Spatio-temporal flow structure investigation of near-wall turbulence by means of Multiplane stereo Particle Image Velocimetry

C J Kähler¹, M Stanislas², J Kompenhans³

1) Institut für Strömungsmechanik, TU-Braunschweig, Bienroder Weg 3, 38106 Braunschweig, Germany
2) Laboratoire de Mécanique de Lille, Bd. Paul Langevin, 59655 Villeneuve D’ASCQ CEDEX, France
3) Deutsches Zentrum für Luft- und Raumfahrt (DLR), Bunsenstrasse 10, 37073 Göttingen, Germany

One of the well established results of turbulent boundary layer research is the characteristic velocity profile, shown in the top row of figure 1, and the maximum of the stream-wise velocity fluctuation u in the buffer layer at \(y^+\approx 15\), see lower left graph. However, when the stream-wise fluctuations are subdivided according to their sign before calculating the rms values, it turns out that the negative fluctuations possess a maximum at \(y^+\approx 20\), while those with a positive component reveal the maximum at \(y^+\approx 10\). This is shown in the lower right graph together with the rms values of the velocity component in wall-normal direction associated with the particular positive and negative stream-wise velocity fluctuation.

![Figure 1](image)

**Figure 1**: Results of high-resolution stereoscopic PIV boundary layer measurements at \(Re_\theta = 7800\). Top: Linear and semi-logarithmic representations of the dimensionless mean velocity profile (average over 2500 velocity fields). Bottom: Root-mean-square values of the stream-wise and wall-normal velocity fluctuations \(u\) and \(v\) (left) and conditional averaged representation (right).

It is not surprising that the maxima do not coincide as the high momentum flow structures originate statistically from large \(y^+\) locations, where \(U^+_\text{mean} \sim \ln(y^+)\), while low momentum flow structures have their origin in the near wall region on average, where \(U^+_\text{mean} \sim y^+\) holds, see upper right figure. However, the fact that the maximum of \(u_{\text{rms}} (u < 0)\) is further away from the wall with respect to \(u_{\text{rms}} (u > 0)\) is not evident. In order to examine the physics behind this experimental result, the statistical properties of a fully developed turbulent boundary layer flow along a flat plate is investigated in stream-wise span-wise planes at \(y^+ \approx 10, 20, 30\) and \(Re_\theta = 7800\) by using the multiplane stereo PIV technique described by Kähler and Kompenhans (2000). First of all, the joint probability density distributions of the velocity fluctuations will be analyzed in order to determine the basic properties of the coherent velocity structures like their occurrence, intensity and main flow direction relative to the wall. Thereafter various spatial correlation and cross-correlation functions are presented in order to compare the mean size and shape of the coherent structures present in the flow field. The dynamics of the dominant structures is investigated by means of spatio-temporal correlation and cross-correlation functions measured in spatially separated planes. Finally, two characteristic flow fields are considered in order to illuminate the relation between the instantaneous structures with respect to the averaged ones and to estimate the contribution of the identified coherent flow structures for the production of turbulence and transport of Reynolds stresses.
Experimental set-up

The experimental investigation described in the following was performed in the closed-circuit wind-tunnel at the Laboratoire de Mécanique de Lille, see Carlier et al (1999) and Kähler et al (2001) for details, with the transportable PIV-equipment developed at the German Aerospace Center (DLR) in Göttingen. The multiplane stereo PIV technique applied for this investigation consists of a four pulse Nd:YAG laser system (BMI model 5013 DNS 10) with approximately 255 mJ output energy per pulse at $\lambda = 532$ nm. The system was arranged in such a way that the differently polarized light pulses leave the laser housing at two different output ports, see Kähler and Kompenhans (2000). Behind each port one optical bench was installed with three appropriate lenses and a $\lambda/2$-retardation plate which allows to rotate the direction of the polarization vector continuously, see left picture of figure 2. By using long focal length lenses, the laser beams could be formed into sheets of about 0.6 mm thickness at the measurement position, corresponding to $\Delta y \approx 5$. The positions of the light-sheets could be smoothly adjusted in vertical direction by moving properly coated mirrors, mounted on a translation table. The determination of the exact distance of the light-sheets from the wall was achieved by illuminating a square metal block, placed on the flat plate and covered with light-sensitive paper.

