Cross Sectional Area Difference Method For Backscatter Particle Sizing

by

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ABSTRACT

This paper reports on possible extensions of the cross-sectional area difference method for sizing particles in backscatter by using commercial laser Doppler equipment. The cross-sectional area difference method was introduced by Albrecht et al. (1993) and Borys (1996). It uses signals from a normal laser Doppler system to estimate the particle size distribution in a statistical manner. The maximum intensity of a burst signal and therefore the burst length or the number of periods depends on the particle size. In the mean larger particles generate signals of higher amplitude and longer duration. The technique is based on the inversion of the particle generated burst-length statistics to the diameter distribution. The monotonic relation between particle size and scattered intensity of the modulated part of a laser Doppler signal is one pre-requisite for the success of this inversion routine. The technique was successfully applied (Borys 1996) to small particles ($x < 1$) and for larger particles at forward scattering angles with monotonic intensity/diameter relation for first-order refraction.

The cross-sectional area difference technique operates just with laser Doppler signals. Therefore it could be attractive to use a commercial backscatter laser Doppler system for velocity and particle sizing only by adding a software component for processing the burst-length statistics. The main problem of the cross-sectional area difference method in backscatter is the fluctuations of the intensity/diameter relation. The paper will present numerical calculations for aperture design. It will be shown that the system can recognize changes in a broader diameter distribution, also in backscatter, whereas the reconstruction of monodispersed particle size distributions is problematic. For forward scattering configurations the system can reconstruct monodispersed and polydisersed particle distributions.

The paper will also present verification experiments for monodispersed and polydispersed sprays, which confirm the measurement principle. The measurements were performed with a commercial laser Doppler backscatter probe which is also used as a transmitting optics of a phase Doppler system. Both systems were simultaneously running and provide the same data for phase Doppler, and cross-sectional area difference method in forward and backscatter direction.

1. INTRODUCTION

The success of the laser Doppler technique in the last decades is mainly due to the development of simple and compact commercial instrumentation, requiring minimal adjustment. Particle sizing with the phase Doppler technique (Albrecht et al. 2003) becomes more problematic and expensive because of the necessary separation of the transmitting and receiving probe. Therefore the phase Doppler instrumentation is connected with difficult adjustment, large and bulky traversing systems and the necessity of having two optical accesses to the measurement point. Therefore, the phase Doppler technique is primarily used where counting techniques are absolutely required, e.g. for velocity, flux and concentration measurements. For the estimation of a simple particle size distribution without any velocity information, ensemble techniques such as those based on laser diffraction are more popular.

This paper will present an extension to the laser Doppler technique for estimating the particle diameter distribution based on monitoring the amplitude, number of periods or burst length of every particle. The technique was first introduced in by Albrecht et al. (1993) and Borys (1996) and uses a standard laser Doppler setup. The statistics of the amplitude, used in Albrecht et al. (1993) and Borys (1996), or burst length, used in this paper, can be inverted to yield the particle diameter distribution under the assumption of a monotonic relation between diameter and scattered intensity. For scattering angles where the phase Doppler techniques works or for particles smaller than the wavelength, this condition is always fulfilled and first measurements were presented in Borys (1996). For particles larger than the wavelength and for the attractive backscatter arrangement the relation between particle diameter and amplitude/burst length can be strongly non-monotonic because of the Mie scattering lobes and the relative contribution of different scattering orders. The whole particle diameter distribution can no longer be calculated by an inversion routine, but a parametric model or the mean value of the burst length based on the Mie scattering calculations can be used for the estimation. More precisely, diameter changes in relation to a polydisperse reference distribution can be monitored. For absolute diameter measurements a calibration with particles of known size is required.

2. THEORY OF THE MEASUREMENT TECHNIQUE

In a laser Doppler system the ellipsoidal measurement volume is defined by the exp(-2) decay of the modulated part of the signal in relation to the modulated signal amplitude at the crossing point of the two laser beams. Because of the Gaussian intensity distribution of the illuminating beams the dimensions of the measurement volume remain independent of the particle diameter and are given by

$$a_0 = \frac{r_w}{\cos\frac{\theta}{2}}, \quad b_0 = r_w, \quad c_0 = \frac{r_w}{\sin\frac{\theta}{2}} \qquad \begin{array}{l} r_w : \text{beam waist radius} \\ \theta : \text{full intersection angle} \end{array} \tag{1}$$

However, only signals exceeding some minimum detection intensity I_d will be registered at the photodetector, thus the detection volume may not coincide precisely with the measurement volume. The minimum detectable intensity will set lower limits for the size of detectable particles and also establishes a relation between the detection volume and the measurement volume. The dimensions of the detection volume can be written as

$$a_d = a_0 F_d, \quad b_d = b_0 F_d, \quad c_d = c_0 F_d \qquad \text{with} \qquad F_d = \sqrt{\frac{1}{2}\ln\frac{I_{ACmax}}{I_d}} \tag{2}$$

