Measurement of the Spatial Coherence of Turbulence in Flows

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ABSTRACT

The prediction of noise generated by turbulent flows is dependent upon knowledge of the spatial coherence of the turbulence. This has traditionally been quantified using the cross correlation function, which gives a correlation length that is typically used in noise prediction models. More recently [1], the use of the frequency domain parameters of coherence and phase have been shown to be more relevant to the development of reliable prediction methods for noise generation as these give information on both the correlation length and the local convection speed as a function of frequency. The use of hot-wire anemometry for quantifying these parameters is problematical especially at the small turbulent scales where the size of the probes can significantly alter the local characteristics. Laser Doppler anemometry offers an alternative that is non-intrusive but has inherent problems associated with the intermittent data acquisition so that the errors associated with the use of two LDA systems can result again in very poor resolution at high frequencies. A compromise is to use LDA together with a hot-wire system with the former located upstream so that the intrusive effect of the hot-wire is minimised.

This paper reports the application of this approach to the measurements of spatial coherence in turbulent flows with narrow and broadband characteristics. A two component DANTEC LDA system based on an F50 BSA processor is used in conjunction with a single hot-wire probe. Procedures for the analysis of the LDA and hot-wire data are developed and corrections for inherent errors associated with the intermittent acquisition of the LDA are given. Results show the effect of frequency on both spatial coherence and phase (and hence convection speeds) of turbulent scales in the flows.

1. INTRODUCTION

For many applications in fluid mechanics, fundamental understanding of turbulence related phenomena depends not only on the correct estimation of the turbulence frequency spectrum but also on accurate estimation of the spatial correlation of the turbulence and on the degree of correlation of the turbulence with other dynamic parameters. This is usually expressed as a normalised cross correlation or cross covariance function but the use of the coherence function or normalised cross-spectrum and phase provides considerably more information as they are frequency dependent measures of interaction. This approach provides information on correlation lengths and convection velocities at each frequency as demonstrated by Durant et al.(2000). Thus, for many measurements, not only the auto spectrum is required but the cross spectrum is also to be determined.

The determination of the auto or power spectrum of flows measured with a laser Doppler anemometer is compounded by the intermittent nature by which the data is acquired from the random passing of particles through the measuring volume. A number of different procedures have been proposed for computing the auto spectrum from LDA measurements but, in all cases, the limiting factors are the average sample rate of the data and the total length of or number of data points in each sample as these dictate respectively the maximum and minimum frequencies that may be resolved. The most straightforward method for analysis is the sample & hold time domain reconstruction procedure although other approaches to time domain signal reconstruction have been proposed (e.g. Host-Madsen & Caspersen, 1995; Veynante & Candel, 1988; Van Mannen and Tullekan, 1994). The most common alternative procedure to time domain reconstruction is the correlation slotting technique proposed by Gaster & Roberts (1975). Comparisons of the various methods for the auto spectrum estimation from LDA data have been conducted by Buchave et al. (1990), Lee & Sung (1994), Britz & Antonio (1996) and Benedict et al. (1996) whilst Tropea (1995) has reviewed the various techniques used for spectrum estimation from LDA data. The errors associated with the sample and hold process have been detailed by Adrian & Yao (1987) and by Boyer & Searby (1986) and a procedure including corrections based on sample and hold has been proposed by Nobach et al. (1998). Despite this effort, there is still no general consensus on the most effective procedures.

Two point correlation measurements using two component LDA systems have been reported by Eriksson & Karlsson (1995) who defined the measurement volume requirements with respect to the Taylor micro-scale in order to obtain accurate correlation estimates at small spacing. Trimis & Melling (1995) have reported an improved method for two point space correlations and suggest the possible use of slot correlation of velocity pairs to obtain the space-time cross correlation. For both of these applications, a classical space correlation approach was used for the estimation of the cross covariance and the main problem was one of coincident measurement. Space and time correlations were used by Cenedese & Romano (1991) to test the Taylor hypothesis in a turbulent flow using a sample and hold approach for the data. A frequency domain approach was shown by Gerosa & Romano (1994) to be capable of determining the critical measurement conditions necessary to obtain coherent data and also that the coherence function properly defined the frequency range over which valid data had been obtained for cross comparison purposes. This frequency domain analysis was implemented by Romano (1995) for the analysis of two point velocity measurements in near wall flows.

In this paper, the errors inherent in the calculation of the auto-spectrum using the sample and hold method are outlined and procedures for correction of these errors are given. The determination of the cross spectrum and associated coherence and phase relationships using LDA generated data with data from other instruments is then considered and a correction technique is developed. The procedures are then used to examine the characteristics of both a wake flow and a turbulent flow.

2. SAMPLE & HOLD RECONSTRUCTION

2.1 Auto Spectrum

The errors associated with the sample and hold process have been detailed by Adrian & Yao (1987) and by Boyer & Searby (1986) and shown to comprise of a step noise which adds a constant bias to the estimated spectrum and a low pass filter effect. Expressions for the errors were obtained by Adrian & Yao (1967) and shown to be functions of the mean sample rate, the maximum frequency to be resolved and the Taylor micro-scale of the flow.