![Figure 2](image)

Figure 2: Experimental set-up for xz-investigation. Left: Double light-sheet optics with $\lambda/2$-retardation plate. Right: Four pulse laser system and multiplane stereo PIV recording unit with detached lenses.

For the data acquisition, four Peltier cooled high resolution PCO cameras with 1280 by 1024 pixel resolution and 12 Bit dynamic range have been employed each connected with a one-axis Scheimpflug adapter in order to adjust precisely the angle between the image-, object- and main-plane of the lens according to the principles described by the Austrian aerial cartographer Theodor Scheimpflug (1865−1911) in his British Patent (GB 1196/1904) from 1904. For magnification and field of view adjustments all Scheimpflug-adapters were mounted on a two-axis linear translation stage which allows high precision translations by thumb screws and the polarizing beam splitter-cubes and mirrors in front of the lenses were connected to two-axis tilt-rotation stages and gimbal mirror mounts. In order to obtain ideal particle images for the image analysis algorithms (bright circles, 2-3 pixel in diameter, surrounded by a dark background), the imaging of the field of view was performed by means of 100 mm Carl Zeiss lenses with an aperture of 8. The arrangement was installed below the wind-tunnel, as shown in the right picture of figure 2. The mean observation distance was 840 mm and the opening angle between the left and right camera systems was 94.4 degree in order to resolve the out-of-plane motion properly. For the calibration of the system a regular grid with 2 mm line spacing was glued on a perfectly plane aluminum plate, and attached with a micrometer translation stage in such a way that a very accurate parallel motion of the grid could be achieved in vertical direction. This grid was precisely aligned with each light-sheet one after another and recorded each time with the four cameras (before and after the experiment in order to proof...
the conservation of the boundary conditions during the experiment). As any translation of the target in the xz-plane could be excluded, a particular xz-position in a plane at y can be precisely related to a xz-position in a plane at y+$\Delta y$. This is extremely important as any uncontrolled movement of the calibration target would appear in the correlation data of the velocity fluctuations and could bias the interpretations of the results, see Kähler (2000). Due to the small spatial separation of only ten wall units between the differently polarized light-sheet planes it was not required to move any camera when changing measurement position as all effects could be uniquely determined by the precise calibration technique along with the calibration validation method described by Kähler (2000). Only the focus was slightly adjusted in order to keep the image contrast and to avoid out-of-focus effects. The determination of the mapping function was achieved by using Hough transformation algorithms as described by Ehrenfried (2001) and the evaluation of the stereo-spicic images was performed on the DLR SUN-cluster by applying the properly normalized free-shape correlation described by Ronneberger et al. (1998). For the displacement estimation with sub-pixel accuracy a two-dimensional Gaussian peak-fit routine was applied as this approach is less sensitive to peak-locking effects due to the small variation of the measurement error with the sub-pixel displacement, see Kähler (2000). By applying the following set of band-pass and gradient filters ($1 < \Delta x < 12$ pixel; $-4 < \Delta y < 4$ pixel and $\Delta x_i - \Delta x_{i+1} < 5$ pixel), the number of correct measurements was on average above 99.9 % and no smoothing algorithm was applied at all. The basic details about the recording and evaluation are summarized in the following table.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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</tr>
<tr>
<td>$Re_5$</td>
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<tr>
<td>$Re_x$</td>
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<td>$\delta$</td>
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<tr>
<td>field of view</td>
<td>0.18 $\times$ 0.09</td>
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<tr>
<td>field of view</td>
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<tr>
<td>spatial resolution</td>
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<tr>
<td>spatial resolution</td>
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<tr>
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<td>Vectors per sample</td>
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<td>Number of samples</td>
<td>4410</td>
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</table>

The tracer particles generated for this experiment were delivered from a smoke generator which produces high concentrations of monodisperse poly-ethylene-glycol particles with a mean diameter of 2 $\mu$m, see Kähler et al. (2001). In order to obtain ideal conditions for accurate PIV measurements, the closed circuit wind-tunnel was completely seeded and continuously operated until no seeding inhomogeneities could be observed at all. During this procedure, all diffuse wall reflections caused by the interaction of the laser-light with the imperfect glass window in the floor of the test section or sticked tracer particles were completely removed by cleaning the window with appropriate liquids and pressurized air while the wind-tunnel was running at a moderate speed. It should be noted that the operation of the wind-tunnel is extremely important for the cleaning procedure, and should never be stopped before the experiment has finished, in order to avoid any settling of the tracer particles which might deteriorate the performance of the measurement.

