same relationship between the respective counterparts of these properties in a turbulent flow with variable density as far as the involved properties can be portrayed by the average properties of single constituents.

The table below summarizes the properties or relations involved in the homomorphism.

Molecular properties in a laminar mixing flow with constant molecular weight :		Counterpart average properties in a turbulent mixing flow such as portrayed by particles with equitable seeding :	
$\frac{N^{(1)}}{r^{(1)}} = \frac{N^{(2)}}{r^{(2)}} = \frac{N_1}{r_1} = \frac{N_2}{r_2} = \frac{N}{r} = b$		$\frac{\langle N_{P1}^{(1)} \rangle}{r^{(1)}} = \frac{\langle N_{P2}^{(2)} \rangle}{r^{(2)}} = \frac{\langle N_{P1} \rangle}{\langle r_1 \rangle} = \frac{\langle N_{P2} \rangle}{\langle r_2 \rangle} = \frac{\langle N_P \rangle}{\langle r \rangle} = k$	
N	ρ	<ρ>	$\langle N_P \rangle$
N_1	ρ_1	$\langle \rho_1 \rangle = \langle \rho Z \rangle$	$\langle N_{P1} \rangle$
N_2	ρ_2	$\langle \rho_2 \rangle = \langle \rho(1-Z) \rangle$	$\langle N_{P2} \rangle$
$N=N_1+N_2$	$\rho=\rho_1+\rho_2$	$\langle \rho \rangle = \langle \rho_1 \rangle + \langle \rho_2 \rangle$	$\langle N_P \rangle = \langle N_{P1} \rangle + \langle N_{P2} \rangle$
$P = \frac{N_1}{N} = \frac{r_1}{r}$	$Z = \frac{r_1}{r}$	$\frac{\langle rZ \rangle}{\langle r \rangle} = \frac{\langle r_1 \rangle}{\langle r \rangle}$	$P = \frac{\langle N_{P1} \rangle}{\langle N_P \rangle} = \frac{\langle r_1 \rangle}{\langle r \rangle}$
$\langle v \rangle_{1+2}$	V	$\frac{\langle rV \rangle}{\langle r \rangle}$	$\langle V_{LDV} \rangle_{1+2}$
$\langle v \rangle_1$	V_1	$\frac{\langle r_1 V_1 \rangle}{\langle r \rangle} = \frac{\langle rZV_1 \rangle}{\langle rZ \rangle}$	$\langle V_{LDV} \rangle_1$
$\langle v \rangle_2$	V_2	$\frac{\langle r_2 V_2 \rangle}{\langle r_2 \rangle} = \frac{\langle r(1-Z)V_2 \rangle}{\langle r(1-Z) \rangle}$	$\langle V_{LDV} \rangle_2$
$V = Z V_1 + (1-Z) V_2$		$\frac{\langle rV \rangle}{\langle r \rangle} = \frac{\langle rZ \rangle}{\langle r \rangle}\frac{\langle rZV_1 \rangle}{\langle rZ \rangle} + \left[1 - \frac{\langle rZ \rangle}{\langle r \rangle}\right]\frac{\langle r(1-Z)V_2 \rangle}{\langle r(1-Z) \rangle}$	
$\langle v \rangle_{1+2} = P\langle v \rangle_1 + (1-P)\langle v \rangle_2$		$\langle V_{LDV} \rangle_{1+2} = P\langle V_{LDV} \rangle_1 + (1-P)\langle V_{LDV} \rangle_2$	
$Z(1-Z)[V_1-V_2] = -D_L \nabla Z$	Þ	$\frac{\langle rZ \rangle}{\langle r \rangle}\left[1 - \frac{\langle rZ \rangle}{\langle r \rangle}\right]\left[\frac{\langle rZV_1 \rangle}{\langle rZ \rangle} - \frac{\langle r(1-Z)V_2 \rangle}{\langle r(1-Z) \rangle}\right] = -D_T \nabla \frac{\langle rZ \rangle}{\langle r \rangle}$	

6.1 Application to turbulent diffusivity

In a laminar mixing flow of molecules with constant molecular weight kinetic theory of gases show that

$$Z(1-Z)(V_1 - V_2) = -D_L \nabla Z \qquad (29)$$

where the laminar diffusivity is given as the average magnitude of the molecular agitation velocity multiplied by the mean free path of the molecules. Since the homomorphism has been extended to the ensemble average quantities associated with the elemental constituents of the two kinds of flows it applies also to D_L. In a turbulent mixing flow the gradient diffusion equation above is mapped into

$$\boxed{\frac{\langle rZ \rangle}{\langle r \rangle}\left[1 - \frac{\langle rZ \rangle}{\langle r \rangle}\right]\left[\frac{\langle rZV_1 \rangle}{\langle rZ \rangle} - \frac{\langle r(1-Z)V_2 \rangle}{\langle r(1-Z) \rangle}\right] = -D_T \nabla \frac{\langle rZ \rangle}{\langle r \rangle} \qquad (30)}$$

where scalar D_T, called turbulent diffusivity, is the map of D_L in the turbulent flow. Then D_T should be given by the product of the Favre velocity fluctuation multiplied by the integral Lagrangian scale of the turbulent flow such as they are portrayed by the statistics for single particles. Such Lagrangian velocity fluctuations have been recently measured by Mordant et al (2003) in a turbulent flow with no mean advection and analyzed in the framework of random walks.