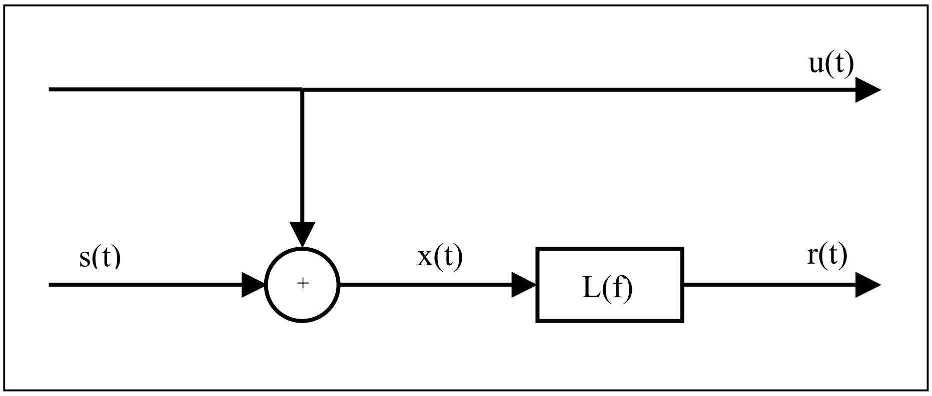


Figure 1 : Schematic of Sample & Hold Procedure

The effect of these errors on the estimation of the auto-spectrum of a signal u(t) can be represented as shown in figure 1 where s(t) represents the step noise and L(f) the low pass filter effect so that r(t) is the reconstructed signal. The relationships for this are given by:

$$G_{xx}(f) = G_{uu}(f) + G_{ss}(f) \qquad (2.1)$$

$$G_{rr}(f) = |L(f)|^2 G_{xx}(f) \qquad (2.2)$$

$$G_{rx}(f) = L(f)\, G_{xx}(f) = G_{ru}(f) + G_{rs}(f) \qquad (2.3)$$

It can be seen from these that, if the characteristics of the low pass filter, *L(f),* are known and if the step noise can be estimated, then the calculated spectrum $G_{rr}(f)$ can be corrected to obtain the true spectrum $G_{uu}(f)$. The low pass filter is a function of the mean sample frequency and has been given by both Boyer & Searby (1986) and Adrian & Yao (1987) as:

$$L(f) = 1/[1 + i(2\pi f/f_s)] \qquad (2.4)$$

where f_s is the mean data rate so that $G_{xx}(f)$ is readily obtained.

The step noise can also be estimated using formulations proposed by Boyer & Searby (1986) and by Adrian & Yao (1987). However, the step noise can only be determined when the actual form of the spectrum is known and this is impossible in nearly all practical cases. Nonetheless, since the step noise is white, it is possible to estimate it in the following manner. The variance of u(t), the turbulent velocity, can be determined from the original time domain data as:

$$\sigma_u^{\,2} = \int_0^T u^2(t)dt/T \qquad (2.5)$$

The variance of the reconstructed signal corrected for the low pass filter effect can be determined from the auto-spectrum as:

$$\sigma_x{}^2 = \int G_{xx}(f)df = \int G_{rr}(f)df/|L(f)|^2 \qquad (2.6)$$

This is equal to the variance of the original signal plus the step noise, so that the variance of the step noise can be found from:

$$\sigma_s{}^2 = \sigma_x{}^2 - \sigma_u{}^2 \qquad (2.7)$$

Since the step noise is white, its spectrum over N points is a constant given by:

$$G_{ss}(f) = (\sigma_s)^2/N \qquad (2.8)$$

so the spectrum of u(t) can be estimated as:

$$G_{uu}(f) = G_{rr}(f) - G_{ss}(f) \qquad (2.9)$$

Thus, for any signal for which a spectrum has been estimated using the sample & hold technique, the correct spectrum can be estimated and the noise to signal ratio α, is given by $G_{ss}(f)/G_{uu}(f)$.

2.2 Cross Spectrum

The cross-spectrum is the frequency domain cross-correlation function, which can be used to determine the frequency response function (and hence the phase) and the degree of correlation between two processes. The effect of the sample and hold process on the estimation of the cross-spectrum between LDA measurements u(t) and those obtained from equi-spaced data z(t) can be assessed in a similar manner. A representation of the system is shown in figure 2 in which *H(f)* represents the Frequency Response Function (FRF) between the velocity fluctuations and the other measurement and *s(t)* and *L(f)* are the step noise and low pass filter inherent in the sample and hold process. The measured data are *r(t)* for the LDA system and *z(t)* for the conventional instrument. From this, the equations relating the various elements can be written in the frequency domain as follows:

$$Z(f) = H(f)\, U(f)$$

$$R(f) = L(f)\{U(f) + S(f)\}$$

So that the spectral relationships are:

$$G_{rz}(f) = \langle R^*(f)\, Z(f)\rangle = L^*(f)\, \{\langle U^*(f)\, Z(f)\rangle + \langle S^*(f)\, Z(f)\rangle\}$$