**Statistical properties of the buffer layer**

It has already been stated that the lower right graph of figure 1 implies that the dominant physical processes responsible for the turbulent mixing in near wall turbulence seem to change when approaching the wall. However, this becomes even more evident when the joint probability density function of the velocity fluctuations $uv$ and $uw$ is considered. This can be clearly seen in figure 3 for two characteristic wall distances (left column: $y^+ = 10$, right column: $y^+ = 20$) by comparing the size and shape of the iso-contour lines and location of the maximum.
First of all, it is not surprising that the functions vary strongly in size and shape for a fixed wall location, as \( u_{\text{rms}} < w_{\text{rms}} < v_{\text{rms}} \), and also the symmetry properties with respect to \( w \) can be explained by the homogeneity of the flow in \( z \)-direction, e.g. \( p(w) = p(-w) \). Secondly, the decreasing size of the distribution towards the wall is clear as the amplitude of the velocity fluctuations is damped due to the presence of the wall and the stretching of the \( p(u,v) \) distribution can be explained by the different asymptotic behavior of the velocity components close to the wall (\( v \sim y^2, u \sim y \)). Furthermore, the slightly inclined orientation of the \( p(u,v) \) function along with the non-symmetrical shape indicate clearly that flow regions associated with ejection (\( u < 0 \) and \( v > 0 \)) and sweeps (\( u > 0 \) and \( v < 0 \)) are more likely and dominate for both wall locations over the regions where both fluctuations possess the same sign simultaneously. This is necessary as the generation and maintenance of turbulence requires that \( \rho \) is positive on average as can be seen from the Reynolds equation in boundary layer approximation. However, surprising is the displacement of the maximum, from positive to negative \( u \)-values, with decreasing wall distance. This implies that close to the wall at \( y^+=10 \), most of the structures (or the large ones) posses a slightly negative stream-wise velocity fluctuation with a rather weak amplitude, as can be seen from the location of the maximum. As the mean value of all fluctuations must be zero per definition, the amplitudes of the structures with a positive stream-wise velocity fluctuation are necessarily larger in order to compensate the maximum at negative \( u \). This explains the surprising fact that the maxima at \( y^+=10 \) in the lower right of figure 1 result from the positive fluctuations and not from the negative as expected. The power-of-two operation, required for calculating the rms-values, increases the weight of the positive large scale fluctuations when the average is taken and decreases the contribution of the negative small scale fluctuations (the argumentation can be reversed for the measurements at \( y^+=20 \)). As the asymptotic behavior of the span-wise velocity amplitude is weaker (\( w \sim y \)) with respect to the \( v \)-component, the displacement of the maximum can be seen more clearly in the lower row of figure 3 where the joint probability density function of the velocity fluctuations in stream-wise and span-wise direction are shown. Another interesting feature, appearing in the lower row, is the asymmetry for the large scale fluctuations, expressed by \( p(u,w) \neq p(-u,w) \) which is visible in the right graph. The fact that large scale fluctuations of the span-wise component \( w \) are frequently associated with positive stream-wise fluctuations and small scale \( w \)-fluctuations with negative \( u \)-fluctuations may be an effect associated with organized motion present in the flow field. This will be considered later.

In order to deduce the typical flow angles associated with the transport of low and high momentum fluid towards and away from the wall, the probability density distribution of the absolute angle between the instantaneous values of the various velocity components was calculated at \( y^+=10, 20, 30 \). The \( w/U \)-distribution, shown in the lower left graphs of figure 4 is exactly symmetrical with respect to the centerline and the range of angles...
increases when approaching the wall. This is clear as the magnitude of the span-wise velocity fluctuations increases toward the wall. The v/U-distribution on the other hand is asymmetric with a large range of scales for positive angles and a maximum located at negative $\alpha$. The right column of figure 4 shows the distribution of the characteristic angles for the velocity structures whose instantaneous stream-wise velocity U is above (lower figure) or below the mean motion. As a stream-wise velocity larger than the mean velocity indicates a positive fluctuation $u$ the positive domain of the lower right plot indicates flow structures which move away from the wall whereas the negative part indicates large momentum fluid moving towards the wall. The asymmetry indicates clearly that the probability of finding a large momentum fluid moving towards the wall (so called sweeps or Q4 events according to the quadrant in the uv-coordinate system) is quite high with respect to high speed fluid moving away from the wall. This implies the dominance of Q4 events over Q1 when large magnitude events are considered and the upper right plots states that Q2 events or ejections dominate over Q3 movements. Furthermore, it should be noted that the largest angles are always associated with Q2 and Q4 events.

