Dynamics of Large-Scale Structures in Turbulent Flow over a Wavy Wall

Nils Kruse¹, Axel Günther² and Philipp Rudolf von Rohr¹ *

¹Institute of Process Engineering, ETH Zurich, CH-8092 Zurich, Switzerland
²Department of Chemical Engineering, MIT, Cambridge, MA 02139, U.S.A.

Abstract. The developed turbulent flow between a train of waves and a flat wall is considered in a wide water channel. The wavelength, \( \Lambda \), of the bottom wall equals the channel height and the wave amplitude is ten times smaller. Previously, a spanwise scale of \( O(1.5\Lambda) \) was found in the velocity and temperature fields (Günther & Rudolf von Rohr, 2002a,b). Present work focuses on the temporal behavior of these scales at a Reynolds number of 3800, defined with the half channel height, and the bulk velocity. Digital particle image velocimetry (PIV) is performed in a horizontal plane with an area of view (AOV) of \( 2.6\Lambda \times 2.7\Lambda \). Ten ensembles of 90 consecutive image pairs are acquired at a rate of 15 Hz, a temporal resolution sufficient to assess how the largest flow scales evolve in time. The streamwise velocity \( u(x, z, t) \) is filtered using the dominant eigenfunctions that are obtained by POD analysis. The very large temporal scales of the meandering motion of the \( O(1.5\Lambda) \) structures could be followed over measurement times of up to 6 s, during which they are convected downstream by a distance of 65 wavelengths.

Figure 1: Coordinate system and schematic of (1) the separation region, and the regions (2) of maximum positive and (3) negative Reynolds shear stress.

*corresponding author, e-mail: vonrohr@ivuk.mavt.ethz.ch
1 Introduction

Longitudinal structures play an important role in a number of transport processes. A wavy bottom wall adds a defined degree of complexity to the turbulent flow between two horizontal walls (e.g. Niederschulte, et al., 1990; Günther, et al., 1998). The mean quantities of the flow vary periodically in the streamwise direction and its boundary conditions are well defined. Since moderate Reynolds numbers are considered, direct numerical simulations (DNS) are capable of resolving the smallest scales of the flows. Since the computational domains of present DNS is of the same order as the $1.5\Delta$ scales, laboratory measurements rather than computational studies seem advantageous to address the dynamics of these large scales. The paper is organized as follows: section 2 describes the experimental facility. Section 3 presents results on the dynamics of the lateral motion of the observed $1.5\Delta$ scales.

Figure 1 shows the coordinate system and schematically illustrates characteristic regions of the mean flow field in the vicinity of the wavy surface. Coordinate $x$ is directed parallel to the mean flow, $y$ is perpendicular to the top wall, and $z$ is the spanwise coordinate. The corresponding velocity components are denoted as $u$, $v$, and $w$. A separation zone (1), bounded by the isosurface of the mean streamfunction $\Psi(x, y) = 0$, is located in the wave troughs. At the uphill side, two regions of maximum (2) and minimum (3) Reynolds shear stress, $-\overline{\partial u \overline{v'}}$, are found. From the DNS results of Cherukat et al. (1998) for $Re_h = 3460$, the locations of regions (2) and (3) are found approximately $0.08\Lambda$, and $0.01\Lambda$ above the wall at the uphill side. The data of Cherukat et al. (1998) and Henn & Sykes (1999) identify the energy of traverse velocity fluctuations, $\overline{\partial w^2}$, to be maximal at a location that is close to region (3). The developed flow between the bottom wall and the bulk is characterized by the ratio of the amplitude, $2a$, to the wavelength, $\alpha = 2a/\Lambda = 0.1$, and the Reynolds number:

$$Re_h = \frac{U_b \cdot h}{\nu},$$

where $\nu$ denotes the kinematic viscosity, and $h$ is the half-height of the channel. The bulk velocity $U_b$ is defined as

$$U_b = \frac{1}{2h} \int_{y_w}^{2h} U(x_\xi, y) dy,$$

where $x_\xi$ denotes an arbitrary $x$-location and $y_w(x) = 0.05\Lambda \cos(x \cdot 2\pi/\Lambda)$ describes the profile of the wavy surface. Reynolds averaging is used to decompose the velocity into a mean and a fluctuating part: $u = \overline{u} + u'$.

Early works described the non-separated, isothermal flow over small amplitude waves by linear stability analysis. With increasing $\alpha$, linear analysis eventually becomes insufficient. Following the original contributions of Motzfeld (1937), Miles (1957), Benjamin (1959), and Hanratty (e.g. Buckels et al. 1984, Hudson et al. 1996), a number of laboratory and numerical experiments were conducted to describe the mean and turbulence quantities of the flow, and turbulence production.

