

# Experimental study of two-dimensional turbulence using Feature Tracking

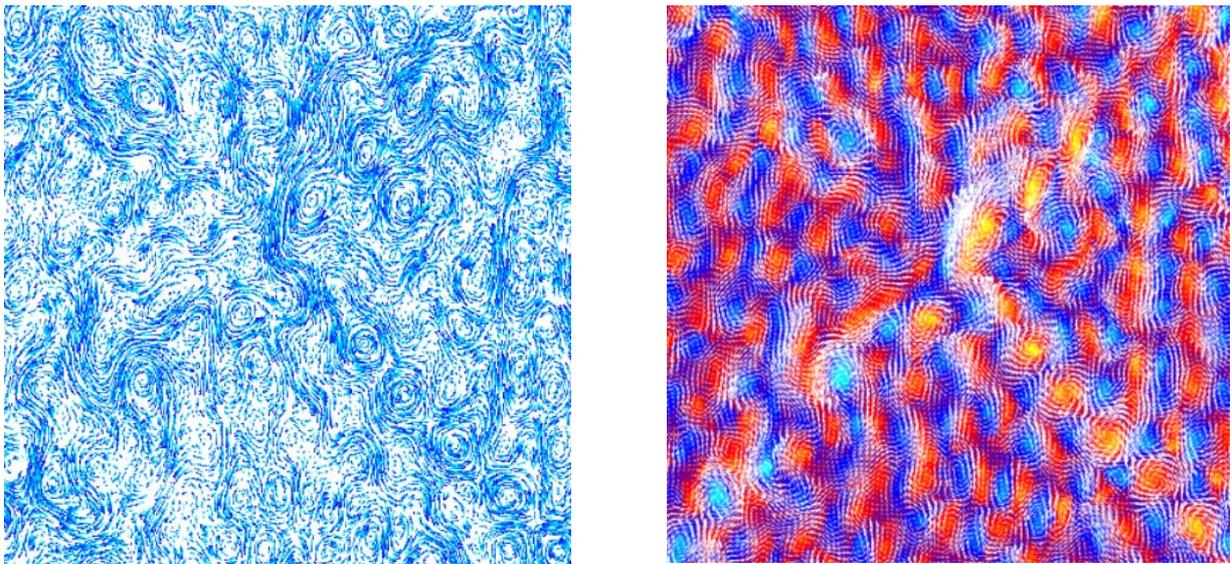
by

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## ABSTRACT

Two dimensional turbulence has been of interest for years because of its applications in geophysics and astrophysics and because it can be seen as a toy model for understanding some aspects of 3D turbulence too. In this context, we present some experimental results on a forced system in which the flow is produced by applying a combination of electric and magnetic fields on a thin layer of an electrolyte solution. The fluid surface is seeded by styrene particles (250  $\mu\text{m}$ ) and flow measurements are performed through image analysis. In particular, by using Feature Tracking (FT) technique, we are able to obtain at the same time a large amount of Lagrangian data (i.e. the seeding density can be very high and trajectories can be followed for large time intervals) and high resolution Eulerian velocity fields. The Feature Tracking technique can be considered as a correlation-based tracking method. As a matter of fact, instead of reconstructing trajectories through evaluating the position of tracer particle centroids within each frame (as in classical PTV), trajectories are reconstructed by analysing the evolution, frame by frame, of the displacements and the deformations of windows surrounding selected features. The feature selection algorithm is defined in such a way that the optimal solution of the tracking algorithm is achieved. In figure 1 a set of reconstructed trajectories (a) and the corresponding instantaneous velocity and vorticity fields (128x128 grid) (b) are shown. Many thousands particles are simultaneously tracked for each frame, and the Eulerian velocity fields are obtained avoiding problems related to spatial or temporal interpolation of low density Lagrangian data over a regular grid. These are two crucial aspects while dealing with experiments: as a matter of fact the lack of statistics, mostly when information at long times are required, can represent a limiting aspect and high spatial resolution velocity fields are required in order to characterized phenomena occurring at different scales.



*Fig. 1. Trajectories reconstruction (a) and instantaneous velocity and vorticity (b) field in a forced 2D turbulence laboratory experiment.*

## 1. INTRODUCTION

The most important results obtained in the study of two dimensional turbulence can be found in several publications and books (among them Kraichnan & Montgomery (1980), Kraichnan (1967, 1971), Frisch (1995), Lesieur (1990)). As a matter of fact, the interest of researchers and the efforts devoted in the understanding and in the characterization of two-dimensional turbulence have been continuously increased in the last decades.

To our opinion, two main reasons are at the basis of this growing interest: the former is related to theoretical aspects being hydrodynamic turbulence still an open problem in several of its characterizing features, the latter is mainly related to applied physics; as a matter of fact the study of the dynamical properties of two dimensional turbulence has a big impact on the characterization of geophysical flows.