Due to the strong dynamic of the velocity fluctuations in the buffer layer and the small mean velocity, it is obvious that the convection term in the conservation equation is not large with respect to the sum of the production, diffusion and dissipation term as required for the observation of coherent structures. However, as the statistical results examined in this section indicate a strong relation between various velocity fluctuations it seems likely that the results are strongly related with the motion of organized or coherent flow structures, e.g. flow regions over which the simultaneous velocity fluctuations are correlated.

**Spatial auto-correlation functions**

In order to link the statistical results of the previous section with the concept of coherent structures, the properties of normalized spatial correlation and cross correlation functions will be considered in this section. The analysis can be seen as a completion of the stereoscopic PIV investigation described by Kähler et al (1998) in which the planes at $y^+=220,100,30$ were investigated at $Re_\theta=950$. The contours of the following plots are spaced in intervals of 0.05 for the primary correlation (excluding 0.05 and 1) and the position and number of the minimum and maximum value is included. Continuous lines indicate positive correlation whereas dotted lines assign regions with negative correlation values.

![Figure 4: Probability density distribution of the flow angle between the instantaneous values of the various velocity components at three different wall distances ($y^+=10, 20, 30$ black, red, green). v/U (upper left), w/U (lower left), v/U with $U<U_{mean}$ (upper right), v/U with $U>U_{mean}$ (lower right). Note: $\tan(\alpha=0.1)=5.7^\circ$.](image)
Figure 5 shows the primary auto-correlations of the stream-wise (left column) and wall-normal (right column) velocity fluctuations measured at $y^+ = 30, 20, 10$ (top to bottom). Clearly visible is the characteristic elliptical shape of both correlations, with the principal axis in stream-wise direction, and the strong dependence of the correlation width from the wall distances in accordance with Kähler et al (1998). However, the most striking feature of the $R_{uu}$ correlation, which could be not established before, is the variation of the size and span-wise displacement of the of the weak negative correlation regions relative to the maximum, when approaching the wall. This is clearly visible in the upper right plot of figure 6 where the span-wise dependence of this correlation at $\Delta x^+ = 0$ is shown in wall-units. The location of the minima are highlighted with symbols and the legend reveals the corresponding displacement of the smallest correlation value from the maximum of correlation, located at $\Delta z^+ = 0$. The range of correlation values indicate a large range of scales and the minima imply a span-wise periodicity whose wavelength increases strongly with increasing wall distance. Furthermore, the existence of stream-wise vortices can be concluded but it should be noted that their length is not several thousand wall units as expected by other authors. This can be seen from the stream-wise extension of the negative correlation region in figure 5. In case of the $R_{uv}$ correlation, shown in the upper left plot of figure 6, only the function measured at $y^+ \approx 10$ changes its sign in span-wise direction and reveals a weak minimum at $\Delta z^+ = 63$, while the others converge slowly to zero with increasing $\Delta z^+$. This indicates a decreasing dynamic velocity range with increasing wall distance and implies that the coherent velocity regions rapidly loose their identity with increasing $y$. 
In order to deduce the statistical properties of the dominant low- and high-momentum flow structures which are convecting downstream, the measured data was subdivided according to the sign of the stream-wise velocity fluctuation before calculating the correlation functions. This conditional correlation approach allows to identify the structures associated with ejection and sweeps for instance. The left plot of the center row of figure 6 reveals the $R_{uu}(u<0)$ correlations, denoted by $R_{uu}(u<0)$ for simplicity as calculated only from the fluctuations which are negative, and the right plot shows the associated correlation of the positive and negative wall-normal fluctuations present in the particular regions where $SuS$ is negative. It is interesting to note that coherent velocity regions defined in this way show nearly the same functional dependence around the maximum (down to correlation values of 0.2 for $R_{uu}(u<0)$ and nearly zero for $R_{vv}(u<0)$) which indicates a high degree of coherence in wall-normal direction. Interesting is also that the value of the minimum increases with increasing wall distance when the $R_{uu}(u<0)$ correlation is considered. This is completely different when the correlations of the wall-normal fluctuations are calculated which can be measured in regions where the stream-wise velocity fluctuations are larger than the mean velocity, e.g. $R_{vv}(u>0)$ shown in the lower right plot of the same figure. In this case the value of the minimum decreases with increasing wall distance. The legends of the $R_{uu}(u<0)$ and $R_{vv}(u>0)$ correlations indicate that the displacement of the minima from the maximum increases with increasing wall distance for both functions but faster for the $R_{vv}(u>0)$, especially close to the wall. Moreover it can be concluded from the functional dependence of the correlations around the maximum that the coherence of the $R_{uu}(u>0)$ structures is rather weak compared to the coherent structures represented by $R_{vv}(u<0)$ and also that the structures which enter in the calculation of $R_{uu}(u<0)$ lose their identity to a large extent with increasing wall-distance as indicated by the strong difference between the graphs.