As the third factor on the left hand side of equation (30) is portrayed by $\langle V_{LDV} \rangle_1 - \langle V_{LDV} \rangle_2$, the validity of this relation has been tested in the near development field (x/d=10) of variable density jets by plotting the difference between the radial components of the conditional LDV data as a function of $-[\partial A/\partial r]/A(1-A)$ where $A = \langle rZ \rangle / \langle r \rangle$. The data shown in Fig. 6 gather reasonably well along a common straight line for both jets.

The slope of this linear evolution ($\approx 3\ 10^{-3}\ m^2/s$) is in good agreement with the order of magnitude of turbulent diffusivity commonly used (Hinze, 1975) in fluid dynamics ($D_T \approx 10^{-2} V_x^{(1)} d$).

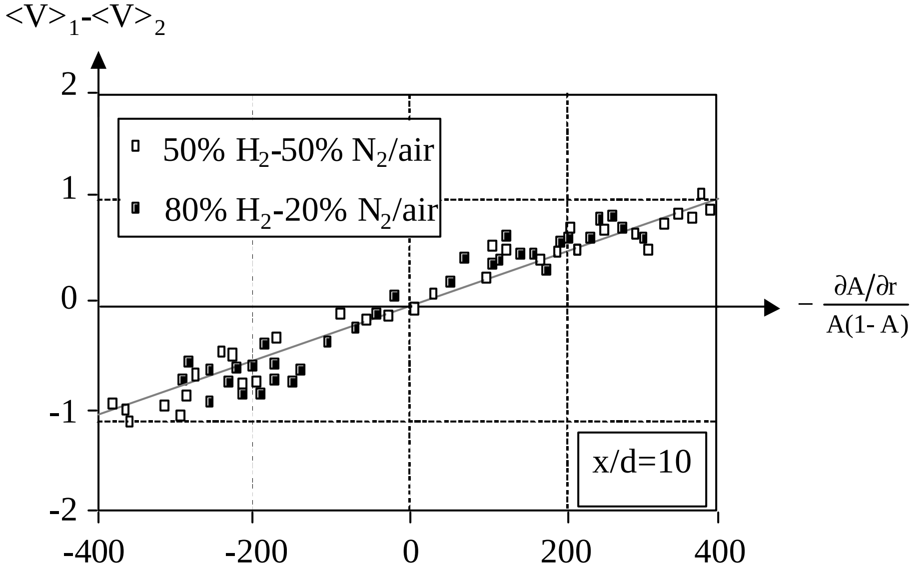

Fig. 6. Diffusivity plot of $\langle V\rangle_1-\langle V\rangle_2$ *where* $A=\langle\rho Z\rangle/\langle\rho\rangle$

The good agreement of the data with equation (30) in the early stages of the jet development was far from being expected because in this region the instantaneous profiles of Z showed strong discontinuities and intermittencies that would not be compatible with the usual assumptions of models. This agreement supports the general validity of equation (30) as being a direct consequence of the homomorphism without any other assumption nor approximations. Instead with of the assumption of quasi steady state for the mixing dynamics, equation (30) reduces to (18) in which turbulent mixing is approximated by the turbulent transport of the mixture fraction.
The generality of relation (30) lies in three features :

- Ensemble averaging accounts for any event including intermittent ones, such as can be sometimes observed in the instantaneous structures. (notice that macroscopic properties in the continuum of a laminar flow are also ensemble averages over the properties of a huge number of molecules in a local control volume)
- As the velocity of the marked fluid is specified by the binary origin of the marker, local mixing dynamic within the control volume is also statistically accounted for, and turbulent mixing is not restrictively considered as the turbulent transport of the scalar state of the mixture.
- Favre averaging - and conditional Favre averaging when the involved property is conditioned on the origin - accounts properly for density fluctuations of the marked fluid. Favre averaging is often considered only as a technique to simplify the form of the averaged conservation equations, under pretence that density fluctuations correlations still remain hidden in the reduced formulation. Actually density-weighted averaging was introduced by Favre (1969) essentially to account for the inertial nature of the interaction processes involved in fluid dynamics and especially to use momentum ρV as a fundamental variable. Simplification of the equations was only a bonus.

7. CONCLUDING REMARKS

In turbulent jets analysis of the marker statistics provides different expressions for the mean LDV data in terms of ensemble conditional Favre averages when only one of the two channels is seeded. When both channels are seeded the mean Doppler velocity $\langle V\rangle_{1+2}$ represents the actual mean velocity of the turbulent flow if the ratio of the two seeding rates is set equal to the ratio of the initial fluid densities. Then, provided that densities are expressed in terms of specific volumes, there is a complete homomorphism between the molecular properties in a laminar mixing flow and the counterpart Favre average properties in a turbulent mixing flow with variable density such as they can be portrayed by those of single particles. With this correspondence the gradient-diffusion law for a laminar flow of identical molecules translates into an equivalent law in terms of conditional Favre average values in a turbulent mixing flow. The general validity of law has been tested by LDV with conditional seeding in the near field of turbulent jets with variable density. Finally, conditional LDV is a relevant observer of turbulent mixing which consider the single particles that mark the motion of the fluid according to its origin with the same point of view *(single point statistics)* as the molecules are considered in the kinetic theory of gases.

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