Application of Particle Image Velocimetry
to High-Speed Supersonic Flows in a Shock Tunnel

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ABSTRACT

Particle Image Velocimetry (PIV) has become an important tool in modern flowfield diagnostics, in particular for subsonic flows. PIV systems have also been used to measure supersonic flows in supersonic wind tunnels, but only recently it has been shown that PIV can also be a valuable tool for high-speed supersonic flows in a shock tunnel (Haertig et al., 2001). At the ISL shock tunnel a PIV system was installed and tested (Fig. 1). A shock tunnel is a transient wind-tunnel facility with short testing times of a few milliseconds and high supersonic flow velocities in the km/s range. The short flow duration time makes a high information density highly desirable. An application of PIV to such kind of flows is considerably more difficult than to a continuous flows, however, because no online adjustments of the particle seeding and the optics can be done.

In this publication, PIV measurement results obtained for flows at Mach numbers of 4.5 and 6 with velocities of up to 1.8 km/s are presented. We measured the freestream flow of nozzles, supersonic wedge flows with an attached shock wave, and the supersonic flow around a sphere with a detached shock wave. To obtain a good spatial resolution a laser pulse separation of less than 0.5 µs is necessary. Solid particles (TiO$_2$ and Al$_2$O$_3$) with a nominal diameter of about 0.3 µm were used for the seeding. The measured particle relaxation across shock waves was found to be consistent with theoretical estimations. Because of the strong particle density gradients in shock waves, the PIV correlation algorithm can contribute to considerable inaccuracies, however.

Fig. 1. ISL shock-tunnel facility with PIV system.
1. INTRODUCTION

Velocity measurements are a difficult task in shock tunnel flows because strong spatial and temporal flow gradients occur in short testing times of typically milliseconds. Additionally, the high-energy flows produced in shock tunnels require the use of non-intrusive measurement techniques due to the formation of shock waves with non-equilibrium zones in front of probes. The short measuring times and the high velocity range make the application of well-established measurement techniques like laser Doppler anemometry (LDA) nearly impossible. Therefore, a particular laser Doppler velocimeter based on a Michelson interferometer was designed by Smeets and George (1978) at the French-German Research Institute of Saint-Louis (ISL) more than 20 years ago. This kind of velocimeter allows accurate velocity measurements even in short-duration high-speed flows. Its single-point measuring character makes systematic flowfield measurements very time- and cost-consuming, however. The PIV technique allows an instantaneous and multi-component measurement of the velocity field, which makes it ideally suited for shock tunnel flows. High-speed PIV measurements are not often found in the literature, however. First PIV measurements performed at the ISL were presented by Haertig and Smigielski (1986) for the 3rd International Symposium on Applications of Laser Anemometry to Fluid Mechanics. In the following, PIV has been applied to blow-down or continuously running wind tunnels with low stagnation states (Johé et al., 1996), which implies relatively low velocities even for high Mach numbers (velocities smaller than 1 km/s for Mach 6, Humphreys et al., 1993). Only recently PIV has been successfully applied to shock tunnel flows for the first time worldwide at the ISL (Haertig et al., 2001). Freestream velocities of up to 1.5 km/s in a Mach-4.5 flow were measured with a high accuracy. The velocity jump across shock waves was studied using wedges and blunt bodies (Havermann et al., 2001). The seeding particle performance was found to be consistent with theoretical estimations.

2. ISL SHOCK-TUNNEL FACILITY STA

A shock tunnel is a short-time-duration wind-tunnel consisting of a shock tube connected to a supersonic nozzle and a test chamber (Fig. 2). The ISL shock tube has an inner diameter of 100 mm and is divided into a 2.7-m-long, high-pressure driver tube and a 18.4-m-long, low-pressure driven tube. The driver gas is a mixture of hydrogen and nitrogen at a pressure of up to 50 MPa, whereas the driven or test gas consists of pure nitrogen at a pressure of up to 0.5 MPa. The two sections are separated by a steel diaphragm with a thickness of up to 4 mm. After pumping up the high-pressure section, the diaphragm bursts and a shock wave is formed. The shock wave propagates at a supersonic speed into the low-pressure test gas, which is consequently heated and pressurized. At the end of the driven tube, the shock wave is reflected at the beginning of the convergent entrance section of the nozzle. The shock travels back through the accelerated test gas, which is decelerated and further heated and pressurized. As a result, a highly heated and compressed stagnation gas volume is created at the nozzle entrance for some milliseconds. This gas is then expanded in the convergent-divergent nozzle to a supersonic velocity. The flow is stationary during the test time of approximately 2 milliseconds.

