The effect of the expansion ratio on a turbulent non-Newtonian recirculating flow

A. S. Pereira
Departamento de Engenharia Química, Instituto Superior de Engenharia do Porto
Rua Dr. António Bernardino de Almeida 431, 4020-072 Porto, Portugal

F. T. Pinho*
Centro de Estudos de Fenómenos de Transporte, DEMEGI, Faculdade de Engenharia
Universidade do Porto, Rua dos Bragas, 4050-123 Porto, Portugal,fpinho@fe.up.pt

ABSTRACT

Measurements of the mean and turbulent flow characteristics of viscoelastic fluids in a sudden expansion with expansion ratio (ER) of 2.0 were carried out by means of laser-Doppler velocimetry and the results compared with those obtained by Pereira and Pinho (2000) in the ER=1.538 expansion, operating with the same fluids. In all cases the inlet condition was that of fully-developed flow and the non-Newtonian fluids were aqueous solutions of xanthan gum at concentrations of 0.1% and 0.2% by weight which were shear-thinning and exhibited elasticity in rheological flows (Coelho and Pinho, 1998). Other evidence of elastic effects was their behaviour in turbulent pipe flow reported by Pereira and Pinho (1999).

The main findings were:

- The higher levels of $\overline{u'^2}$ in the initial stages of the shear layer, due to advection of upstream wall turbulence and higher $\overline{u'^2}$ production, reduce the recirculation length, but these effects are less intense in the ER=2 geometry than in the ER=1.538 case;
- The earlier development of $\overline{u'^2}$ and the subsequent faster decay of this turbulent component relative to that of the Newtonian fluids is more intense in the ER=2 expansion. Similarly, for the radial normal Reynolds stress ($\overline{v'^2}$) the maximum turbulence occurs earlier with xanthan gum than with water, and this effect is more pronounced in the ER=2 case, but the maximum values of this stress for the two types of fluids do not differ significantly in contrast to what happened in the ER=1.538 expansion;
- For the tangential component of turbulence the effect of the expansion ratio is reversed: with the increase in the expansion ratio the difference between the Newtonian and non-Newtonian characteristics is reduced. Thus, the differences between the Newtonian and non-Newtonian behaviour of $\overline{u'^2}$ and $\overline{v'^2}$ contribute for the decrease of the recirculation length of the xantham gum solutions, but the relative behaviour of the tangential turbulence goes against this variation, and consequently, although the overall effect is still a reduction in recirculation length, the effect is less pronounced in the ER=2 geometry than in the smaller expansion.

1. INTRODUCTION

Knowledge on non-Newtonian sudden expansion flows is still rather limited although these are frequently encountered in the process industries. For dilute solutions the flow is likely to be turbulent but there are only a few quantitative studies in the literature. Pak et al (1991) measured the loss coefficient of polyacrylamide solutions in expansions having D/d of 1.39 and 1.90 after a preliminary visualisation study in Pak et al (1990) aimed at defining the ranges of laminar, transitional and turbulent regimes of various non-Newtonian fluids. Those investigations concluded that the recirculation length of viscoelastic turbulent flows are longer than those involving purely viscous fluids, but the reasons were not fully understood. The lack of detailed information on the mean and turbulent flow fields of Pak et al's (1990,91) works promoted more recent research to shed light on the subject.

The detailed velocity measurements of Castro and Pinho (1995) with tylose solutions for turbulent fully-developed inlet flow conditions showed reductions of the normal Reynolds streses of up to 30%,especially in the radial and tangential directions, but only small variations in the recirculation bubble. The more recent investigations of Pereira and Pinho (2000) with significantly more elastic fluids based on xanthan gum showed a shortening of the recirculation length and a mixed variation of the normal Reynolds stress relative to the corresponding stresses for Newtonian fluids. The differences were attributed, to a large extent, to the effect of viscoelasticity upon the upstream pipe Reynolds stress field which was advected downstream. This was confirmed by a similar work of Escudier and Smith (1999) who probed the same solutions in the same geometry, but with a uniform, low turbulence inlet velocity profile. Although viscoelasticity reduced the turbulent kinetic energy, these changes were not sufficiently strong to affect the mean flow.

There is clearly the need to pursue the detailed investigation of this class of flows in various ways: to extend it to other expansion ratios using the same fluids and to bring in significantly more elastic fluids, in particular those that were previously studied by Pak et al (1991).