Where the operator < > denotes an ensemble average and * is the conjugate. Since the last term is zero as s(t) and z(t) are uncorrelated, this gives:

$$G_{rz} = L^*(f)\, G_{uz}(f)$$

So that the cross spectrum between u(t) and z(t) is obtained as:

$$G_{uz}(f) = G_{rz}(f) / L^*(f) \qquad (2.10)$$

And the frequency response function is obtained from:

$$H(f) = G_{zz}(f)/ G^*_{uz}(f) \qquad (2.11)$$

By using the output, the step noise errors in the measurement of u(t) do not need to be considered. The coherence function, or frequency domain normalised cross-covariance can be estimated using the low pass filter corrected LDA measurement, x(t), and the equi-sampled data, z(t), to give:

$$\gamma_{xz}^2(f) = [G_{xz}^*(f)\ G_{xz}(f)]/[G_{xx}(f)\ G_{zz}(f)] \quad (2.12)$$

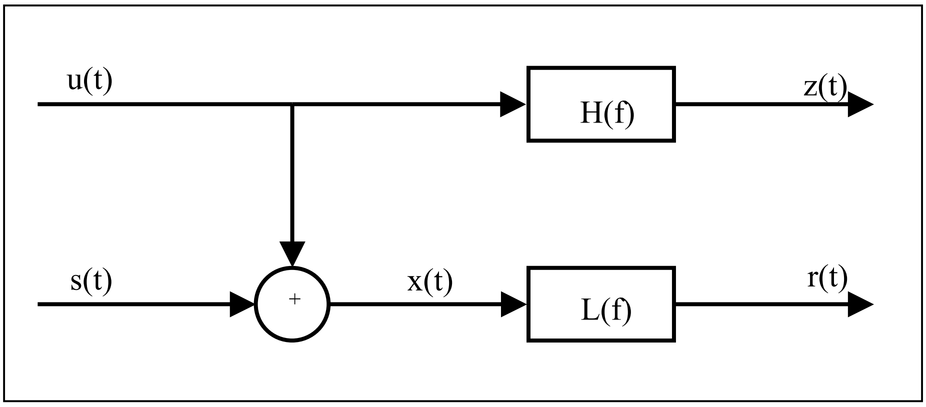
so that the actual coherence function is given by:

$$\gamma_{uz}^2(f) = \gamma_{xz}^2(f)\ [\ 1 + G_{ss}(f)/G_{uu}(f)] = \gamma_{xz}^2(f)\ [\ 1 + \alpha] \quad (2.13)$$

where α is the noise to signal ratio. The main problem is that, as the step noise is constant across the spectrum, α increases substantially at high frequencies so the estimated coherence contains substantial bias.

Thus a bias free estimate of the frequency response function of the measured parameters can be obtained. For the case of LDA and hot-wire measurements, this enables calculation of the phase (and hence convection speeds) and the dissipation. Estimates of correlation length that are related to the coherence can also be made once the noise to signal ratio (α) has been determined as detailed in 2.1.

Figure 2. Schematic of input/output system



3. EXPERIMENTAL SET-UP & ANALYSIS PROCEDURES

3.1 Experimental Configuration & Instrumentation

The tests were performed in a low turbulence wind tunnel with a working section of 125mmx125mm and three flow configurations were investigated. The first two were in the wake of horizontal cylinders of diameter D = 12.7mm & 6.35mm, mounted perpendicular to the flow so that regular vortex shedding with a relatively narrow band frequency dominated the flow characteristics. For the third set up, the flow behind a regular wire grid 3x3mm and a blockage ratio of 30% was installed upstream of the test section so that a broader band frequency spectrum was generated.

Velocity measurements were obtained using a Dantec LDA system and a single hot-wire. The LDA acquisition system consists of a continuous Argon -Ion laser (514.5*nm*) and a burst spectrum analyser (BSA50) with the data for the stream-wise component of velocity used. For the cylinder tests, the position of the LDA measurement volume was kept constant throughout the experiments at 1 diameter above the cylinders and 1.5 diameters behind the cylinder center. The hot-wire system consisted of a single Dantec 55P01 probe located at a position of 1diameter vertically above the cylinder center and traversed in the downstream of the cylinder. Data was recorded at 4 stream-wise locations 0.5 D apart, starting at 1.5 D from the cylinder center. For the grid, the LDA was located 200mm downstream and the hot wire was traversed from this position to 18.8mm downstream in 6.25mm steps.

3.2 Data Acquisition & Analysis

Figure 3 shows a schematic of the measurement set-up and the acquisition system. Each 1s of LDA data was mapped to an array of size 81920 for sample and hold, low pass filtered and re-sampled at 8192 Hz for subsequent signal processing. The hot wire data was acquired directly using a National Instruments 2024E 12-bit PCI-DAQ acquisition card. The card was operated using the LabView programming environment and the commencement of the acquisition was digitally triggered using an output of the LDA system. The data was acquired in time records of 10 seconds in length and processed using a 1024-point FFT implementation in MATLAB. The sampling frequency of the A/D converter was f_h = 8192 Hz so that the re-sampling of the LDA data was also at this frequency with the resulting maximum frequency resolution of 4 kHz at 8 Hz intervals. The analysis procedures detailed in section 2 were then implemented to determine the auto and cross spectra for the measurements.