Concerning the first underlined motivation, it can be noticed that there are several phenomena i.e. cascades and coherent structures emergence observed both in three-dimensional and in two-dimensional turbulent flows. Considering that a simulation can be easily performed in a two dimensional domain, two-dimensional turbulence has been often considered as a toy model for the understanding of three-dimensional complex phenomena.

Concerning the second underlined motivation, it can be observed that in geophysical flows the combined effects of earth rotation and of stratification induce the inhibition of vertical displacements so that large scale (i.e. characteristic length scales ranging from  $L \sim 1000\text{km}$  up to  $10000\text{km}$ ) atmospheric (Lacorata *et al* (2004), Koh & Legras (2002)) and oceanic (Bower *et al.* (1994)) flows can be considered quasi two dimensional flows.

In order to analyze briefly the main aspects of two dimensional turbulence, consider Navier-Stokes equations for a two dimensional systems in terms of vorticity:

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega + f_\omega \quad (1)$$

where  $\nu$  is the fluid viscosity and  $f_\omega$  represents the injection of energy in the systems (forcing).

It is known that the same equation, written for a three dimensional system, contains a further term  $\boldsymbol{\omega} \cdot \nabla \mathbf{u}$  i.e. *vortex stretching* term which represent the mechanism of generation of small scale and intense vortex structures. This mechanism is absent in two dimensional turbulence, as a matter of fact Helmholtz theorem states that vorticity is conserved if the fluid is inviscid and the forcing is equal to zero.

Considering then the kinetic energy per unit mass:

$$E = \frac{1}{2} \langle \mathbf{u}^2 \rangle \quad (2)$$

where the operator  $\langle \cdot \rangle$  represents the average over a representative set of independent realization, the time evolution equation for (2) can be written as:

$$\frac{dE}{dt} = -\nu Z \quad (3)$$

where  $Z = \langle \omega^2 \rangle$  represents the enstrophy of the system whose time evolution can be expressed using the following equation as:

$$\frac{dZ}{dt} = -\nu \langle (\nabla \omega)^2 \rangle \quad (4)$$

This equation indicates that the initial vorticity distribution tends to an increment of vorticity gradients until viscosity vanishes these gradients. In analogy with the energy cascade in a three dimensional system (Kraichnan (1971)), it is possible to identify a stationary state corresponding to a constant enstrophy dissipation rate independent from viscosity.

In a two dimensional system then, enstrophy decreases with time (4) while energy is essentially conserved when viscosity tends to zero. Instead, in a three dimensional system, the enstrophy increases until viscosity become important while energy decrease at a constant rate independent from viscosity.

As a consequence in a two dimensional system the energy can not be dissipated at the small scales and a direct energy cascade can not be observed while enstrophy can be dissipated at the small scales and a direct enstrophy cascade can be observed. The analogy between two and three-dimensional turbulence considers enstrophy instead of energy and vorticity instead of velocity.

Having observed the existence of these invariant i.e. energy and enstrophy, Kraichnan (1967) theorized the existence of two inertial ranges for two dimensional inviscid flow: the former from the injection scale towards largest scales (smallest wave numbers) characterized by a constant energy flux called inverse energy cascade, the latter from the injection scale towards smallest scales (largest wave numbers) characterized by a constant enstrophy flux called direct enstrophy cascade.

This *double cascade* is clearly evident in a Fourier domain and is shown in Figure 2; scale arguments (Kraichnan (1967)) have easily allowed evaluating the two slopes of the spectrum.

The spectra obtained both in numerical and in laboratory simulations of forced and decaying two dimensional turbulence reveals similar features; an interesting review of the main numerical and experimental evidence of two dimensional turbulence can be found in Tabeling (2002).

Due to the accuracy and to the high space and time resolution of the actually performed flow measurements, laboratory experiments have furnished a fundamental contribution in the understanding and in the characterization of these features.

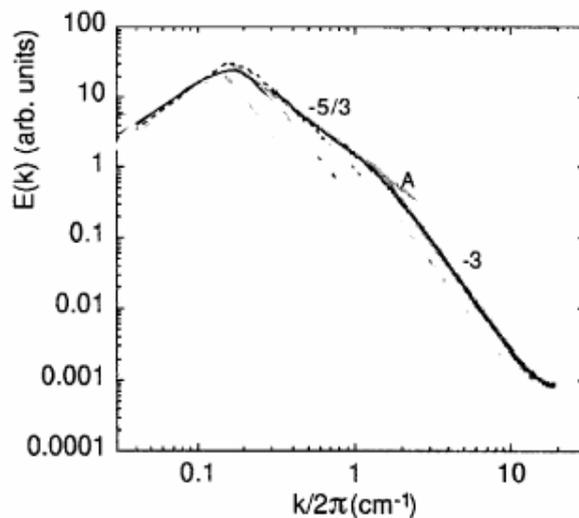


Fig. 2. *Double cascade in two dimensional turbulence in Fourier domain. From Tabeling (2002).*

In particular, in laboratory experiments, two dimensionality (or quasi two dimensionality) is essentially achieved using rotating and/or stratified systems (Hopfinger & van Heijst (1994), Linden et al. (1995)), thin layers of electrolyte solution (Jullien (2003), Jullien et al. (1999)) or soap films (Kellay et al. (1995)). In this work the flow is generated using the same conditions described in Jullien (2003), i.e. by applying an electromagnetically forcing based on the superposition of electric and magnetic fields on a thin layer of an electrolyte solution. The measuring technique, substantially different from the techniques usually described in literature, will be described in the following section.