This paper is a contribution to the first objective in that it extends the previous work of Pereira and Pinho (2000) to a larger sudden expansion having a ratio ER of 2, using the same xanthan gum solutions and water. The next section presents the experimental setup and is followed by a brief description of the rheology of the fluids. Then, the hydrodynamic results are presented, discussed and compared with previous measurements.

2. EXPERIMENTAL SETUP

The flow configuration is that used by Pereira and Pinho (2000) in their sudden expansion experiments, except for the transparent test section and the downstream pipe. The rig consisted of a vertical closed loop with a 100 litre tank and a centrifugal pump located at the bottom. The descending pipe before the test section was 26 mm in diameter and more than 90 diameters in length, leading to the sudden expansion from 26 mm to 52 mm in diameter and 630 mm in length, thus defining a diameter ratio of 1:2, in contrast to the ratio of 1:1.538 of Pereira and Pinho (2000). The 52 mm diameter pipe leads the flow back to the tank. The test section is represented schematically in Fig. 1 with the coordinate system and it had a square outer cross section to reduce diffraction of light beams. To help ensure a fully developed flow at the inlet of the sudden expansion , a honeyconb was placed at the inlet of the descending 26 mm pipe, i.e., at more than 90 diameters upstream of the sudden expansion plane. The rising pipe had an electromagnetic flowmeter and two valves which, together with a bypass circuit, allowed the flow rate to be properly monitored.

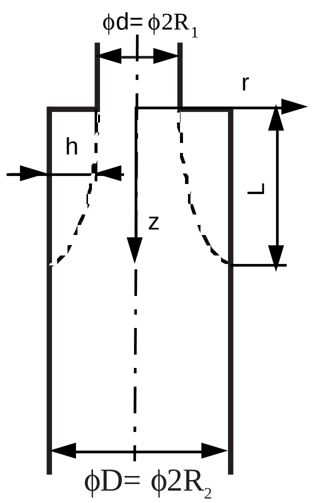

Fig. 1. Schematic representation of the sudden expansion test section.

For the velocity measurements a miniaturised fiber optics laser-Doppler velocimeter from INVENT, model DFLDA, similar to that described by Stieglmeier and Tropea (1992), was used, with a 100 mm front lens mounted onto the 30 mm diameter probe. Scattered light was collected by a photodiode in the forward scatter mode, and the main characteristics of the anemometer are listed in Table I and described by Stieglmeier and Tropea (1992). Measurements of the radial velocity component were limited to the inner 70% of the pipe radius due to excessive refraction of light beams outside that region.

Table I - Laser-Doppler characteristics

Laser wavelength	827 nm
Laser power	100 mW
Measured half angle of beams in air	3.68
Size of measuring volume in water (e^{-2} int.)	
minor axis	37µm
major axis	550µm
Fringe spacing	6.44µm
Frequency shift	3.0 MHz

The signal was processed by a TSI 1990C counter interfaced with a computer via a DOSTEK 1400 A card, which provided the statistical quantities. The data presented in this paper have been corrected for the effects of the mean gradient broadening. The maximum uncertainties in the axial mean and rms velocities, at a 95% confidence level, are of 1.0% and 2.2% on axis respectively, and of 1.1% and 5.2% in the wall region. The uncertainty of the radial and tangential rms velocity components is 2.5% and 5.9% on axis and close to the wall, respectively.

The velocimeter was mounted on a milling table with movement in the three coordinates and the positional uncertainties are of ± 200 µm and ± 150 µm in the axial and transverse directions, respectively.

3. FLUID PROPERTIES

Water and aqueous solutions of xanthan gum grade Keltrol TF from Kelco, a polysaccharide of high molecular weight ($2*10^6$ kg/kmol), at weight concentrations of 0.1% and 0.2% were used. These were the same fluids used previously by Pereira and Pinho (2000) who, together with Pereira and Pinho (1999), reported on their rheology. The solutions were shear-thinning over a wide range of shear rates and tap water was used to prepare them, with the addition of 0.02% by weight of the biocide Kathon LXE from Rohm and Haas to help prevent bacteriological degradation. The rheological characterisation was carried out in the Physica MC100 rheometer implementing a double gap concentric cylinder geometry, described previously by Coelho and Pinho (1998).