Two contoured Laval nozzles with an exit diameter of 220 mm and nominal Mach numbers of 4.5 and 6 are available. The test chamber contains the models to be studied and catches the shock-tube gases after the experiment. Additionally, the test chamber has optical access from three sides to apply in particular flow visualization methods. After each shot, the freestream flow conditions are recalculated using a one-dimensional shock-tube code, which requires the measured shock wave speed in the driven tube as an input. By varying the tube pressures, the freestream flow can be adjusted to duplicate flow conditions in the lower atmosphere down to height of 2.5 km for the Mach-4.5 nozzle, and down to a height of 13 km for the Mach-6 nozzle. At these heights, the corresponding flow velocity is about 1.5 km/s and 1.8 km/s, respectively.

Fig. 2. ISL shock-tunnel facility STA.
3. PARTICLE IMAGE VELOCIMETRY SYSTEM

A double-frame/single-exposure digital PIV system was installed at the ISL shock tunnel STA (Fig. 1). The light source consisted of a frequency-doubled Nd:YAG double-pulse laser (Quantel Twins) with a nominal pulse energy of 150 mJ each. A laser-light sheet (300 mm wide, 0.2 mm thick) perpendicular to the nozzle axis was created by means of a telescope and reflected vertically into the test section (Fig. 3). The imaging CCD-camera was mounted on the horizontal axis to view the illuminated flowfield behind the nozzle axis (Fig. 1), and it can acquire two images within a pulse delay in the microsecond range. The first experiments were conducted with the Mach-4.5 nozzle and at that time we used a TSI PIVcam (1000 x 1016 pixels). In the following, a sharpVISION 1300 DE camera (1280 x 1024 pixels) by IDT was available, which was used for the Mach-6 flows studied later. Nikon zoom objectives were mounted to the cameras to change the field of view. The laser and camera synchronizer are triggered by a heat flux sensor in the shock tube wall. The synchronizer separation time was checked by a fast-response photodiode and timing errors were found to be less than 1 %. Solid particles were chosen for the seeding because they had to withstand the high stagnation temperatures of up to 1700 K after the shock reflection at the nozzle entrance. For this reason, TiO$_2$ or Al$_2$O$_3$ particles dispersed by a fluidized-bed seeder were seeded into the low-pressure tube together with the nitrogen test gas before the experiment started. The PIV images were analysed after the experiment by a cross-correlation algorithm using the IDT software with adaptive interrogation window sizes down to 24 x 24 pixels. This software allowed a high spatial resolution analysis due to a mesh-free algorithm (Lourenço and Krothapalli, 2000).

4. EXPERIMENTAL RESULTS

4.1 Freestream Nozzle Flow

First, the freestream flowfields of the Mach-4.5 and -6 Laval nozzle are shown. The Mach-4.5 Laval nozzle flowfield was recorded with TSI’s PIVcam at a laser pulse separation of 1.5 µs with a field of view of 200 x 200 mm, a camera lens focal length of 50 mm, and an aperture of f-4. The optical calibration factor was determined to 201 µm/pixel so that a maximum particle displacement of about 11 pixels could be expected for a velocity of 1500 m/s. The data analysis was performed with TSI’s Insight 2.0 software using a correlation window of 64 x 64 pixels with a 50 % overlap shifting so that the spatial resolution was 32 x 32 pixels (~6.4 x 6.4 mm). The PIV picture (Fig. 4a) shows a rather homogeneous TiO$_2$ particle seeding except for the upper and lower part of the nozzle where the interaction of the freestream boundary layer with expansion waves creates a higher turbulence. The data analysis yields a homogeneous flowfield (Fig. 4b) with a measured mean horizontal velocity of 1440 ±17 m/s that is very close to the calculated flow velocity of 1455 m/s for an atmospheric altitude of 5 km.

The Mach-6 Laval nozzle flowfield (atmospheric height of 16 km) was recorded with IDT’s sharpVISION CCD camera at a pulse separation of 0.4 µs for a field of view of 180 x 140 mm, a camera focal length of 80 mm, and an aperture of f-5.6. The optical calibration factor of 140 µm/pixel allowed a maximum particle displacement of about 5 pixels for a velocity of 1800 m/s. The two images were analysed using IDT’s PIV software with a
correlation window of 64 x 64 pixels and a 50 % overlap shifting, which lead to a spatial resolution of 4.5 x 4.5 mm corresponding to 32 x 32 pixels.

In this case, $\text{Al}_2\text{O}_3$ particles were used for the seeding and the PIV picture (Fig. 5a) shows again a homogeneous seeding density except for the upper part of the nozzle with the expansion fan.