## 2. EXPERIMENTAL APPARATUS AND MEASURING TECHNIQUES

### 2.1 Experimental apparatus

The experimental apparatus consists of a squared plexiglass tank whose dimensions are 50cm×50cm filled with an electrolyte solution of water and sodium chlorite. Figure 3 shows a transversal section of the apparatus. In particular the tank is filled by injecting the fluid from four holes opened in the bottom of the lateral reservoirs surrounding the test section. These reservoirs have been designed to keep the product of the electrolysis process out of the test section in order to not alter the flow measurements.

Two fluid characterized by different densities (*i.e.* salt concentration) are subsequently injected from those holes starting from the lightest one; at the end of the filling procedure a stable stratification with controlled fluids thickness is then gained and a two dimensional system is obtained (Cardoso *et al.* (1994)). In the performed experiments the fluid thickness for both the fluids has been maintained equal to 3mm.

The flow is forced using the superposition of electric and magnetic fields *i.e.* the effects of Lorentz force. The magnetic field have been generated by using several Neodymium permanents magnets located on a metallic plate just below the bottom surface of the tank. They have been grouped in four triangles according to their sign, in particular each triangle contains opposite sign magnets respect to close triangles. The electric field is created using a power supply able to generate a current signal of maximum amplitude of 0.1 mA oscillating with random frequency; the duration of these oscillation is random too and value are selected considering a normal distribution with assigned mean and variance.

Figure 4 shows a scheme of the forcing imposed on the system.

The combined action of electric and magnetic forcing induces the continuous formation of opposite signed vortical structures whose vorticity is positive or negative according to the verse of the resulting Lorentz force, whose characteristic length scale is related to the distance between magnets and whose characteristic time scale is related to the current oscillation duration.

Experiments have been performed considering the following steps: the forcing is activated, the free fluid surface is seeded using styrene particles ( $d_m \sim 250 \mu m$ ), the test section is illuminated using an array of lamps placed orthogonally to one side of the tank and the fluid flow is recorded using a standard speed video camera ( $f_{acq} = 25 Hz$ ) placed orthogonally to the tank surface. The mean duration of the acquisition is 6 minutes: during this time interval both the density stratification in the fluid and two dimensionality of the flow are maintained (Cardoso *et al.* (1994)). After the acquired images have been digitised, flow measurements have been performed by means of techniques based on image analysis. In particular we use the Feature Tracking approach. The application of this methodology, developed in the context of Computer Vision, is quite new in the context of fluid dynamics applications (Miozzi (2004)).

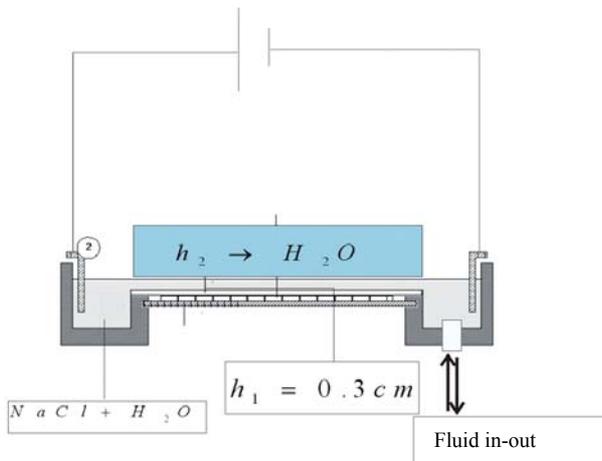


Fig. 3. Schematic drawing of experimental apparatus.

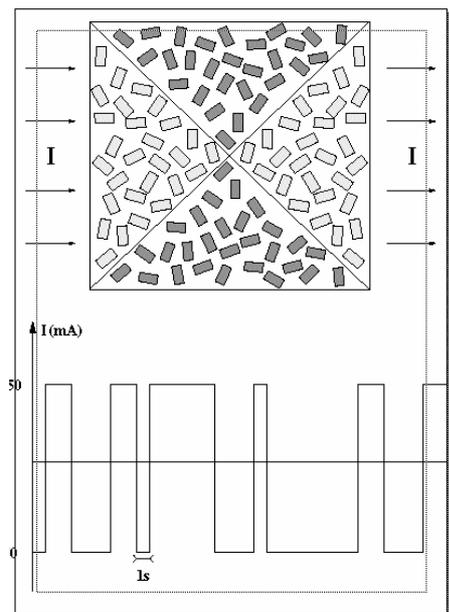


Fig.4. Schematic drawing of the forcing imposed to the system to obtain a two dimensional turbulent flow.