The maximum signal amplitude I_{ACmax} occurs when the particle traverses the center of the measurement volume and takes the value (Albrecht et al. 2003)

$$I_{ACmax}(d_p) = \frac{8}{\pi c}\frac{q\eta_q}{h}\frac{\lambda P}{d_w^2 k^2} g(d_p) G(d_p) \tag{3}$$

where P is the power of the laser beams, η_q is the quantum efficiency of the detector, q denotes the elemental charge, c is a speed of the light, h is Planck's constant, d_w is the diameter of the beam waist, k denotes wave number, $g(d_p)$ is the particle dependent visibility function and $G(d_p)$ the integral scattering function which also depends on the particle diameter.

Therefore, these equations establish a unique relation between particle size and detection volume size as long as $g(d_p)\,G(d_p)$ is a monotonic function of particle diameter. In the micrometer and sub-micron range the signal amplitude and the detection volume size increase monotonically with particle size (Rayleigh scattering). One measure of the detection volume size is the maximum signal intensity I_{ACmax} as shown in Eq. (2); however the amplitudes of the individual signals I_{max} are statistically distributed between $I_d = I_{max} = I_{ACmax}$ because of the different particle trajectories through the detection volume. Therefore only a statistical relation between signal amplitude statistics and particle size distribution can be expected.

The amplitude of individual signals can be measured either directly or indirectly. Indirectly either the burst length l or the number of periods N in the signal (for systems without shift frequency) can be used as a measure of the signal amplitude. Such a system was introduced in Albrecht et al. (1993) and Borys (1996), in which both the signal amplitude and the number of signal periods were used for sizing particles smaller than 1µm.

The particle size distribution will now be examined as a function of the burst-length distribution, because the burst length is always available in commercial laser Doppler systems by multiplying the transit time with the velocity ($l = t_T v_P$). The burst length of a particle crossing the detection plane (projection of the detection volume in the direction of the trajectory) at the location (y_P, z_P) in the main flow direction x is given by

$$l = 2a_d \sqrt{1 - \frac{y_p^2}{b_d^2} - \frac{z_p^2}{c_d^2}} \tag{4}$$

From this equation the detection area for burst lengths larger than l can be derived as

$$A_d = \pi b_0 c_0 \frac{l_{max}^2 - l^2}{l_0^2} \tag{5}$$

where $l_{max} = 2a_d$ is the maximum burst length for this specific particle diameter when crossing the center of the measurement volume, and $l_0 = 2a_0$. Monodispersed particles with burst lengths larger than l contribute to the total signal rate $\dot{N}$ or particle flux Q as

$$Q = \bar{n}_p v_p \pi b_0 c_0 \frac{(2a_d)^2 - l^2}{(2a_0)^2} \tag{6}$$

The contribution of a respective burst length class $l_i - Dl_i/2 < l \le l_i + Dl_i/2$ ($i = 1,2,\ldots,n$) to the total particle flux of monodispersed particles is therefore given by

$$DQ_i = \frac{1}{2}\bar{n}_p v_p \pi b_0 c_0 \frac{l_i}{a_0^2} Dl_i \qquad l_1 = \frac{Dl_1}{2} > 0\ , \ l_1 < l_2 < \ldots < l_n\ , \ l_{i+1} - l_i = \frac{1}{2}\left(Dl_i + Dl_{i+1}\right) \tag{7}$$

and is a linear function of the burst length as shown in Fig. 1.

For the case of polydispersed particles, different “monodispersed” classes C contribute with their own burst-length histogram to produce the total burst-length histogram as

$$DQ_i = \frac{\pi}{2} \frac{b_0 c_0}{a_0^2} l_i Dl_i \sum_{p=0}^{C} \bar{n}_p v_p\ , \quad i = 1,2,\ldots,n \tag{8}$$

which is a system of n linear equations for the particle size dependent concentrations $\bar{n}_p$, if the velocity is independent of particle size and aligned with the main flow direction. Since the smallest particle size generates only burst lengths of l_1, the matrix described by Eq. (8) has a triangular structure and can be easily inverted. An example for a hypothetical trimodal particle size distribution is shown in Fig. 2.