The viscometric viscosity of the solutions are plotted in Fig. 2 together with the curve-fitted Sisko model equation (Eq. 1), whose parameters are listed in Table II.

$$\eta = \eta_{ref} \left(\lambda_s \dot{\gamma} \right)^{n-1} + \eta_\infty \qquad (1)$$

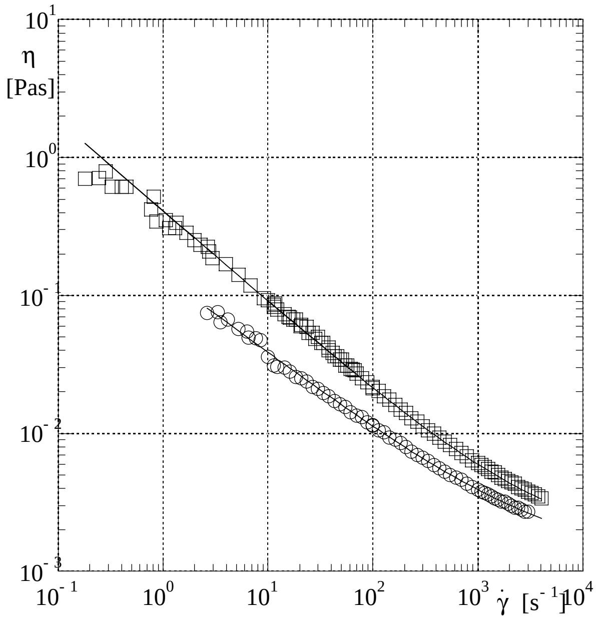

Fig. 2. Variation of the viscosity of xanthan gum solutions with the shear rate at 25°C, and the corresponding curve-fitted Sisko models: O 0.1% xanthan gum; ¤ 0.2% xanthan gum. *From Pereira and Pinho (2000)*

Creep and oscillatory shear flows were used to characterize the extent of viscoelastic behaviour. A similar aqueous solution of 0.2% XG (same grade) was also investigated in a sudden expansion by Escudier and Smith

(1999) and they report other rheological measurements such as the first normal stress difference. The only difference between the 0.2% XG solutions used here and by Escudier and Smith (1999) was the solvent, in both cases local tap water. The differences in rheology were fairly small and are assessed in the comparative work of Escudier et al (2000).

Table II- Sisko model parameters for Keltrol solutions at 25°C. From Pereira and Pinho (2000)

Solution	η_{ref} [Pas]	η_∞ [Pas]	λ_s [s]	n
0.1%	10.52	0.0012	1970	0.4299
0.2%	58.06	0.001589	1900	0.3434

Prior to the investigation of the sudden expansion flow the fully developed turbulent pipe flow was studied in terms of friction factor versus Reynolds number behaviour by Pereira and Pinho (1999). The solutions exhibited drag reduction in a 26 mm diameter pipe and for maximum wall Reynolds numbers of 40,100 and 28,100, the measured drag reductions of 45% and 59% for the 0.1% and 0.2% xanthan gum solutions represent over 50% and 75% of the maximum drag reduction predicted by Virk's asymptote (Virk et al, 1970).

4. RESULTS AND DISCUSSION

Measurements of the three components of the mean and root-mean-square of the fluctuations of the velocity of the xanthan gum solutions and water were carried out by means of laser-Doppler velocimetry in a new sudden expansion with an expansion ratio (ER) of 2, and compared with the previous measurements of Pereira and Pinho (2000) in the expansion of ER=1.538 with the same fluids. The flow conditions were the same as in Pereira and Pinho (2000), i.e., similar fully-developed inlet conditions and equal Reynolds numbers based on the upstream pipe flow.

Table I- Flow conditions, recirculation length and maximum turbulent quantities.

Run	*D/d*	Fluid	Re_w	x_R/h	$\left(\overline{u'^2}/U_1^2\right)_{max}$	$\left(\overline{v'^2}/U_1^2\right)_{max}$	$\left(\overline{w'^2}/U_1^2\right)_{max}$	$\left(k/U_1^2\right)_{max}$
1	1.538^+	water	135,000	8.43	0.0466	0.0275	0.0326	0.0533
2	1.538^+	water	50,300	8.71	0.0423	0.0271	0.0279	0.0478
3	1.538^+	0.1%XG	19,600	6.93	0.0400	0.0247	0.0321	0.0481
4	1.538^+	0.2%XG	27,200	6.78	0.0502	0.0263	0.0325	0.0540
5	1.538^+	0.2%XG	19,400	7.14	0.0447	0.0242	0.0325	0.0499
6	2.0	water	134,000	9.30	0.0556	0.0255	0.0344	0.0555
7	2.0	water	50,400	10.00	0.0527	0.0244	0.0284	0.0521
8	2.0	0.1%XG	19,600	9.08	0.0482	0.0251	0.0304	0.0490
9	2.0	0.2%XG	27,100	8.73	0.0507	0.0254	0.0311	0.0510
10	2.0	0.2%XG	19,400	8.77	0.0490	0.0253	0.0311	0.0505