## 2.2 Experimental Techniques

In this section a brief description of the technique used for measuring the flow field both in a lagrangian than in an eulerian frame of reference is given. More details on the presented method can be found in Miozzi (2004).

The Feature Tracking method is based on the Lukas-Kanade (1981) algorithm and on its following version given by Shi and Tomasi in 1994; it can be defined as a tracking technique based on correlation windows; the method defines a matching measure between fixed-sized feature windows (*i.e.* interrogation window surrounding a feature) in two consecutive frames and the window displacement is then evaluated by considering the best correspondence between

subsequent images as in classical PIV. In this case the window displacement is evaluated by minimizing the Sum of Squared Differences (SSD) between the image intensity in two subsequent images instead of by maximizing the inner product (i.e. the correlation) between intensities.

For small displacements it can be showed that a linearization of the image intensity leads to solve a Newton-Raphson style minimization problem (Shi & Tomasi (1994)).

The Feature Tracking routine can be subdivided into the following steps: feature extraction and feature tracking. The feature extraction algorithm is defined in such a way that the optimal solution of the tracking algorithm is achieved. Rather than defining a subjective, a priori, notion of a good feature, this definition is based on the method used for tracking itself: “a good feature is one that can be tracked well”. With this approach, one overcomes a subjective definition of the object that has to be tracked (such as particle centroids in classical PTV or eulerian grid points in classical PIV) and of its corresponding definition parameters. Moreover, one knows that a feature is selected only if it is good enough for the purpose; it follows that the selection criterion is optimal by construction. In particular, it can be proved that the solvability of the SSD minimization problem is guaranteed if the eigenvalues of the correlation matrix of the gradients of the intensity values corresponding to a small window around the feature are both real and positive (this condition is always guaranteed by the construction rules of the correlation matrix) and if the minimum eigenvalue of this matrix is greater than a minimum prefixed threshold (Shi and Tomasi, 1994).

It follows that the solution of this problem has to be searched where image intensity gradients are not null both along  $x$  and  $y$  direction. As a consequence, the parameters to be set for the feature description are: the size of the interrogation window and the minimum eigenvalue of the corresponding intensity matrix.

Two subsequent models of motion have been adopted for the interrogation window: in a first step, a pure translational model allows to detect a rigid displacement of the interrogation window; in a second step, an affine motion model is adopted, using the previous translational results as initial seeding. In this way, window velocity components and the corresponding spatial derivatives are gained (Miozzi, 2004). Moreover, the use of this two-step procedure enforces the numerical stability of the solution that rises when evaluating velocity derivatives: the translational model of motion furnishes a good initial seed that permits a rapid and efficient convergence of the affine motion model minimization problem (Miozzi, 2004). At the end of the tracking procedure, a high spatial resolution set of feature velocity and velocity gradient are gained for each couple of frames.

After the tracking procedure, lagrangian data obtained evaluating the feature position in different time instants are embedded into a Delaunay triangulation (1934). This geometric construction represents a uniquely defined spatial domain subdivision obtained from relative feature spatial position. A Natural Neighbours interpolation type (Watson (1985)) is applied in order to extract velocity data on an arbitrary chosen regular grid. Natural Neighbours method has the following interesting properties: 1) the interpolated quantity is a continuum function; 2) the original values are recovered exactly at the reference points; 3) the interpolation is entirely local and 4) the derivatives of the interpolated functions are continuous everywhere except at reference points.

A complete description of the NN algorithm can be found in Sambridge et al. (1995); the application of the NN method in a fluid dynamic context is reported in Miozzi (2004).

### 3. RESULTS AND DISCUSSION

In the following set of figures the output of the FT algorithm are shown in terms of reconstructed trajectories over a sequence of frames (Figures 5), of selected ‘very long’ (according to the characteristic scales of the investigated flow) trajectories (Figure 6a) and of the corresponding Lagrangian vorticity (Figure 6b), of Lagrangian velocity (Figure 7a) and of Lagrangian vorticity evaluated for the first couple of frames, embedded in a Delaunay suport (Figure 7b). This plot has been obtained by filling each triangle with the centroid triangle vorticity value obtained using a linear interpolation of vorticities at the triangle vertexes. (Figure 8).

Figures clearly show that many thousands of particles are simultaneously tracked for each frame, and that the Eulerian velocity fields are obtained avoiding problems related to spatial or temporal interpolation of low density Lagrangian data over a regular grid. Both of these aspects are crucial when dealing with experimental complex flow measurements. Considering the Lagrangian viewpoint, often the evaluation of significant statistics on particle displacement is conditioned by the lack of long trajectories being the mean length of reconstructed trajectories using classical tracking methods usually smaller than the characteristic scales of the investigated flow. As a consequence robust statistics on particle displacements are often evaluated considering synthetic trajectories obtained by integrating the equation of motion based on the measure of the velocity field (Jullien (2003)). The lack of correctness of this procedure is intrinsically connected with the non-trivial relation between the Eulerian and Lagrangian aspect of the motion i.e. to the possible onset of Lagrangian chaos (Crisanti *et al.*, (1991)).