The measurement result for the horizontal flow velocity (Fig. 5b) shows a homogeneous flow in the nozzle core ($y < 125 \text{ mm}$), which gives an average value of 1755 m/s with a mean standard deviation of $\pm 16 \text{ m/s}$.

Both measured velocities are compared to theoretical velocities that were calculated with a one-dimensional shock tube / nozzle code (Tab. 1). A good agreement can be observed and the difference between measurements and calculation can be explained by the nozzle code, which did not include boundary layer effects. This means that the effective nozzle area is reduced so that the experimental velocity is lower than the theoretical one.

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**Fig. 4. Mach-4.5 Laval nozzle flow.**
(a) PIV picture with $\text{TiO}_2$ particles. (b) Horizontal flow velocity.

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**Fig. 5. Mach-6 Laval nozzle flow.**
(a) PIV picture with $\text{Al}_2\text{O}_3$ particles. (b) Horizontal flow velocity.

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**Table 1. Horizontal freestream velocity: Comparison of PIV measurement with calculated values.**

<table>
<thead>
<tr>
<th>Nominal Mach number</th>
<th>PIV: $u$ (m/s)</th>
<th>Calculated velocity (m/s)</th>
<th>Difference measurement-calculation, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>1440 ± 17</td>
<td>1455</td>
<td>-1.2</td>
</tr>
<tr>
<td>6</td>
<td>1755 ± 16</td>
<td>1790</td>
<td>-2.0</td>
</tr>
</tbody>
</table>
4.2 Supersonic Flow Over a Wedge

A wedge is a good test body in supersonic flows because a shock wave formed at the wedge tip causes a strong velocity gradient. For the present flow conditions and wedge geometry the shock wave is attached at the tip and the flow within the shock wave is uniform so that it can be calculated analytically using the gasdynamic shock relations.

For these reasons, a 20°-halfangle wedge was mounted into the test chamber and first measured for the Mach-4.5 flow (atmospheric height of 9 km) using TSI’s PIV cam (Fig. 6a). For the Mach-6 flow (atmospheric height of 21 km), IDT’s sharpVISION camera was used with a shorter pulse delay (Fig. 6b). To avoid laser light reflections, the wedge was painted black and slightly inclined (1°) in the spanwise direction.

Mach-4.5 wedge flow

In the Mach-4.5 PIV picture (Fig. 6a), the shock wave is very well visible on the upper part of the wedge where the particle density is increased. The fluid density increases by a factor of 3.1 across the shock. For a laser pulse delay of 0.8 µs, a field of view of 90 x 90 mm was imaged with a camera lens focal length of 135 mm at f-5.6. The optical calibration factor was calculated to 92 µm/pixel and the IDT software with a correlation window of 32 x 32 pixels and 50% overlap shifting was used for the analysis so that the spatial resolution can be estimated to 1.5 x 1.5 mm. The maximum particle displacement for a velocity of 1500 m/s was 13 pixels, which exceeded the maximum value of 8 pixels allowed by the correlation window size. An artificial shift of –8 pixels was applied to fulfill this condition.

The measured velocity distribution is shown in Figs. 7a and 7b for the horizontal and vertical velocity component, respectively. The correlation shows a good confidence level with only 10 interpolated vectors out of 2813. It can be seen that particle relaxation occurs within a small area after the shock. The averaged two-dimensional velocity distribution on the wedge corresponded very well to the calculated velocities (Tab.2).
Mach-6 wedge flow

For the Mach-6 flow, the IDT system was used with a shorter laser pulse delay of 0.4 µs. Again, the shock wave is well visible on the upper part of the wedge (Fig. 6b), where the fluid density increases by a factor of 3.7. A field of view of 111 x 89 mm was imaged with a camera lens focal length of 150 mm at f-5.6. The optical calibration factor was calculated to 86.9 µm/pixel. The IDT software allowed a minimum correlation window size of 24 x 24 pixels so that the spatial resolution was about 1 x 1 mm. An artificial shift of –6 pixels was used to account of the expected maximum particle displacement of 8 pixels for a velocity of 1800 m/s. The measured velocity distribution is shown in Figures 8a and 8b for the two velocity components. The correlation was good showing only 9 interpolated vectors out of 6178. The thickness of the particle relaxation zone at the shock wave is smaller compared to the Mach-4.5 wedge flow, which is due to the higher spatial resolution. Again, the averaged two-dimensional velocity distribution on the wedge corresponded very well to the calculated velocities (Tab. 2).

Table 2. Calculated and measured freestream velocity before the shock and horizontal (u) and vertical (v) velocities of the flow over a wedge after the shock.