The inversion algorithm begins with the largest burst length for $i = n$ and assumes a linear burst-length distribution as shown in Fig. 1. This distribution results in the concentration of the largest particle sizes $p = C$ and is then subtracted

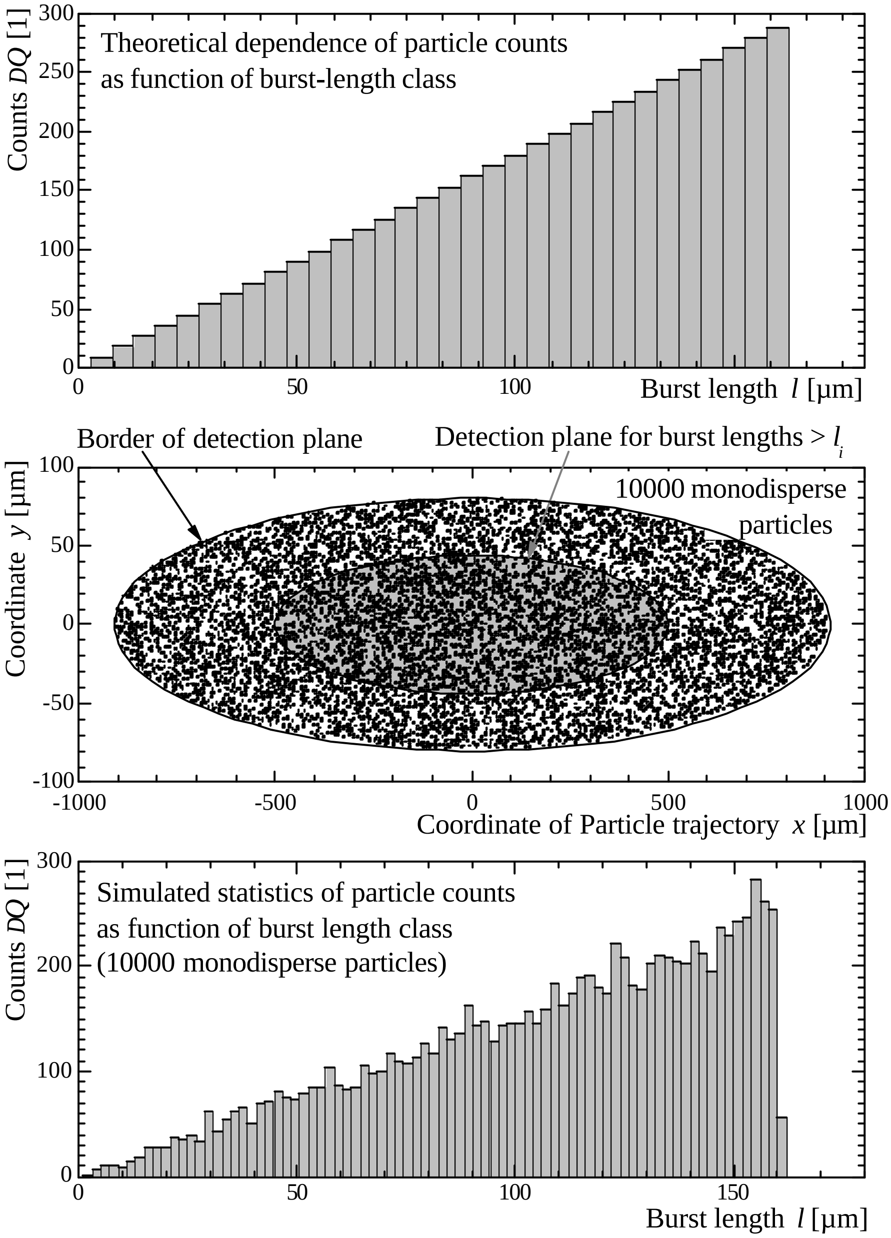

Fig. 1: Burst-length statistics for monodisperse particles. a) Theoretical dependence, b) Simulated statistical particle trajectories, c) Statistical burst-length distribution.

from the original burst-length distribution. In a similar manner the particle concentrations for the following size classes $p = C-1,\dots,0$ can be calculated. Each inverted burst-length class can be related to a particle diameter by using Eqs. (2) and (3). To obtain absolute diameters a calibration with particles of known diameter is necessary, because at least one point of the transfer function from laser intensity to particle size for the specific laser Doppler system has to be known.

3. SIMULATION OF THE TECHNIQUE

Recognizing that realization of the cross-sectional area difference technique requires a monotonic scattering function for the oscillating component of the signal $g(d_p)G(d_p)$, calculations of this quantity were carried out with the Lorenz-Mie theory for a backscatter and forward scatter configuration of a laser Doppler system. The scattering function for the given wavelength (514 µm) and beam intersection angle is influenced by the receiving angle, polarisation, distance to the receiver and by the size and shape of the aperture. The scattering functions in the particle size range of 0.5 – 30 µm have been studied.