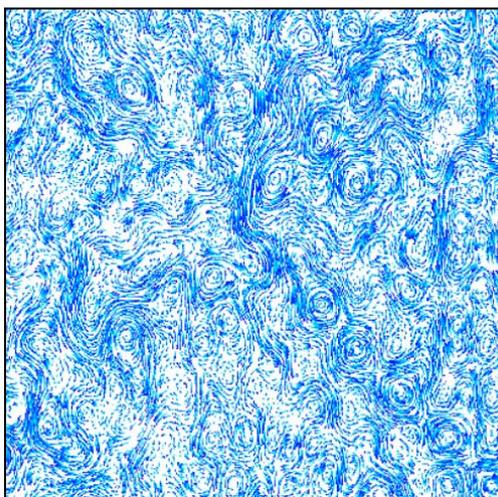


Fig.5 Reconstructed trajectories over a frame sequence of the acquisition.

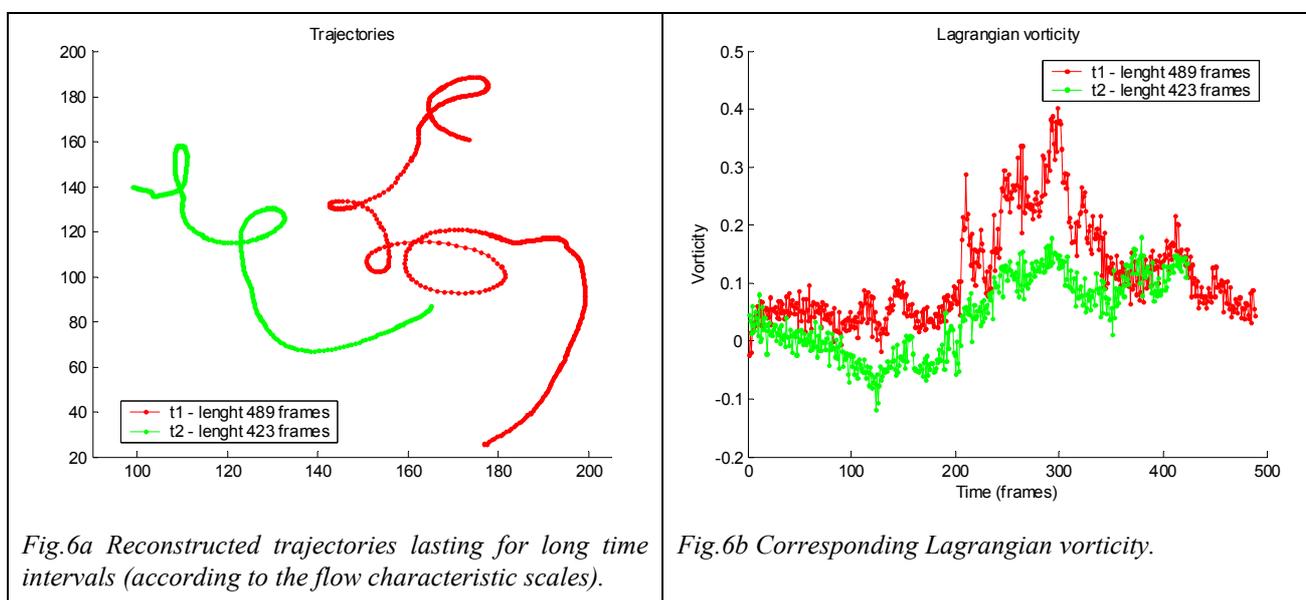


Fig.6a Reconstructed trajectories lasting for long time intervals (according to the flow characteristic scales).

Fig.6b Corresponding Lagrangian vorticity.