**Figure 6:** One dimensional spatial correlation function of fluctuating stream-wise and wall-normal velocity components measured at $y^+=10$ (solid graph), $y^+=20$ (dotted graph) and $y^+=30$ (dashed graph) as a function of the span-wise coordinate. The symbols indicate the location of the minimum and the legend the corresponding span-wise coordinate in wall-units.
The exact values of the span-wise periodicity of the structures, indicated by the minimum in the correlations, is still a point of discussion in the literature, especially at high Reynolds numbers. One reason for this controversy is the dependence of this number from the method applied for the determination of the wavelength. However, it should be noted that the conditional correlation approach, applied here, is fully based on mathematical grounds and yields only one well defined value for the wavelength, instead of a streak-spacing distribution from which a mean has to be calculated in an appropriate way.

Spatial cross-correlation functions

For phenomena associated with the production of turbulence the cross correlation $R_{vu}$ is more important than the primary components of the correlation tensor, because this quantity reflects clearly the size and shape of the structures responsible for the transport of relatively low-momentum fluid outwards into higher speed regions and the movement of high-momentum fluid toward the wall and into lower speed regions.

This correlation was first studied by Tritton (1977) by using a pair of hot-wire probes 2.1 m behind a trip-wire at free-stream velocities between 6.75 m/s and 7.1 m/s. The boundary layer thickness was 60 mm and the Reynolds number $Re_\delta = 22000$. Due to technical reasons, his results were measured only along the three principal axes and at larger wall-locations, so that a direct comparison with the work presented here is questionable. However, it is important to keep in mind that the cross-correlation function is not necessarily an even function with a maximum

![Figure 7](image.png)
at $\Delta x=0$, in contrast to the primary correlations, but there is still an important symmetry property when the random variables $u$ and $v$ are interchanged, namely $R_{uv}(\Delta x) = R_{vu}(-\Delta x)$.

The left column in figure 7 shows the $R_{vu}$ correlation function whereby the $v$ component was fixed while $u$ was shifted in the two homogeneous directions ($x$ and $z$). First of all it should be noted that the sign of $R_{vu}$ indicates again that the transport of relatively low-momentum fluid outward into higher speed regions ($u<0$ and $v>0$) and the movement of high-momentum fluid toward the wall into lower speed regions ($u>0$ and $v<0$) are the predominant processes in the near wall region, as expected. In addition, the strong elliptical shape implies that the turbulent mixing in the wall-normal direction is related to the low-speed structures represented by $R_{uu}(\Delta x>0)$. But obviously only a small part of a low momentum structures shows a correlated motion in both stream-wise and wall-normal direction with sufficient strength (lower curve of figure 7) as the total length of the low-speed structures are several thousand wall units in length according to the size of the $R_{uu}$ correlation shown in figure 4. This is fully consistent with the size of the $R_{vu}$ correlation and the results in Kähler et al (1998). The span-wise extent of the flow-structures associated with the production of turbulence can be estimated best from the upper right graphs of figure 8 and the phase relation between both orthogonal fluctuations from the location of the maximum shown in the upper left graphs of the same figure.

Apart from the structural details of the flow regions associated with the turbulent exchange in wall-normal and stream-wise direction, it is important to consider the cross-correlation with the span-wise velocity fluctuations as any outflow of fluid induces an organized span-wise flow motion towards the lifting structure due to continuity.