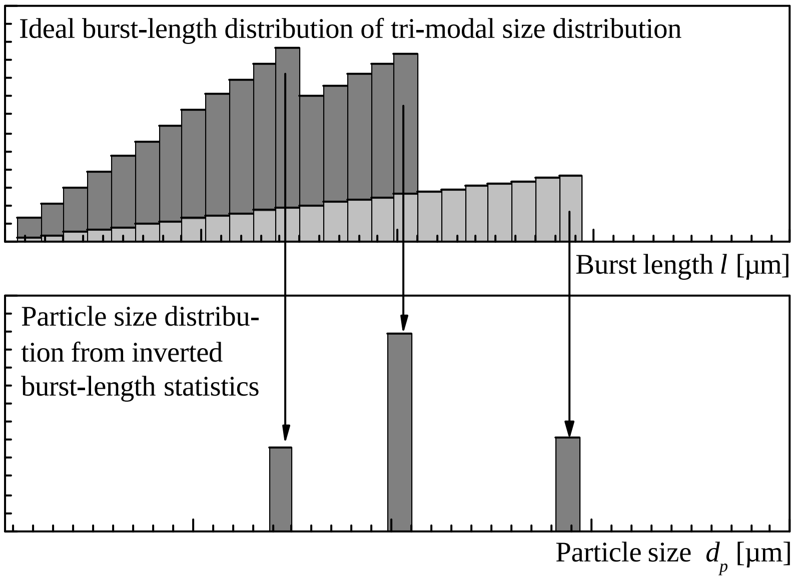

Fig. 2: Ideal burst-length statistics of a tri-modal particle size distribution and their statistical inversion.

To obtain a monotonic and smooth intensity-diameter relation in the forward scatter the scattering angle J_r must be in the range 20deg < J_r < 60deg, with the optimum around 30deg, because the first-order refraction dominates in this angular region. Examples of scattering functions for $J_r = 30\ \deg$ are shown in Fig. 3a.

In the standard backscatter laser Doppler arrangement the central part of the front lens defines the aperture. Nevertheless a mask can be installed to vary the size and position of the aperture.

The calculations carried out for the various sets of system parameters have shown that the monotonic behaviour of the intensity-diameter relation can be obtained neither directly in backscatter nor in the off-axis arrangement. The scattering function exhibits very large oscillations in all cases. The modulated part of the scattered intensity for such a backscatter arrangement is shown in Fig. 3b.

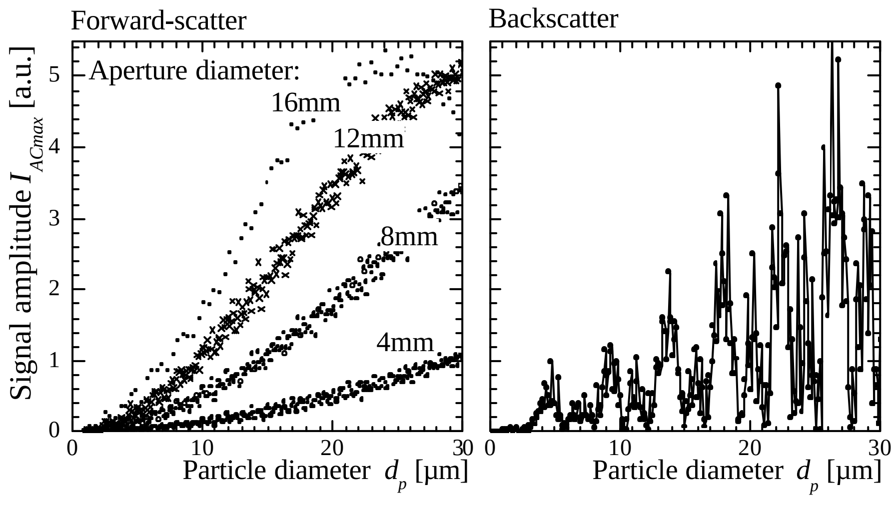

Fig. 3: Maximum modulated signal intensity as a function of the particle diameter. a) Forward scatter configuration with a nearly monotonic relation, b) Backscatter configuration exhibiting strong oscillations.

In the next step a flow of monodispersed particles moving with a constant velocity in the main flow direction was simulated; only the coordinates of the particle trajectory were randomly distributed. The burst-length distribution of such a simulation was already shown in Fig. 1c. The burst-length distribution is no longer smooth, due to the fact that the tra-