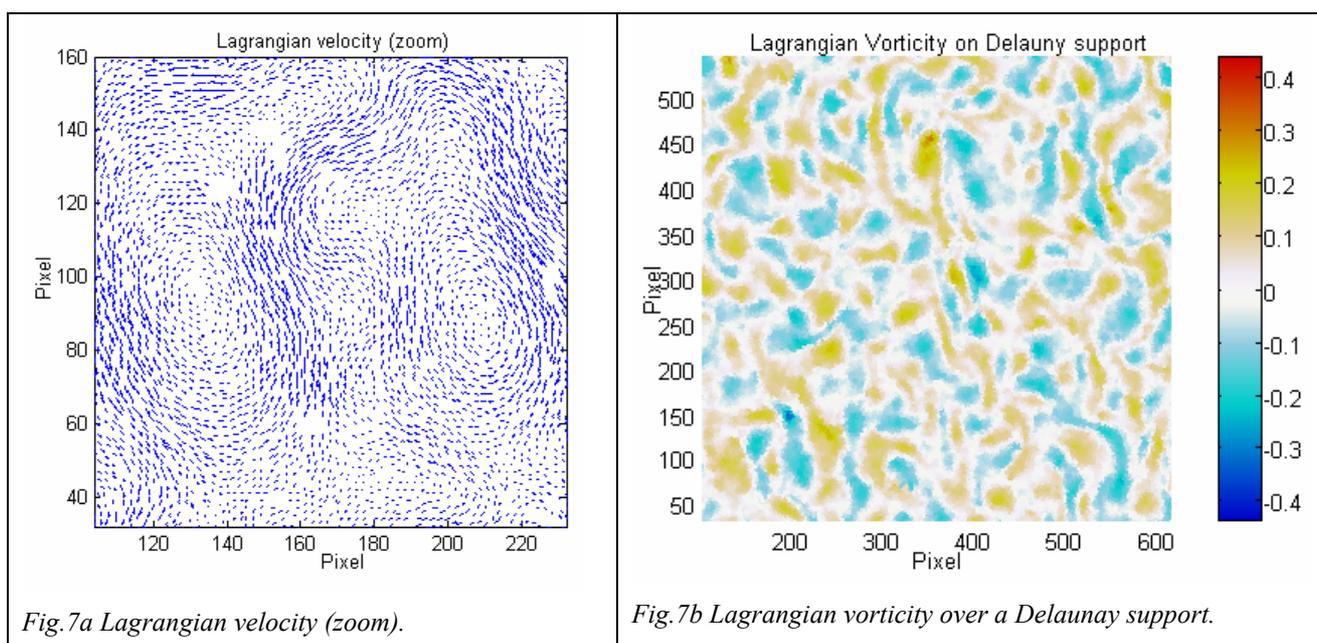


Fig.7a Lagrangian velocity (zoom).

Fig.7b Lagrangian vorticity over a Delaunay support.

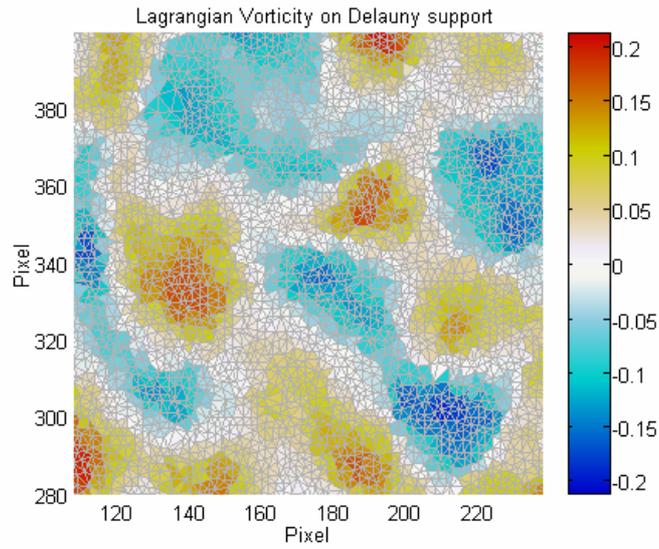
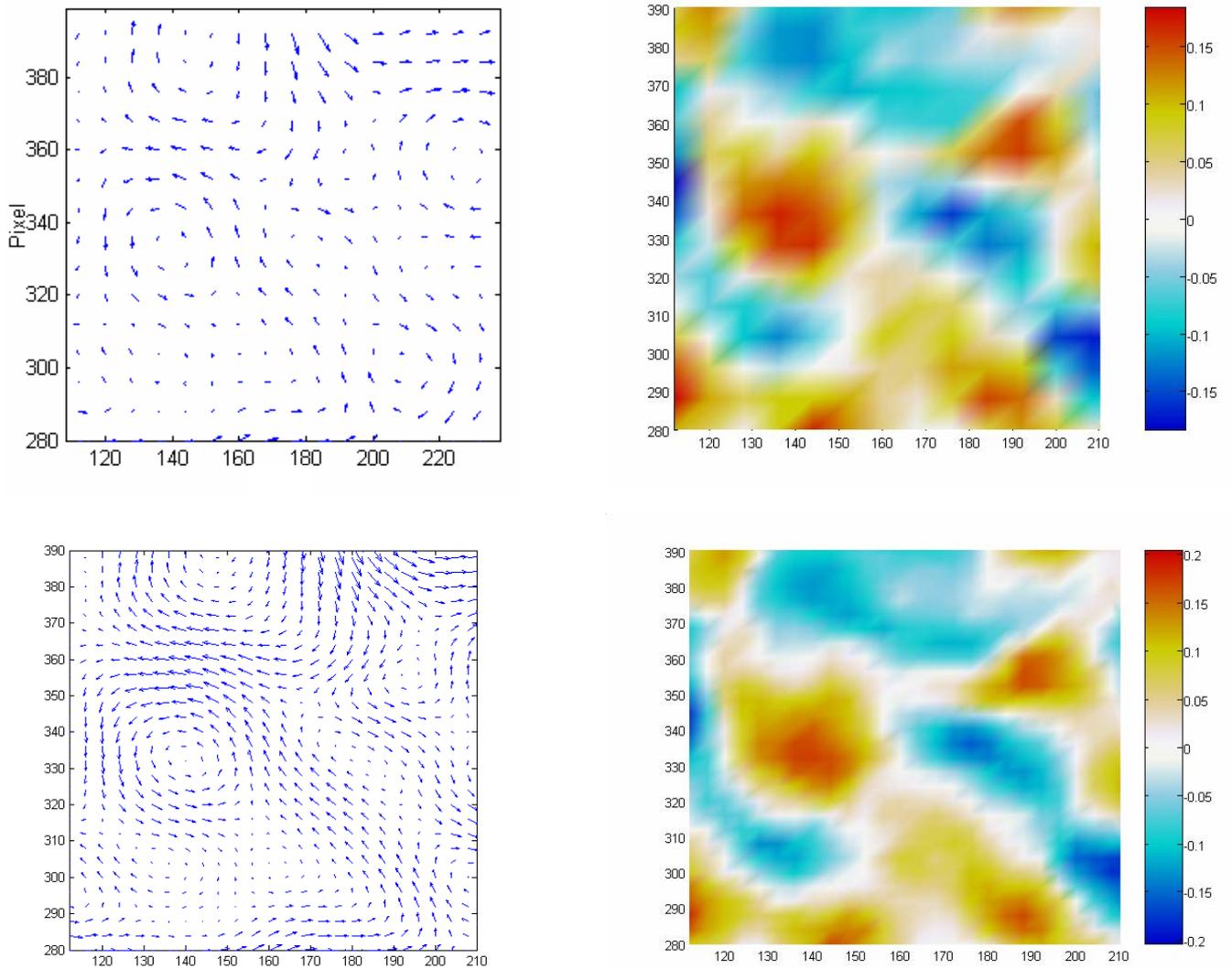