The literature on the stability of an isothermal, sheared flow over rigid waves suggests the Görtler (Görtler 1940, Saric 1994) and the Craik-Leibovich type 2 (CL-2) instability (Phillips & Wu 1994) to produce, or catalyse, spanwise-periodic longitudinal vortices. Even though such structures are of three-dimensional nature, the mostly qualitative vi-
ualizations were restricted to observations in the \((x, y)\)-plane so far. Only recently, the attention was drawn to the effect of the wavy wall with respect to three-dimensional, large-scale structures. Gong et al. (1996) concluded the presence of a CL-2 mechanism from a spanwise variation of the mean streamwise velocity in a low aspect ratio wind tunnel. Günther & Rudolf von Rohr (2002a) addressed the existence of \(O\{1.5\Lambda\}\) scales in a wide channel. In \((x, z)\)-planes above the wave crests, the perturbations in the velocity field were found to be the largest, a reason why we consider the same vertical position in this paper. Due to the wavyness of the bottom wall, the mean flow is weakly inhomogeneous in the \(x\)-direction in this plane, where the spanwise direction, \(z\), can be considered homogeneous at the measurement location in the channel center. At turbulent conditions and a position \(y/\Lambda = 0.26\), the observed longitudinal structures do not have fixed spanwise locations but they meander laterally. A discussion of characteristic scales was based on a POD analysis of the streamwise velocity component. The dominance of eigenvalues \(\lambda_{1,2}\) is limited to the lower half of the channel and increases with increasing Reynolds number. Scale \(\Lambda_z = O\{1.5\Lambda\}\) agrees with the one reported by Gong et al. (1996). It was concluded that the non-developed flow does not already contain the structures we observe for the developed flow situation, but they develop and grow when passing a periodic train of waves in the streamwise direction.

2 Experimental

Measurements are carried out in a channel facility. The flow loop is designed for low Reynolds number turbulence measurements with light sheet techniques. It contains approximately \(0.280\) \(m^3\) of de-ionized and filtered water. The entire facility is made of black anodized aluminum, PVC, and Schott BK-7 glass. For a detailed description we refer to Günther (2001) or Günther & Rudolf von Rohr (2002a).

The full height of the channel, \(H\), is 30 mm, and its aspect ratio, \(B/H\), is 12:1. The wavelength \(\Lambda\) of the sinusoidal wall profile is equal to the channel height. Optical access is provided at four streamwise locations of the wavy channel section through viewing-ports at both side walls and at the flat top wall. Measurements are performed for a hydrodynamically developed flow after the 50th wave crest. The maximum AOV for the top windows is \(3.3 \cdot \Lambda(\text{streamwise}) \times 3.3 \cdot \Lambda(\text{spanwise})\). To determine the fluid viscosity, the water temperature is monitored downstream of the test section. We use digital PIV (Adrian, 1991; Westerweel, 1995; Raffel et al., 1998) to assess the temporal behavior of large-scale structures in the velocity field. For the velocity measurements with an large AOV of \(2.6\Lambda \times 2.7\Lambda\), the flow is seeded with 100 \(\mu m\) polyamide particles. The measurement system consisting of the laser, the laser optics, and the camera, is positioned on a traversing system that allows the vertical position to be changed. The accuracy of adjusting the \(y\)-position with the traverse is approximately 10 \(\mu m\). A flashlamp-pumped dual Nd:YAG laser provides the pulse light source. A 8 bit CCD camera with a resolution of \(1008 \times 1016\) pixels\(^2\) is used for the velocity measurements. Measurements are taken in the \((x, z)\)-plane for the vertical distances \(y/H = 0.26\).
3 Results

**Instantaneous velocity fields.** Measurements in the \((x, z)\)-plane are expected to reveal information that is related to large-scale, longitudinal flow structures. The left row of Fig. 5 shows a sequence of six contour plots of the instantaneous streamwise velocity in the \((x, z)\)-plane acquired with a frame rate of 15 Hz. The images correspond to a Reynolds number of 3800. In this plane above the wave crests at the wall-normal distance \(y/H = 0.26\) the perturbations in the velocity field were previously found to be the largest ( Günther 2001). Due to the wavyness of the bottom wall, the mean flow is weakly inhomogeneous in the \(x\)-direction in this plane, where the spanwise direction, \(z\), can be considered homogeneous at the measurement location in the channel center. Already from the instantaneous plots, the existence of large-scale longitudinal structures become obvious. Large fluid columns with a characteristic distance of \(O\{1.5\Lambda\}\) in the spanwise direction can be observed. However, quantitative contributions of the different scales, or the dominant scale cannot be found by such means. The longitudinal structures observed do not have fixed spanwise locations but they meander laterally. To address the qualitative contribution of the different scales, we now perform a POD analysis with an ensemble of \(M = 900\) realizations of the velocity \(u(x, y + 0.26\Lambda, z, t)\). The ensemble contains 10 sequences of 90 consecutive velocity fields that were acquired at 15 Hz.