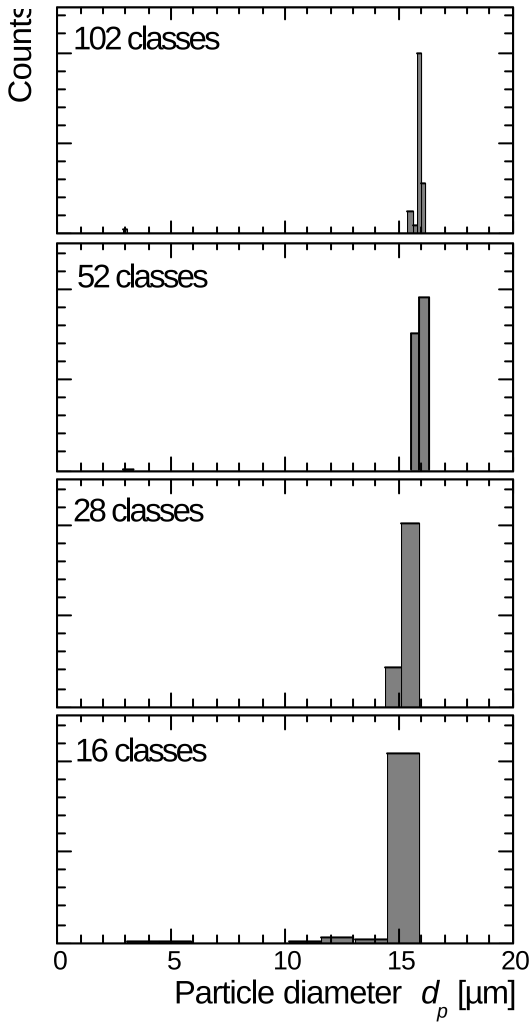

Fig. 4: Computed particle size distributions using various numbers of size classes.

jectory of the particle through the measurement volume is randomly chosen. Consequently, the computed size distributions demonstrate scatter, particularly for small size intervals and a large number of classes, as used in Fig. 4.

In a further simulation step the case of random velocities, particle sizes and coordinates was considered. For these simulations a Gaussian particle size probability density function with a standard deviation of 1µm and different mean values was assumed. This corresponds to the adjustable particle size distribution of the commercial monodispersed particle generator MAG 2010 (PALAS) used for experimental verification of the technique. This particle size distribution will be considered 'monodispersed'. The velocity was considered independent of particle size, aligned with the main flow direction and prescribed using a Gaussian probability density with a mean of 10m/s. Furthermore, a log-normal particle size probability function with different mean diameters was also applied to simulate the system response to polydispersed sprays. The scattering functions in forward scatter and in backscatter were used to calculate the burst length and to convert the inverted burst-length distribution into a particle size distribution.

The estimated particle size distribution in forward scatter shows good agreement with the input particle size distribution, as can be seen in Fig. 5.

The program for the simulation of laser Doppler bursts was also used to obtain the system response in backscatter for 'monodispersed' particles and a log-normal particle size distribution. Because of the non-monotonic scattering dependence on particle diameter, as seen in Fig. 3, the estimated particle size distribution exhibits large scatter and oscillations. Therefore the cross-sectional area difference method cannot be used for measuring narrow particle size distributions in backscatter. Nevertheless, the mean value of the inverted burst-length statistics could be considered a measure of the particle size distribution.

An additional aim was to check the statistical stability of the measured mean value, depending on the number of the processed bursts. The response in backscatter for 'monodispersed' particles is shown in Fig. 6. A large number of data sets were used to obtain estimates of the mean and standard deviation, as shown in the diagram by vertical 'error' bars. As can be seen, the statistical reproducibility is very good.

The results for narrow Gaussian ('monodisperse') distributions measured in backscatter are shown in Fig. 6a,b for different numbers of simulated samples. As expected, the influence of the oscillating scattering function from Fig. 3 is still visible and prevents particle sizing. The variance of the estimated mean value reduces with the number of collected samples. In forward scatter, not shown here, the relation between particle diameter and mean inverted burst length is monotonic because of the monotonic relation between signal amplitude and particle diameter.

The sizing response for the log-normal particle size distribution is shown in Fig. 6c,d for different number of samples. The statistical stability in this case is a bit lower, because of the randomly generated particle size distribution. However more important is that the overall form of the curve is monotonic, even the slope is smaller in comparison to the 'monodispersed' distribution. The monotonic dependence is due to the integration of the scattering function over all intervals of the distribution. The particle size distribution effectively averages over the oscillations seen in Fig. 3. Because the scattering function increases in mean, also the inverted burst length increases monotonically. Thus, the simulated sizing response for polydispersed particles demonstrates the ability of the technique for estimating the mean particle size of a wider particle size distribution in backscatter.