Fig.8 Lagrangian vorticity over a Delaunay support showing the triangles' structure (Zoom of Figure 7).



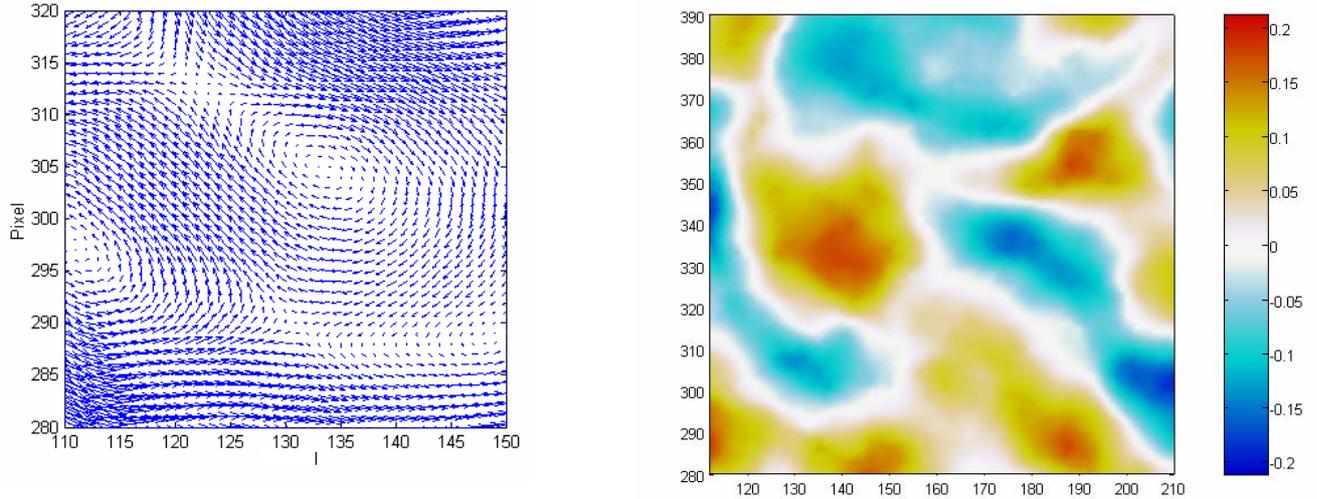


Fig.9 Eulerian velocity and vorticity fields for different grid size, respectively: 8px, 4px, 1px. Velocity field evaluated on 1px grid resolution is represented for a zoomed area.

After the NN interpolation, Eulerian velocity fields are obtained. In Figure 9 instantaneous velocity and vorticity fields respectively corresponding to a spatial resolution of 8px, 4px, 1px are shown. Considering the Eulerian viewpoint, it has to be noticed that the characterization of small scales phenomena is related to a high spatial resolution measure of the velocity and vorticity fields; the NN method underlines the existence of a unique solution: nesting grids with different cell sizes sharing a set of grid points, exhibits the same interpolated values in the common points as shown in Figure 10. In those pictures the vertical profiles of horizontal component of the velocity and vorticity fields are shown for respectively 1px, 4px and 16px cell sizes. The superposition of these profiles clearly shows how a smallest grid resolution is more suitable for a precise definition of velocity and vorticity signal.