**Figure 8**: One dimensional spatial cross-correlation function of Reynolds stress components measured at $y^+=30, 20, 10$ (Solid graph: $y^+=10$. Dotted graph: $y^+=20$. Dashed graph: $y^+=30$) as a function of $\Delta x^+$ and $\Delta z^+$. The symbols indicate the maximum of correlation and the legend shows the exact position in wall-units.
In addition, as in case of isotropic turbulence this quantity must be zero, these correlations indicate any departure from this ideal situation which is mathematically accessible.

Figure 9 shows the two dimensional spatial cross-correlation function of fluctuating stream-wise with span-wise velocity components $R_{uw}(\Delta x, y, \Delta z)$ (left) and correlation of wall-normal with span-wise components $R_{vw}(\Delta x, y, \Delta z)$ (right) measured at $y^+=30, 20, 10$ (top to bottom). First of all, it can be concluded from the general size, shape and intensity of both correlations, that a very well organized span-wise motion exists, especially at $y^+=10$ where the intensity is above the $R_{vu}$ correlation. This organized motion implies that the concept of local isotropy, which links the turbulent-energy dissipation in shear flows with the dissipation of isotropic turbulence, is inadequate in near wall turbulence. Secondly, it should be noted that the degree of organization increases with decreasing wall distance while the opposite holds for the $R_{vu}$ correlation as can be estimated from the intensity of the maximum. This is important for the performance of numerical flow simulations. Thirdly, it should be noted that the spacing between the extrema is increasing with larger wall distances but in addition it can be seen that the location of these particular points moves to negative stream-wise displacements, particularly if the $R_{vw}$ correlation is considered. For clarity, the two lower rows of figure 8 reveal the functional dependence of the correlations along the axis of symmetry. This representation allows to compare the height of the correlation and the location of the maximum in detail.
Spatio-temporal correlation functions

In this section the spatio-temporal flow structures of the buffer layer will be described quantitatively whereby the convection and decay of turbulent fluctuations are of particular interest. The first investigation of this type was performed by Favre et al (1958) who studied the spatio-temporal structure of the stream-wise velocity component $R_{uu}$ at $y^+>40$ and $Re_\theta=1600$, by using a pair of spatially separated hot wire probes. Their measurements were performed 0.79 m and 1.94 m behind a tripping device at free-stream velocities of 12 m/s with $\delta\approx17$ mm and $\delta\approx34$ mm. Here, the structure at $y^+=30$ and below will be investigated at $Re_\theta=7800$ by analyzing the primary correlation $R_{uu}$, the cross correlation $R_{uv}$ and the conditional correlations where the sign of the fluctuations is considered ($R_{+uv}$, $R_{-uv}$ and others). The conditional correlations yield information about the space-time structure of the bursting phenomenon and allows to estimate the mean convection velocity of the coherent velocity structures present in the near wall region. In contrast to all correlations considered so far which where calculated at a fixed $y^+$ location here the spatial-temporal correlation between separated planes is considered. The spacing is always 10 wall-units but the position of the lower measurement plane is altered between $y^+=10$ and 20 wall units. When the temporal delay between a pair of measurements is varied, this approach allows to investigate the structures moving toward and away from the wall, depending on the temporal order of the measurements. In other words, when a structure is moving towards the wall, this structure can be investigated best when the first measurement is performed at $y^++\Delta y$ and the second at $y$ while flow-structures moving away from the wall yield higher correlation values when the first measurement is performed at $y$ and the second at $y^+\Delta y$. Basic requirement for the interpretation of the two-dimensional spatial cross-correlation functions is clear understanding of the symmetry properties as their intensity, shape and position may strongly depend on the order of the fluctuating quantities considered for the correlation $R_{uv}\neq R_{vu}$ as well as their exact position $R_{uv(y^+\Delta y)}\neq R_{uv(y,y^+\Delta y)}$.

Figure 10: Combination of possible two dimensional spatial cross correlation functions of fluctuating stream-wise and wall-normal velocity components extracted simultaneously at $y$ and $y^+\Delta y$. $R_{uy+(y^+\Delta y)uy}$ (upper left), $R_{vy+(y^+\Delta y)vy}$ (lower left), $R_{vy+(y^+\Delta y)uy}$ (upper right), $R_{uy+(y^+\Delta y)vy}$ (lower right).