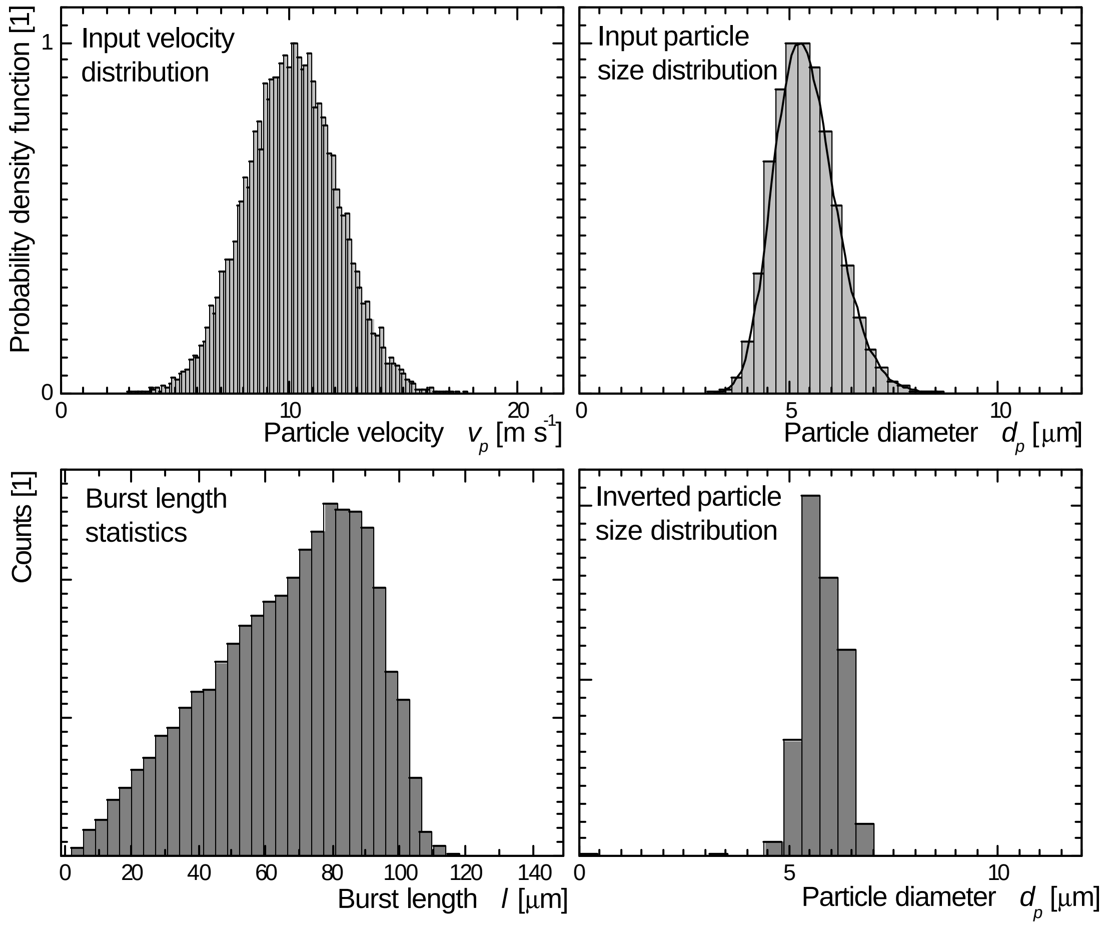

Fig. 5: Example of a simulated polydispersed spray and the estimated particle size distribution from the burst-length statistics.

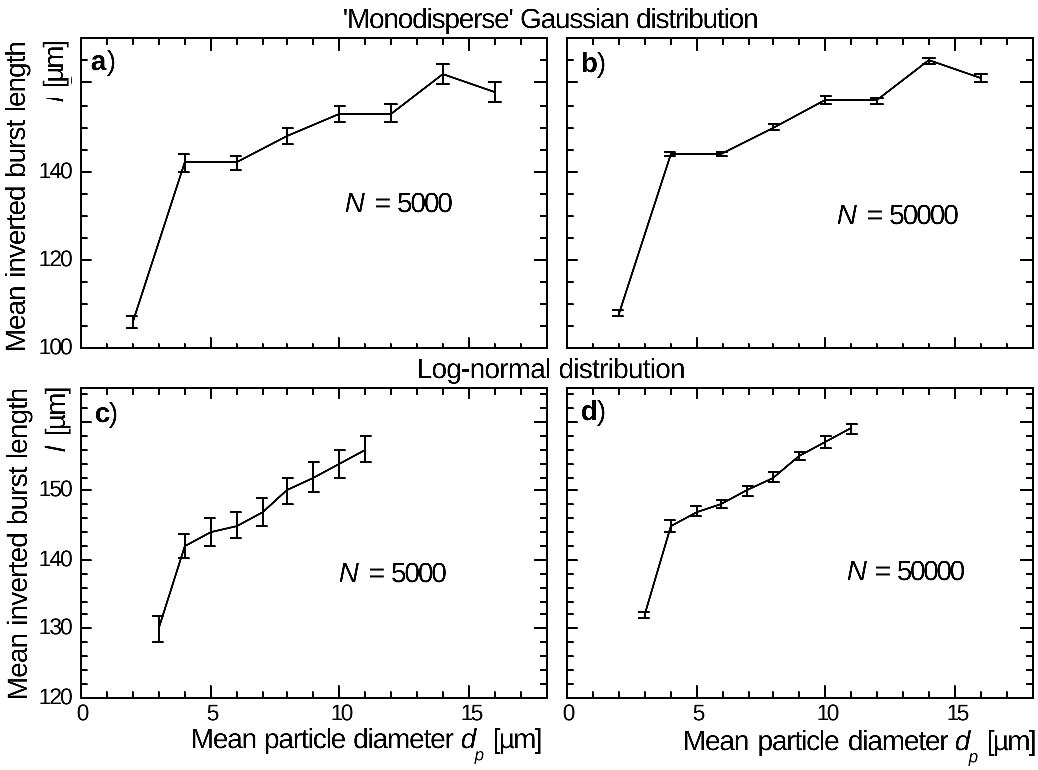

Fig. 6: Estimated mean inverted burst length and its variance for different number of samples as a function of the mean diameter of 'monodispersed' and log-normal particle size distributions.