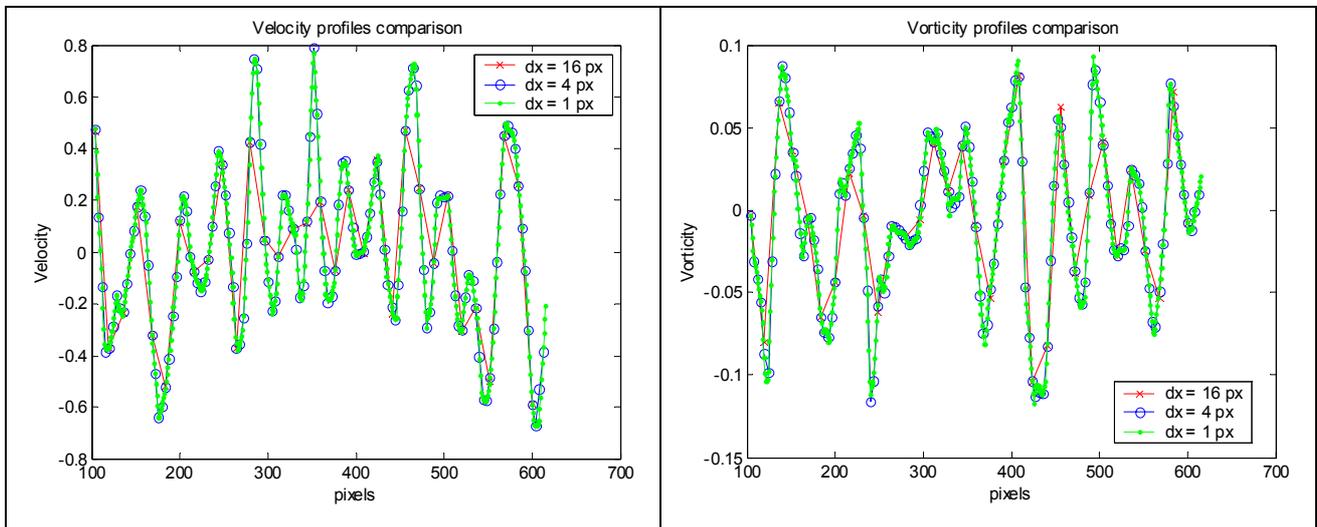


Fig.10 Vertical profiles of horizontal velocity for different grid cell sizes.

These set of reconstructed high-resolution Eulerian velocity fields, has allowed the characterization of the phenomena in the wave number domain (Figure 11). As discussed in section 1, the spectrum clearly evidences two different slopes in correspondence of scales greatest than the scale of the forcing (red) and scales smallest than the scale of the forcing (green). The slope corresponding to the direct cascade range (-3.4), has found to be greater than the expected value (-

3). This evidence has already found in numerical simulation of forced two-dimensional turbulence (Boffetta et al. (2002)) and is representative of a non trivial influence of drag on the enstrophy transfer towards smallest scales.

A future investigation of this flow is then aimed to the analysis of the influence of the thickness of the fluid on the spectrum steepness.

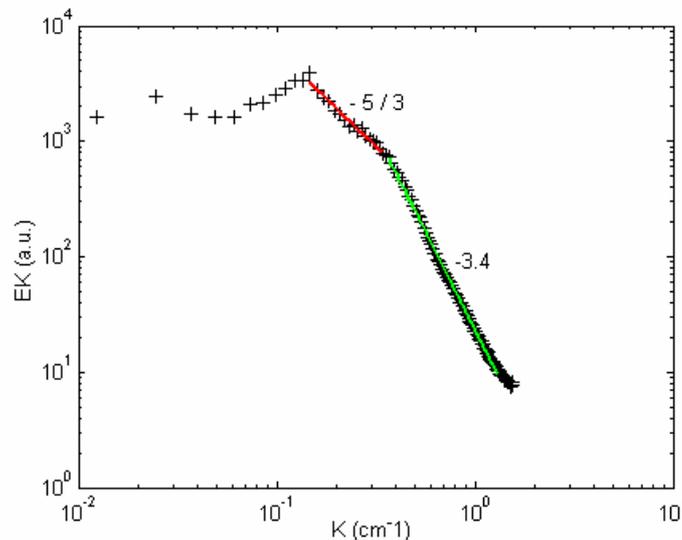


Fig. 11 Energy spectrum, the slopes corresponding to direct (green) and inverse (red) cascades are shown.

#### 4. CONCLUSIONS

The main feature of a two dimensional turbulent flow has been recovered using an experimental apparatus based on the electromagnetic forcing on a thin layer of electrolyte solution. Lagrangian and Eulerian flow have been measured using FT technique, in particular the possibility of obtaining high resolution velocity fields has allowed to characterize the smallest spatial scale of the flow in the wave number domain and to evidence the occurrence of both direct and inverse cascade regimes in the investigated flow

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