**POD analysis.** We use the method of snapshots and perform a Karhunen-Loève (KL) or proper orthogonal decomposition (POD) of the streamwise velocity component (Liu et al. 2001, Berkooz et al. 1993). A single coordinate \(1, \ldots, N\) with \(N = n \cdot m\) is used to distinguish between the different positions in the \((x, z)\)-plane and we write the set of spatiotemporal velocity data as:

\[
U = \{U_i\}_{i=1}^M = \begin{bmatrix}
    u_{11}, u_{12}, \ldots, u_{1M} \\
    u_{21}, u_{22}, \ldots, u_{2M} \\
    \vdots \\
    u_{N1}, u_{N2}, \ldots, u_{NM}
\end{bmatrix}
\]  

with \(U_i = [u_1, u_2, \ldots, u_N]^T\). We obtain the mean streamwise velocity by averaging over the columns:

\[
\overline{U} = \frac{1}{M} \sum_{i=1}^M U_i, \quad i = 1, \ldots, M.
\]  

For the velocity fluctuations then follows:

\[
U'_i = U_i - \overline{U}, \quad i = 1, \ldots, M.
\]  

Using the method of snapshots, the \(M \times M\) covariance matrix becomes:

\[
C_{ij} = \langle U'_i U'_j \rangle, \quad i, j = 1, \ldots, M,
\]  

where \(\langle \cdot, \cdot \rangle\) is the Euclidean inner product. Since the matrix is symmetric its eigenvalues, \(\lambda_i\), are nonnegative, and its eigenvectors, \(\phi_i\), \(i = 1, \ldots, M\), form a complete orthogonal
set. The orthogonal eigenfunctions are:

\[ \Pi^{[k]} = \sum_{i=1}^{M} \phi_i^{[k]} u_i, \quad k = 1, \ldots, M, \]  

(7)

where \( \phi_i^{[k]} \) is the \( i \)-th component of the \( k \)-th eigenvector. Figure 2 shows the eigenfunctions that correspond to modes 1-8. The results suggest that, for the considered Reynolds numbers, the eigenfunctions \( \Pi_1 \) and \( \Pi_2 \) have a characteristic scale \( \Lambda_z = O\{1.5\Lambda\} \) in the spanwise direction.

Figure 2: Eigenfunctions of modes 1-8 from a decomposition of \( u(x, y/\Lambda = 0.26, z, t) \) with \( AOV=2.6\Lambda \times 2.7\Lambda, Re_h = 3800 \).

We can associate an energy with the velocity fluctuations and write:

\[ E = \sum_{i=1}^{M} \lambda_i. \]  

(8)

The fractional contribution of one eigenfunctions associated eigenvalue is

\[ \frac{E_k}{E} = \frac{\lambda_k}{E}. \]  

(9)

Since the streamwise velocity is homogeneous in the spanwise direction, note that the POD analysis is identical to a Fourier decomposition in this direction. However, this is not strictly true in the weakly inhomogeneous \( x \)-direction, where POD analysis is the correct way of decomposing the velocity field. Figure 3 shows the eigenvalues \( \lambda_1, \ldots, \lambda_{05} \) being ranked in a decreasing order of their fractional contribution to the turbulent kinetic energy \( E \) and confirms the dominance of modes 1 and 2 with a cumulative contribution of 0.25\( E \).
Temporal evolution. We now have a closer look at the ten sequences of 90 image pairs (frame rate 15 Hz) that are contained in the ensemble we used for the POD analysis. Since the AOV is rather large, $2.6\Lambda \times 2.7\Lambda$, the time-resolution of the camera is sufficient to assess the largest flow scales evolve in time. To analyse the temporal evolution, the POD coefficients $a_i(t)$ of the dominant modes have been determined for each instant in time. Of particular interest is the temporal behaviour of modes 1 and 2, since the corresponding eigenfunctions have previously been related to the $O\{1.5\Lambda\}$ scales. Figure 4 shows the time history of the coefficients $a_i$ for modes 1, 2, 10, and 40. Where $a_1(t)$ and $a_2(t)$ – which we expect to resemble deterministic contributions – show a relatively slow temporal variation, the high frequency variations of $a_{10}(t)$ and $a_{40}(t)$ are, as expected, much less deterministic.