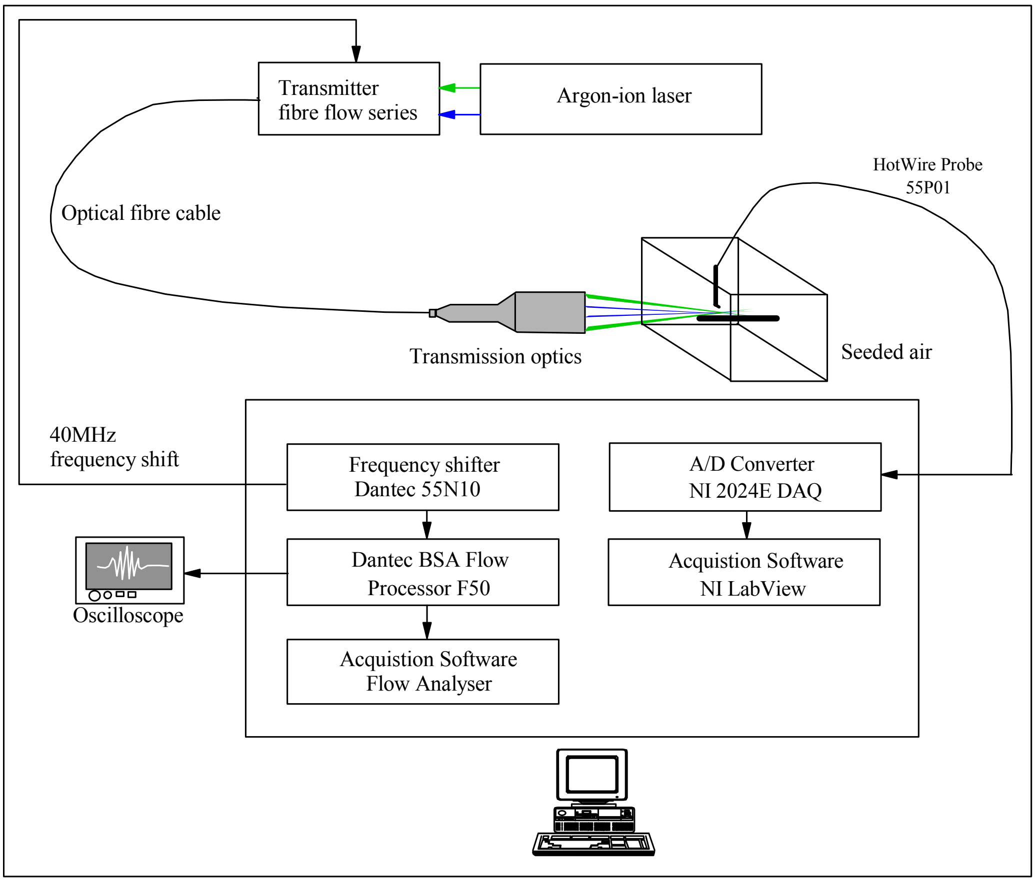
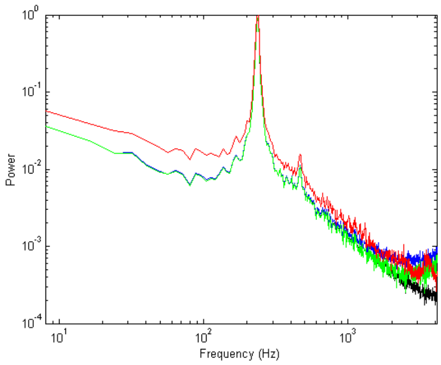


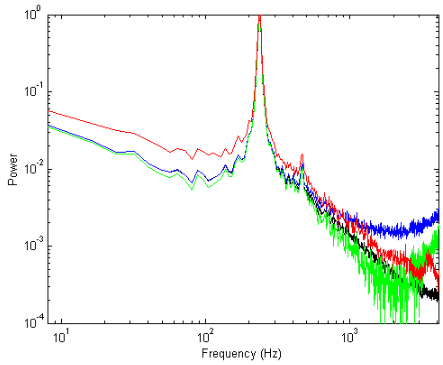
Figure 3. Schematic of measurement set-up & data acquisition system

4. RESULTS

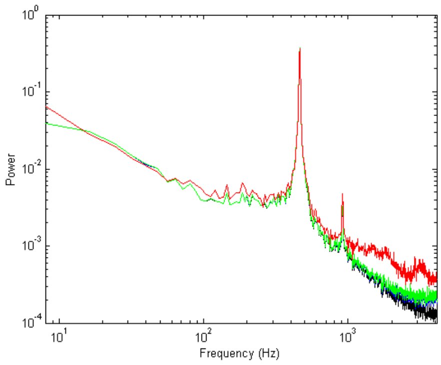
In the first instance, evaluation of the performance of the correction algorithms was performed for each configuration with the LDA and hot wire in close proximity. All tests were performed at a velocity of 15 m/s and with mean data rates of 16kHz for the LDA acquisition. Figures 4(a)-(c) show the power spectra of the hot-wire measurements together with the raw and corrected spectra for the LDA data. It should be noted that the hot wire measurements were contaminated at high frequencies due to oil droplets striking the probe. For each case, the data rate was high so that the influence of the errors is only observed at higher frequencies as can be seen from the results. The LDA data was then re-sampled using only every 2^{nd} point so that effective data rate was reduced substantially and the procedures repeated. The results from this are shown in figures 4(d)-(f) where the various errors inherent in the