<table>
<thead>
<tr>
<th>Nominal Mach number</th>
<th>Before shock</th>
<th>After shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated velocity / (m/s)</td>
<td>Measured velocity / (m/s)</td>
</tr>
<tr>
<td></td>
<td>u / (m/s)</td>
<td>u / (m/s)</td>
</tr>
<tr>
<td>4.5</td>
<td>1415</td>
<td>1380±18</td>
</tr>
<tr>
<td>6</td>
<td>1793</td>
<td>1768±44</td>
</tr>
</tbody>
</table>

4.3 Supersonic Flow Around a Sphere

In a supersonic flow a detached bow shock wave is formed in front of a sphere. Inside the shock a three-dimensional flow with a subsonic, a transonic, and a supersonic zone is established, which make this kind of flow important and interesting for the validation of computational fluid dynamics codes.

A sphere 120 mm in diameter was installed in the shock tunnel and experiments at the Mach-4.5 and Mach-6 flow conditions (atmospheric heights of 10 and 16 km, respectively) were conducted. Consistent with the wedge flows, the Mach-4.5 flow was imaged by TSI’s PIVcam using TiO₂ particles, whereas the Mach-6 flow was studied using IDT’s sharpVISION camera and Al₂O₃ particles.

Mach-4.5 sphere flow

For this experiment the same optical and laser pulse separation adjustments were used as for the Mach-4.5 wedge flow condition (cf. section 4.2). The bow shock is very well visible in Fig. 9a at the edge of the increased particle density. The PIV analysis was performed by IDT’s software with a 32 x 32 pixels correlation window size and yielded 98 interpolated vectors out of 2212. The corresponding result of the PIV correlation process is
shown in Fig. 9b where the subsonic and transonic zone (blue) can be well distinguished from the supersonic zone (green and red) behind the bow shock.

![Fig. 9. Mach-4.5 flow around a 120 mm diameter sphere. (a) PIV image. (b) Velocity field.](image1)

**Mach-6 sphere flow**

The same optical and laser settings were used for the Mach-6 sphere flow as for the Mach-6 wedge flow (cf. section 4.2). The bow shock is very well visible in the PIV picture (Fig. 10a) and it is closer to the sphere compared with the Mach-4.5 flow case. The reduced laser pulse separation of 0.4 µs allowed a minimum correlation window size of 24 x 24 pixels. The PIV correlation yielded a good result with no interpolated vector out of 4895 (Fig. 10b). The sub-and transonic region inside the bow shock is characterized by the blue colour.

![Fig. 10. Mach-6 flow around a 120 mm diameter sphere. (a) PIV image. (b) Velocity field.](image2)

**5. SEED PARTICLE PERFORMANCE**

The seed particle performance is important for all laser measurement techniques based on light scattering. A good light scattering behaviour (large particle diameter) competes with a high frequency response, which is particularly important in supersonic flows. These flows exhibit strong velocity and density gradients due to shock waves so that seed particle dynamics plays an important role for estimating the flow tracing capability. A simplified approach of the equation of motion leads to the following differential equation for the particle velocity $u_p$ adapting to a fluid velocity $u$, if Stokes’ drag law for spheres is used:

$$ u_p = \frac{C_d}{\rho_f} \frac{1}{2} \rho_f u^2 A_p $$
\[
\frac{du_p}{dt} = \frac{u-u_p}{\tau} 
\]  

Here \( \tau \) represents the particle relaxation time defined as 

\[
\tau = \frac{\rho_s d_p^2}{18 \mu},
\]  

where \( \rho_s \) is the particle density, \( d_p \) its diameter, and \( \mu \) the fluid viscosity (1.8 \( \cdot \) 10\(^{-5} \) kg/m/s).

If a sharp velocity drop like across a shock wave is considered (from \( u_1 \) down to \( u_2 \)), the solution of Eq. 1 reads:

\[
u_p = (u_1 - u_2) \exp \left( -\frac{t}{\tau} \right) + u_2
\]  

This solution shows that after one particle relaxation time the relative particle velocity \( \frac{u_p-u_2}{u_1-u_2} \) reduces to \( 1/e \), and after two particle relaxation times to \( 1/e^2 \), which is a good measure of the particle relaxation length. An integration of the particle velocity yields the particle trajectory

\[x_p = (u_1 - u_2) \left( 1 - \exp \left( -\frac{t}{\tau} \right) \right) + u_2 t
\]  

which for \( t=2\tau \) gives the particle relaxation length. This length can be estimated by analysing the stagnation streamline of the supersonic sphere flow (section 4.3). On this streamline, the bow shock can be assumed to be a normal shock, which can be calculated using the Rankine-Hugoniot relations.