4. EXPERIMENTAL VERIFICATION

Experimental testing of the technique was carried out using a commercial monodispersed particle generator MAG 2010 (PALAS). The so-called Sinclair-LaMer principle generates monodispersed and highly concentrated test aerosols using heterogeneous condensation.

To produce particles of desired size the boiler temperature must be set up in accordance with a calibration curve. Since the particle generator borrowed for the experiment from the PALAS company was not calibrated, it was necessary to measure the size of the generated particles. This was accomplished using a two-velocity component phase Doppler instrument. For the present measurements, a DantecDynamics MultiPDA system was used. This system was configured for the standard operational mode, which is advantageous for sizing very small particles.

For the burst-length measurements in backscatter the phase Doppler system was additionally equipped with a separate laser Doppler receiver and processor (DantecDynamics BSA F80), operating on-line with a second PC. The backscatter signal was supplied to the detector by means of a backscatter fiber in the optical probe. For burst-length measurements in forward scatter one of the phase Doppler signals was connected to the laser Doppler processor instead of the backscatter receiver. The experimental arrangement is shown schematically in Fig. 7.

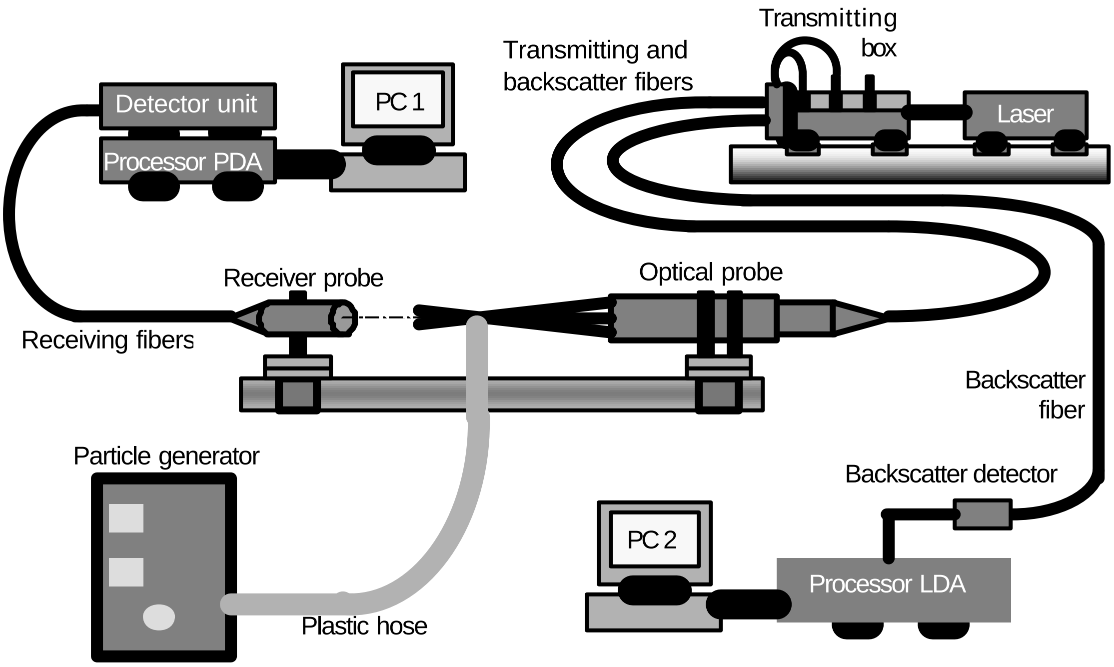

Fig. 7: Experimental setup

Data acquired at various temperature settings of the boiler, corresponding to specific particle diameters, consists of 3 different data sets: phase Doppler data, laser Doppler data set for backscatter and laser Doppler data for forward scatter. Processing of the phase Doppler data was carried out using the standard DantecDynamics software to obtain the mean particle diameter and the particle size distribution. Both laser Doppler data sets – forward and backscatter – were processed with customized software to obtain in the first step the burst-length distribution. In the next step these distributions were processed to obtain the reduced burst-length (particle size) distribution and finally the mean value of the reduced burst length was estimated.

The processing was performed off-line. One of the first results of the processing revealed that some measurements were taken with saturation of the signal in the processor. In this case the detected particle density is too high and the measured burst length is underestimated because of possible multiple burst signals from one particle. In principle however, an on-line processing would allow the amplification to be adjusted accordingly, using either the photomultiplier high-voltage or the input gain on the signal processor. These first test experiments showed that the cross sectional area difference method is very sensitive to the required single-realisation of the laser Doppler technique.