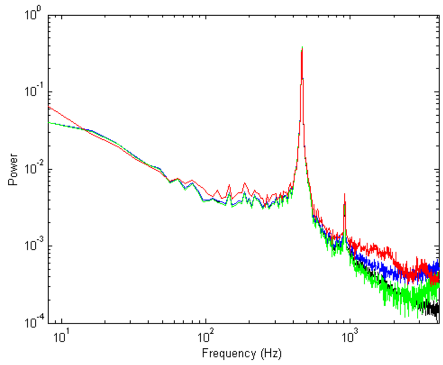
(a) test 1 (high data rate)



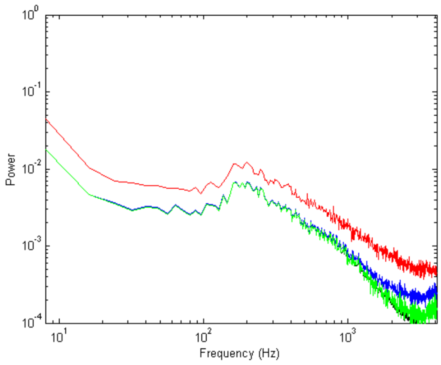
(d) test 1 (low data rate)



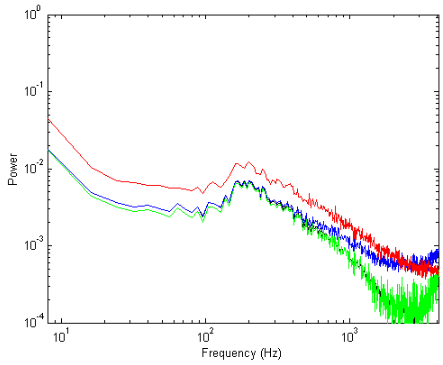
(b) test 2 (high data rate)



(e) test 2 (low data rate)



(e) test 3 (high data rate)



(f) test 3 (low data rate)

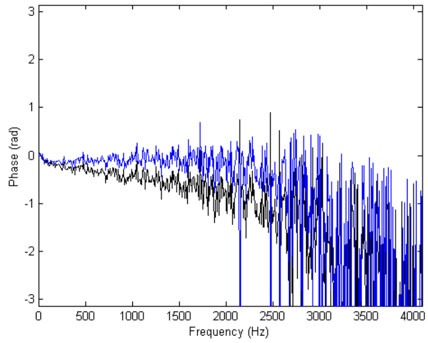
Figure 4 : Hot wire and LDA Spectra

(red – hot wire spectra: black – raw LDA spectra)

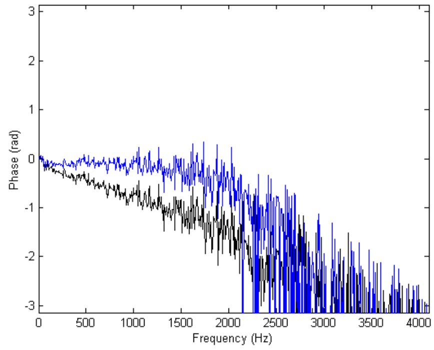
(blue – low pass corrected: green – step noise corrected)

sample and hold procedures at higher frequencies are quite apparent. The effect of the low pass filter and step noise corrections are evident in both sets of results. Good estimates of the spectra can be observed up to some 3 kHz for the original data and 1.5 kHz for the decimated data. This indicates that a mean data rate of some five times the Nyquist frequency is required, or approximately twice the normal sampling rate for equi-spaced data as has observed by other authors. The differences observed between the hot wire and LDA spectra for cases 1 & 3 are due to the normal hot wire measuring the vector component of the flow whereas the LDA is sensing the stream-wise a single component of velocity only.

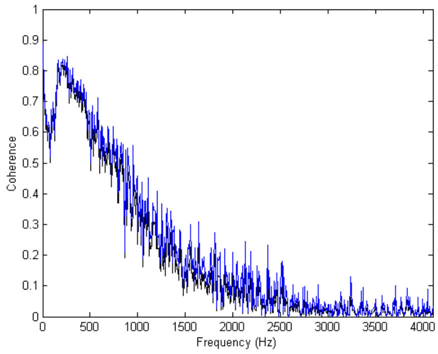
The results from the cylinder cross flows show narrow band frequency spectrum characteristics. For the larger cylinder, the major peak at 250 Hz corresponds to the vortex shedding frequency with a secondary peak at evident at 500 Hz whilst for the smaller cylinder, the main peak was at 500 Hz. As stated above, the difference in spectra due to influence of other velocity components on HWA response is noticeable. For the coherence and phase measurements, coherence peaks close to unity at the vortex shedding frequencies were measured and phase information on vortex convection speeds were readily obtained.