The measured horizontal and vertical flow velocity component along the stagnation streamline are plotted in Fig. 11 for the Mach 4.5 flow. For comparison, the shock location found from the PIV image with the corresponding theoretical horizontal velocities across a normal shock is shown. It can be seen that the shock, which theoretically has a negligible thickness, is smeared to \( x/D=0.04 \) (4.8 mm) both in the upstream as well as in the downstream direction. Because there is no physical reason for an upstream thickening effect, this is attributed to the PIV correlation algorithm, which seems to have problems in correctly treating the strong increase of particle density across the shock. The spatial resolution, however, is further limited by the correlation window size of 32 x 32 pixels (1.5 x 1.5 mm), which was a result of the laser pulse separation of 0.8 \( \mu \)s. The particle relaxation length estimated only from the downstream thickening is of the order of \( x/D=0.015 \) (1.8 mm), which is in good accordance with the calculated length using Eq. 4 (Tab. 3).

\[\text{Figure 11. Stagnation point streamline of Mach-4.5 sphere flow (x/D=0 corresponds to the body surface).}\]
For the Mach-6 flow case, an improved spatial resolution of the shock region was possible compared with the Mach-4.5 case due to a smaller correlation window size of 24 x 24 pixels. In Fig. 12, the measured horizontal and vertical velocity components are plotted with the theoretical velocity drop across a shock. Additionally, the results from a CFD calculation (Berner, 2002) are plotted, which show a good agreement with the measurements downstream of the shock. Up- and downstream shock thickening from the PIV measurement is about x/D=0.03 (3.6 mm) and the estimated particle relaxation length from the downstream thickening is comparable to the Mach-4.5 case (x/D=0.015). This corresponds to an aerodynamic particle diameter of about 0.28 µm, which is close to the nominal diameter (Tab. 3).

Table 3. Seed particle relaxation for the supersonic sphere flows.

<table>
<thead>
<tr>
<th>Nominal Mach number</th>
<th>Particles</th>
<th>Density / (kg/m³)</th>
<th>Nominal diameter / (µm)</th>
<th>Relaxation time / (µs)</th>
<th>Relaxation length / (mm)</th>
<th>u1 / (m/s)</th>
<th>u2 / (m/s)</th>
<th>Estimated Relaxation length / (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>TiO₂</td>
<td>4260</td>
<td>0.32</td>
<td>1.35</td>
<td>2.0</td>
<td>1370</td>
<td>270</td>
<td>1.8</td>
</tr>
<tr>
<td>6</td>
<td>Al₂O₃</td>
<td>3960</td>
<td>0.3</td>
<td>1.10</td>
<td>2.1</td>
<td>1780</td>
<td>310</td>
<td>1.8</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

Particle Image Velocimetry was applied to high-speed supersonic flows in a shock tunnel. First, two freestream nozzle flows with Mach numbers of 4.5 and 6 were measured. The PIV measurements agreed very well with the calculated nozzle velocities and showed less than 2 % difference. After that the supersonic flow over a 20°-halfangle wedge was studied. This flow has a strong velocity and density gradient caused by an oblique shock wave at the wedge tip. To resolve the shock wave, both a good spatial resolution and a good particle tracking capability are necessary. The spatial resolution was improved by reducing the laser pulse separation down to 0.4 µs and by using a sophisticated PIV correlation algorithm by IDT. Thereby a minimum correlation window size of 24 x 24 pixels could be achieved giving a spatial resolution of 1 x 1 mm for a field of view of 111 x 89 mm. The PIV measurement of the supersonic flow around a sphere clearly showed the detached bow shock and the complex flowfield between the shock and the sphere surface. The stagnation point streamline was analysed in detail to study the seed particle tracking behaviour. It was found that the PIV correlation algorithm has some difficulties in treating the strong density gradients at shock waves. This effect together with particle relaxation lead to an artificial thickening of the shock. The particle relaxation length was estimated from the downstream shock thickening and found to correspond with theoretically predicted values.

Particle image velocimetry has proven to work efficiently and accurately in transient high-speed supersonic flows. Compared to the first measurements the spatial resolution has been improved considerably by using a better CCD camera suited for short laser pulse separations and an efficient correlation software. It is planned to improve the spatial resolution further by reducing the laser pulse separation down the smallest value possible (0.2 µs). PIV will be then used in the ISL shock tunnel to study more complex flows than shown here.
REFERENCES

Berner, C. (2002), Personal communication, ISL.