This is highlighted in figure 10 which reveals all possible combinations of the two dimensional spatial cross correlation functions of fluctuating stream-wise and wall-normal velocity components extracted simultaneously at $y$ and $y^+\Delta y$. Important parameter for the analysis of the cross-correlations functions are their size, shape, intensity, sign and location of the maxima as these values furnish basically information about the average dimensions of the moving flow structures, their degree of organization and propagation direction relative to the mean motion. The particular example shown in figure 10 indicates that the correlation function is rather weak and relatively small in spatial extent when the $v$ component, measured in the plane located at $y$, is correlated with the $u$ component from the plane located at $y^+\Delta y$ (see upper figure). The correlation of the $v(y^+\Delta y)$ with $u(y)$ on
the other hand results in a quite strong and well extended correlation (see lower figure). The location of the maximum on the other hand is symmetrically located with respect to the x=0 axis when the two components are altered. In the lower left figure, the u(y) component is shifted for the calculation of the cross correlation function and for the lower right calculation the v(y+\Delta y) component was shifted relatively to v(y). In case of the two-dimensional functions in figure 10 for instance, it can be stated from the sign that the ejections and sweeps must be the predominant structures and the different size, shape and intensity of the functions (R_{u(y),u(y+\Delta y)} > R_{v(y),u(y+\Delta y)}) implies the dominance of ejections at this wall locations. In addition it can be estimated from the location of the maximum in the lower left graph that the low momentum region appears as a shear layer in y-direction while the strong positive side peaks in the same figure indicate that the outflow of low momentum fluid is associated with a secondary motion. Of fundamental importance are the differences in the fine structure when the correlation is considered as a function of the temporal delay and the structure dependent convection velocity highlighted quantitatively in figure 11 and 12 whereby only a line of the data extracted from the two dimensional correlation functions along the x-coordinate through the maximum of correlation is shown here. First of all it can be seen that the shift of the maximum (indicated by the symbols and the legend) depends strongly on the sign of the stream-wise velocity fluctuation considered for the calculation of the conditional correlation and from the wall-distance for all correlations. This is clear as structures whose velocity is larger than the mean velocity are traveling faster. Secondly it turns out that the height of the conditional correlations is always lower than the correlations based on all fluctuations. Finally, it should be noted by comparing the values in the legend that the movement of high-speed structures towards and away from the wall differs to a large extent.

Figure 11: One dimensional spatio-temporal correlation function of stream-wise velocity fluctuation $R_{uu}$ (top), the low-momentum fluctuations in stream-wise direction $R_{-u(-u)}$ (center) and the high-momentum stream-wise fluctuations $R_{+u(+u)}$ (bottom) measured at $y^+=10$ (left column) and $y^+=20$ (right column) and different time delays between a pair of measurements ($\Delta \tau = -5, 0, +5$ ms). The symbols indicate the maximum of correlation and the legend shows the exact position in wall-units.
Figure 12: One dimensional spatio-temporal cross correlation function of stream-wise with wall-normal velocity fluctuation $R_{vu}$ (top), correlations of negative stream-wise with wall-normal fluctuation $R_{v(-u)}$ (center) and correlations of positive stream-wise with wall-normal fluctuation $R_{v(+u)}$ (bottom) measured at $y^+ = 10$ (left column) and $y^+ = 20$ (right column) and different time delays between a pair of measurements ($\Delta \tau = -5$, $0$, $+5$ ms). The symbols indicate the maximum of correlation and the legend shows the exact position in wall-units.

Instantaneous velocity structures

As the various correlations do not yield any information about the real organization and structural details of the individual flow regions, two instantaneous velocity fields will be analyzed in this section in order to illuminate the complexity of the turbulent motion and to prove the interpretations of the statistical results. In order to display the properties of dominant flow structures which contribute to the production of turbulence to a large extent the vector field in figure 14 and 15 reveal structures whose instantaneous Reynolds stress component $uv$ is below $-0.2$. However, it should be noted that these sturctures do not occur frequently, according to figure 13 where the probability density function of the Reynolds stress component $uv$ ($y^+ = 20$) is shown in linear (left) and semi-logarithmic representation. In addition, it can be seen from the cumulative sum, that their contribution for the total production of turbulence is quite small compared with structures which are dominant with respect to their occurence also their intensity of the Reynolds stress component $uv$ is quite small. These structures are also visible in figure 14 and 15.
The lower image of figure 14 shows a representative instantaneous flow field measured at $y^+=20$ whereby the local mean velocity of 1.4 m/s was subtracted in order to display the fluctuating velocities $u$ (stream wise) and $w$ (span-wise) clearly. The mean flow direction is from left to right and the color behind the velocity vectors indicates the out-of-plane motion (red: $v>0$, blue: $v<0$).