The sizing response in forward scatter and backscatter for two amplification levels is shown in Fig. 8. These diagrams demonstrate that in the range of particle sizes studied, the system amplification can be chosen in such a way, that the curve for forward scatter shows a monotonic increase with the sizing parameter – the mean of the reduced burst-length distribution - with the actual particle size. For backscatter the influence of the oscillating scattering function is clearly seen by the non-monotonic response. This was expected for the 'monodispersed' particle size distribution and confirms very well the simulations from section 0. The experiment shows that the length of the burst evaluated from the transit time and velocity of the individual particle measured by the laser Doppler processor, follows the theoretically predicted behaviour. The mean value of the reduced burst-length distribution, obtained after processing with the software, can be used for particle sizing in forward scattering. Taking into account the monotonic nature of the scattering function, this behaviour can also be expected for larger particles. Therefore this technique can be used to validate phase Doppler measurements and to extend the size range of a phase Doppler configuration to particle sizes smaller than 1µm. Particle sizes smaller than 0.5µm can be measured in backscatter, because then the scattering function is strictly monotonic. For the case of wider particle size distribution the different particle sizes are integrated over the oscillations of the scattering functions and a monotonic relation is expected. Nevertheless, these conclusions for backscatter should still be confirmed with further experiments. One difficulty is the availability of a controllable polydispersed particle generator.

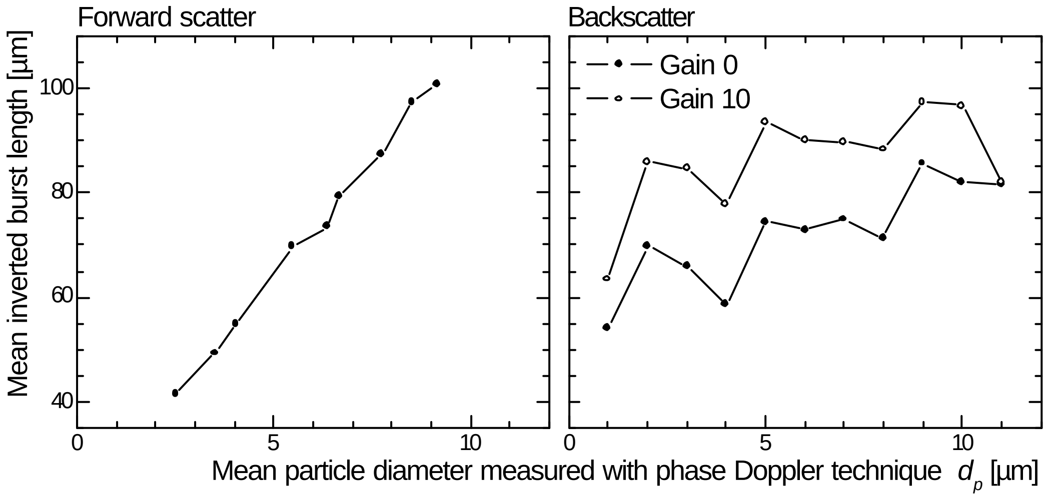

Fig. 8: Measurements of 'monodispersed' Gaussian particle size distributions. The mean inverted burst length is shown for forward and backscatter measurements.

5. SUMMARY

The purpose of this work was to investigate the feasibility of using the cross-sectional area difference method for sizing droplets larger than 1µm in forward and especially in backscattering by using commercial laser Doppler equipment. The prerequisites for this technique are as follows: no correlation between particle size and velocity and a monotonic relation between particle size and amplitude of the modulated signal part. Furthermore measurement of absolute particle size requires a calibration with particles of known diameter and of the same medium. Whether operating in forward scatter or backscatter, a stable estimation of the particle size distribution requires a rather large sample size, since the information content of the signal arises out of the burst length increments between size classes and thus represents the difference of two large numbers.

The feasibility study consisted of theoretical considerations, numerical simulations and laboratory experiments. The overall conclusion is that the technique is feasible for sizing particles smaller than 0.5µm in forward and backscatter direction and for sizing particles larger than 1µm in forward scattering. The technique can also been used in backscatter for measuring the mean particle diameter for an assumed wider particle size distribution, and for monitoring temporal deviations in the size distribution. The backscatter arrangement is preferable due to handling and operational considerations.

The implementation of this technique builds on existing optical instruments and modifies the data processing software. This has been accomplished in the present study using a provisional program operated off-line. The experiments indicate that on-line processing is highly desirable, since a correct adjustment of the instrument, in particular the gain and high voltage settings, is necessary and can only be determined when viewing the processed results. Therefore, an on-line version of the processing software should be introduced at the next possible stage.

Measurements with the monodispersed droplet generator and adjustable polydisperse sprays will be performed in the future to further verify the operation of the instrument in backscatter.

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