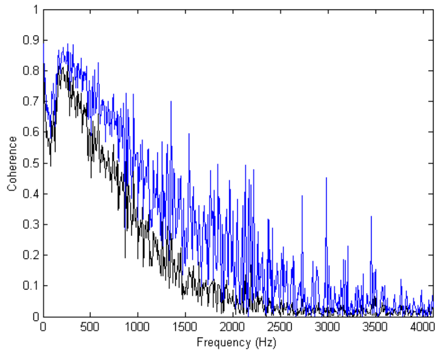
(a) Phase (high)



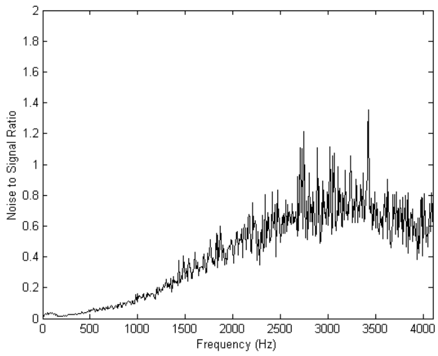
(b) Phase (low)



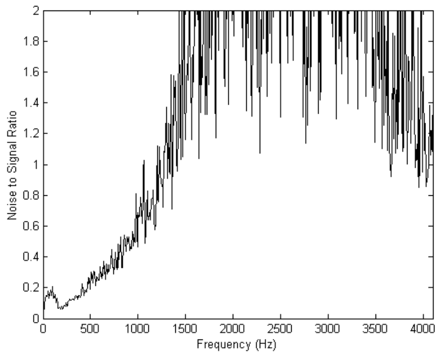
(c) Coherence (high)



(d)Coherence (low)



(e) Noise/Signal (high)



(f) Noise/Signal (low)

Figure 5 : Coherence, Phase and Noise/Signal Ratio (d = 12.7mm)

((a), (b), (c) & (d): black - uncorrected : blue – corrected)

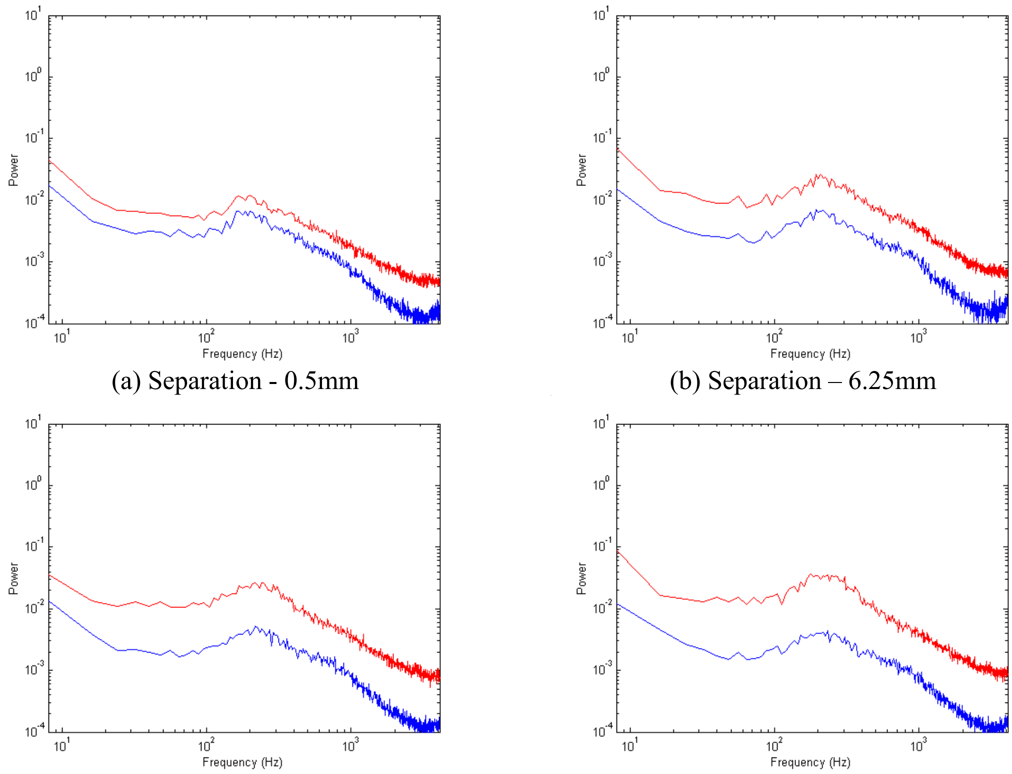
The results for both the original and corrected estimates of phase and coherence together with an estimate of the noise to signal ratio for the grid turbulence are shown in figure 5 for the LDA and hot wire probe separated by 0.5mm. The effect of the phase correction is apparent in figures 5 (a) & (b). For the higher data rate, the phase estimate is reasonable up to 2 kHz whereas for the lower sample rate, 1.5 kHz is again the limit. These estimates would be improved with additional ensemble averaging. The correction of the coherence function for the higher data rate seems appropriate but for the lower data rate the step-noise has been substantially overestimated.