Predominant structures are the elongated low-speed fluid regions which are convecting downstream with approximately half the local mean velocity. It can be clearly seen that the $u$ component of the velocity is frequently low ($u<0$) in regions where the fluid moves away from the wall ($v>0$) and the stream-wise extent of the wall-normal motion is quite short with respect to total streaks length as stated before. This illustrates again that in general no strong stream wise vortices flank the low speed regions, as proposed by Kim et al (1971). Only
a weak vortex-like flow motion can be seen as the lifting of low speed regions into higher momentum flows must be compensated due to continuity. However, it seems that these vortices are only a secondary motion which is produced locally. The upper image of figure 14 shows the spatial distribution of the instantaneous Reynolds stress component $uv$ which highlights again the importance of low-speed streaks for the production of turbulence. The strong production region in the upper right corner seems to be a result of an interaction between a stream-wise vortex and a low-speed streak whereby the vortex is moving above the streak with a higher convection velocity. This phenomena is described by Schoppa and Hussain (2002) in detail.

**Figure 15**: Distribution of the Reynolds stress component $uv$ (top) and velocity vector field (bottom). The out of plane component $v$ is color coded (red: $v>0$, blue: $v<0$).

Figure 15 does not reveal any extended streak pattern but the intensity of the Reynolds stress component is very large as can be estimate from the upper image. However, by visual inspection of the velocity field it can be seen that the regions of strong production is associated with counter rotating vortex pairs which indicate the presence of a hairpin like structures, Kähler et al (1998). Moreover it seems that these structures are convecting as a package as described by Zhou et al (1995) but it should be keep in mind that such arrangements of hair-pin like structures can be only observed when the intensity of the structure is very large.

**Conclusions**

The statistical properties of a fully developed turbulent boundary layer flow along a flat plate is investigated in stream-wise span-wise planes at $y^+ = 10, 20, 30$ and $Re_\theta = 7800$ by means of multiplane stereo PIV. The occurrence, intensity and main flow direction of the coherent structures is deduced from the analysis of the joint probability density function of the velocity fluctuations. It is shown that the largest flow angles ($\alpha>10^\circ$) in wall-normal direction are usually associated with Q2 and Q4 events. In order to obtain information about the structural features of the coherent structures, the size, shape and intensity of various spatial correlation, cross-correlation
and conditional-correlation functions were examined in detail. It could be shown that the range of scales and span-wise periodicity of the coherent structures present in the flow depends strongly on the exact wall-distance. The mean streak-spacing is 92 wall-units at $y^+ = 10$ when estimated from the conditional correlation and the span-wise size of the stream-wise vortices associated with sweeps measures 27 to 53 wall units at $y^+ = 10, 20, 30$ while those associated with ejections are 35 to 42 wall units in size for the same wall locations. However, the stream-wise size of these vortices is short relative to the length of the low-speed streaks, and it seems that these vortices are induced locally by the lift-up of low-speed streaks. The dynamics of the dominant structures is investigated by means of spatio-temporal correlation, cross-correlation and conditional correlations functions measured in spatially separated planes. The conditional correlations yield information about the space-time structure of the bursting phenomenon and allows to estimate the mean convection velocity of the coherent velocity structures present in the near wall region. The analysis of instantaneous velocity fields along with the probability density function of the Reynolds stress component $uv$ implies finally, that most of the production of turbulence is associated with low-speed streaks, but the magnitude of the instantaneous Reynolds stress component $uv$ associated with streaks is relatively small. The flow structures associated with large values of $|uv|$ on the other hand are frequently hair-pin like. However, as the likelihood of these structures is quite small with respect to the lifting streaks, they do not contribute to the total Reynolds stress to a large extend.

Reference

Carlier J, Foucaut JM, Dupont P, Stanislas M (1999) Caractérisation d'une couche limite turbulente à grand nombre de Reynolds par anémométrie à fils chauds et par Vélocimétrie par Images de Particules, 35ème colloque d'Aérodynamique Appliquée. Analyse et contrôle des écoulements turbulents, Lille France

Ehrenfried (2001), Hough transformation, 4th International Symposium on Particle Image Velocimetry, Göttingen, Germany, September 17–19