Using only the first $K$ most energetic eigenfunctions, the original data can be approximated as:

$$U'_j = U + \sum_{i=1}^{K} a_i \Pi^{[i]}$$

where the coefficient is computed from the projection of the sample vector $U'_j$ ($N \times 1$ matrix) onto eigenfunction $\Pi^{[i]}$:

$$a_i = \frac{U'_j \cdot \Pi^{[i]}}{\Pi^{[i]} \cdot \Pi^{[i]}}.$$  \hspace{1cm} (11)

This operation corresponds to a low-pass filtering and is used to study the temporal behavior of the largest scales. Representative samples of the raw and the corresponding filtered velocity fields are shown in Fig 5. Six snapshots of instantaneous fluctuating flow fields are projected onto eigenmodes 1-2 and modes 1-8 (containing 25% and 49% of...
Figure 4: Temporal behavior of the coefficients $a_i(t)$ for POD modes 1, 2, 10, and 40 for a decomposition of velocity $u(x, y/\Lambda = 0.26, z, t)$ with $\text{AOV} = 2.6\Lambda \times 2.7\Lambda$, $Re_h = 3800$.

the total energy). The longitudinal structures in Fig. 5 meander laterally. Subsequent realizations are separated by 0.067s. For one of the ten sequences, Fig. 6 shows the temporal behavior of the spanwise maximum/minimum locations of the streamwise averaged velocity field. Eigenfunctions $\Pi_{1-2}$ and $\Pi_{1-8}$ are used for the reconstruction. The total length of the sequence is 6s. In order to estimate the extension of the $\mathcal{O}(1.5\Lambda)$ scales in the streamwise direction, we use Taylor’s hypothesis and introduce a second abscissa:

$$\hat{x} = t \cdot \frac{\langle U \rangle_{xz}}{\Lambda},$$

where $\langle U \rangle_{xz}$ denotes a spatial average of the streamwise mean velocity in the $(x, z)$-plane. Projecting onto modes 1-2 vs. modes 1-8 has no qualitative effect and relatively little quantitative effect on detecting the spanwise location of the extrema in $u(x, y+0.26\Lambda, z, t)$. We therefore decide to filter the instantaneous velocity fields acquired at 15Hz by projecting them onto eigenfunctions $\Pi_{1-2}$ and follow the lateral motion of the corresponding $\mathcal{O}(1.5\Lambda)$ scales in time. The temporal scales of the meandering motion are of the order of seconds, during which they are convected downstream over several tens of wavelengths. The observed large coherence lengths are important for computational investigations of the dynamics of the $\mathcal{O}(1.5\Lambda)$ scales, since the required streamwise extension of the computational domain seems to exceed the capabilities of current DNS and LES.

Figures 7 and 8 show the temporal evolution of the obtained maximum and minimum locations for four sequences of 90 velocity fields. Note that extrema in figures (b) and (c) travel more than $2\Lambda$ in the spanwise direction during the measuring time of 6 s, corresponding to a streamwise distance of 65$\Lambda$. 

\[ \text{[Continued on next page]} \]
Figure 5: Sequence of instantaneous velocities \( u(x, y/\Lambda = 0.26, z, t)/\langle u \rangle \). The temporal separation is \( \Delta t = 1/15 \) s. Raw data (left column), projection on modes 1-2 (middle) and modes 1-8 (right) with AOV=2.6\( \Lambda \times 2.7\Lambda \), \( Re_h = 3800 \).
4 Concluding Remarks

We extended our study on large-scale structures in a low Reynolds number turbulent flow over a train of periodic waves. The temporal behavior of the previously observed $O\{1.5\Lambda\}$ scales has been addressed. The meandering motion of these scales could be followed over measurement times of up to 6 s, during which they are convected downstream by a distance of 65 wavelengths. The observed very large coherent lengths in the streamwise direction are significantly larger than the streamwise domain extensions of all LES and DNS conducted so far. The obtained data improves the available structural information on the considered reference flow case. The meandering motion of the $O\{1.5\Lambda\}$ scales provides a mechanism for momentum and mass transfer between the wavy wall and the bulk fluid.

Acknowledgements

We gratefully acknowledge financial support from the Swiss National Science Foundation (SNF). Measurement technology is partially provided by TSI.
Figure 7: Temporal behavior of velocity minima and maxima for four sequences of 90 velocity fields $u(x, y/\Lambda = 0.26, z, t)$ that are reconstructed from POD modes 1-2. AOV=2.6$\Lambda$ x 2.7$\Lambda$, $Re_h = 3800$. 
Figure 8: For caption see facing page.
References


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