For the grid turbulence, the hot wire was moved to 3 positions downstream of the LDA system and the measurement procedure was repeated. Table 1 summarises the positions and the mean data rates for the LDA acquisition.

Table 1: Mean Data Rate, f_s (Hz) – Grid Turbulence

Hot-wire position	*0.5mm*	*6.25mm*	*12.5mm*	*18.75mm*
LDA data rate (Hz)	16000	15000	21000	21000

The resulting spectra are shown in figure 6 with the turbulence intensity increasing in the stream-wise direction as can be seen from the hot wire measurements. The differences between the hot wire and corrected LDA spectra at the higher frequencies are clearly evident.



(a) Separation - 0.5mm (b) Separation – 6.25mm

(c) Separation – 12.5mm (d) Separation – 18.75mm

Figure 6: Power spectra (grid turbulence)
(red – hot wire: blue – corrected LDA)

The results for the coherence, phase and correlation function measured at each point are shown in figure 7. It can be seen that the stream-wise coherence is reduced with increased separation, so that for the 3rd and 4th positions, there is virtually no coherence above 1 kHz. The adjacent phase estimates give the convection speed at each frequency. These plots show the phase lag increasing with both separation and frequency. From these plots, the convection speed can be estimated up to 3 kHz for the minimal separation and 1Khz for the maximum separation. The normalized cross-correlation function (R_{uz}) at each location calculated using the inverse DFT of the corrected cross-spectrum is also shown in figure 7. This data is the biased estimate which has been normalized by the number of samples. The location of the cross-correlation peak is shifted by an increasing amount in time as the distance between the probes is increased. This behaviour is as expected for a steadily convecting flow field.

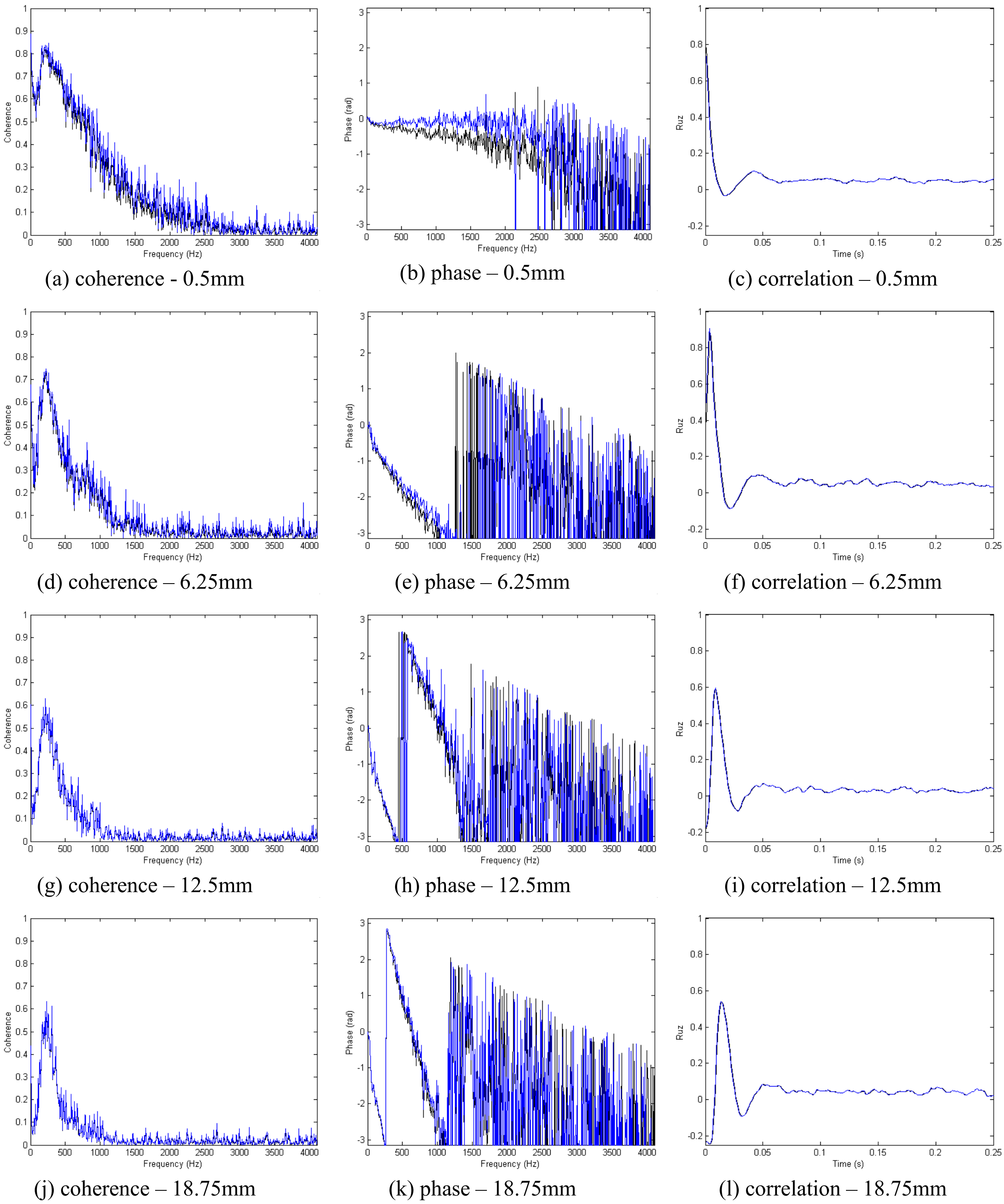


Figure 7: Coherence, phase and cross-correlation (grid turbulence)

(black - uncorrected : blue – corrected)

5. DISCUSSION & CONCLUSIONS

The results provide an indication of the effect that the LDA data acquisition rate has on the estimation of coherence and phase between the LDA data and measurements from a conventional acquisition. The most significant effect of the correction process is the increase in the value of the coherence function at high frequencies, and the change in the phase. The change in the phase estimate effected by the corrections is important, as the phase data is often used to calculate the convection velocity of coherent structures. The variation in the phase estimate caused by the corrections was up to 10% of the uncorrected phase measured at the lowest data rate. However, the correction did not significantly change to the shape or magnitude of the cross-correlation function. This is because the cross spectrum is dominated by the lower frequencies so that differences at the higher frequencies will not appear to play a significant role when the cross correlation is determined via inverse Fourier transform.

For the coherence, at high data rates the step noise is reduced in the re-sampling process and therefore the benefit of the step-noise correction is also reduced. The increase in the numerical value of the coherence is not immediately helpful in this instance due to the increased variation at high frequencies. However, the use of more ensemble averaging for these measurements would be likely to produce a more consistent estimate of the coherence and it is thought that the correction will boost the signal of high frequency events and facilitate their detection. It is anticipated that where the coherence is less than 0.1, some 1000 ensemble averages are required to obtain a good statistical estimate of the coherence as indicated by Durant et al. (2000). Thus, to obtain reasonable estimates of the correlation lengths at high frequencies will require very long data records with high data rates for the LDA measurements. The step-noise estimate can be further reduced by the judicious choice of low-pass filter during sample-and-hold. In these experiments a 6^{th} order Butterworth filter was used with a cut-off at 0.2 times the resampling frequency (i.e. the Nyquist frequency for the equisampled data).

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REFERENCES

Adrian R.J.; Yao C.S. (1987) Power spectra of fluid velocities measured by laser Doppler velocimetry. Exp Fluids 5: 17-28.

Ajmani D.B.S.; Roberts J.D. (1990) Improved Spectral Estimation for Irregularly Sampled Data using Digital Filters. Mechanical Systems and Signal Processing 4: 77-94.

Benedict, L.H.; Nobach, H, Tropea,C. (1996) benchmark tests for the estimation of power spectra from LDA signals. Paper 32.6, 9^{th} Int. Symp. On Applications of Laser Techniques in Fluid Mechanics.

Boyer, L., Searby, G. (1986) Random Sampling: Distortion and reconstruction of velocity spectra from fast Fourier transform analysis of the analog signal of a laser Doppler processor. J. App. Physics 60, 2699-2707.

Britz, D.; Antonia,R., (1996) A comparison of methods of computing power spectra of LDA signals. Meas.Sci. Technol. 7: 1042-1053.

Cenedese. A.; Romano,G.P.; Di Felice, F. (1991) Experimental testing of Taylor’s hypothesis by L.D.A. in highly turbulent flow. Exp Fluids 11: 351-358.

Durant, C.; Robert; G., Filipi, P.J.T.; Mattei, P.O. (2000) Vibroacoustic response of a thin cylindrical shell excited by a turbulent internal flow: comparison between numerical prediction and experimentation. Journal of Sound & Vibration 229: 1115-1155.

Eriksson, J.G, Karlsson, R.I. (1995) An investigation of the spatial resolution requirements for two-point correlation measurements using LDV. Exp Fluids 18: 393-396.

Gaster, M.; Roberts, J.D. (1975) Spectral analysis of randomly sampled signals. J. Inst. Maths. Appl.15: 195-216.

Gerosa, S.; Romano, G.P. (1994) Effect of noise in laser Doppler anemometry. Mechanical Systems and Signal Processing 8: 229-242.

Host-Madsen,A, Caspersen,C. (1995) Spectral estimation for random sampling using interpolation. Signal Processing 46: 297-313.

Romano,G.P.; (1995) Analysis of two-point velocity measurements in near-wall flows. Exp Fluids 20: 63-83.

Trimis, D.; Melling,A. (1995) Improved laser Doppler anemometry techniques for two-point turbulent flow correlations. Meas Sci Tech 6: 663-673.

Tropea,C. (1995) Laser Doppler anemometry: recent developments and future challenges. Meas. Sci. Technol. 6: 605-619.

Van Maanen, H., Tulleken, H. (1994) Application of Kalman reconstruction to laser Doppler anemometry for estimation of turbulent velocity fluctuations. Paper 23.1, 7th Int. Symp. On Applications of Laser Techniques in Fluid Mechanics.

Veynante,D.; Candel,S.M. (1988) Application of non-linear spectral analysis and signal reconstruction to laser Doppler velocimetry. Exp Fluids 6: 534-540.