$^+$ from Pereira and Pinho (2000)

The flow conditions, the normalised recirculation length and the maximum values of the normal Reynolds stresses and turbulent kinetic energy k are summarised in Table I which includes data from Pereira and Pinho (2000) for the ER=1.538 runs. The Newtonian results, and their variations with Reynolds number, are consistent with the findings of Khezzar et al (1985) for the sudden expansion.

As in the smaller expansion, the addition of polymer reduced the recirculation length relative to that of the pure solvent. For the ER=2 expansion, the reduction in x_R/h on going from water to 0.1% XG was 10%, which is half that found for the smaller expansion by Pereira and Pinho (2000), as is well shown in the x_R/h plot of Fig. 3. Further increase in polymer concentration to 0.2% reduces the recirculation length. For the smaller expansion the value of x_R/h of the 0.1% XG solution was intermediate to that of the two runs with 0.2% XG in contrast to what is happening now in the ER=2 geometry. However, these variations are within the 3% uncertainty in the measurement of x_R/h (c.f. Fig. 3), consequently no conclusions can be drawn.

The smaller polymer effect on x_R/h for the ER=2 geometry is in agreement with a higher rate of variation of the vorticity thickness (δ_ω). For the ER=1.538 expansion Pereira and Pinho (2000) found that δ_ω varied linearly with the longitudinal coordinate according to

$$\delta_\omega/d = 0.15\,x/d + 0.1 \tag{2}$$

where d represents the inlet pipe diameter. For the larger (ER=2) expansion, the variation of δ_ω/d is no longer linear as can be seen in Fig. 4. The growth of the vorticity thickness is initially fast, it becomes linear with a slope of 0.16 in the range $0.75 \le x/d \le 2$ and later decreases due to the curvature and wall effects. A more appropriate law of variation of the vorticity thickness is of second order

$$\delta_\omega/d = 0.09 + 0.24\,x/d - 0.025(x/d)^2 \tag{3}$$

and is also plotted in Fig. 4. This change in the behaviour of the vorticity thickness is a consequence of the lower degree of confinement of the shear layer for the ER=2 geometry relative to the smaller expansion.

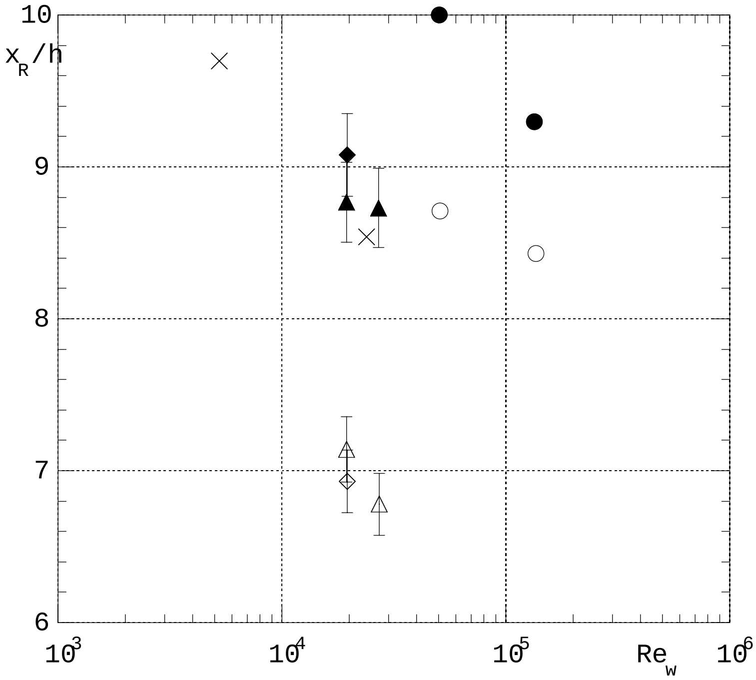

Fig. 3. Effect of expansion ratio and upstream wall Reynolds number on normalised recirculation length with 3% error bars: O *water;* ◊ *0.1% XG;* Δ *0.2% XG. Open symbols: ER=1.538; Closed symbols: ER=2;* X *0.2%XG in ER=1.538 from Escudier and Smith (1999).*

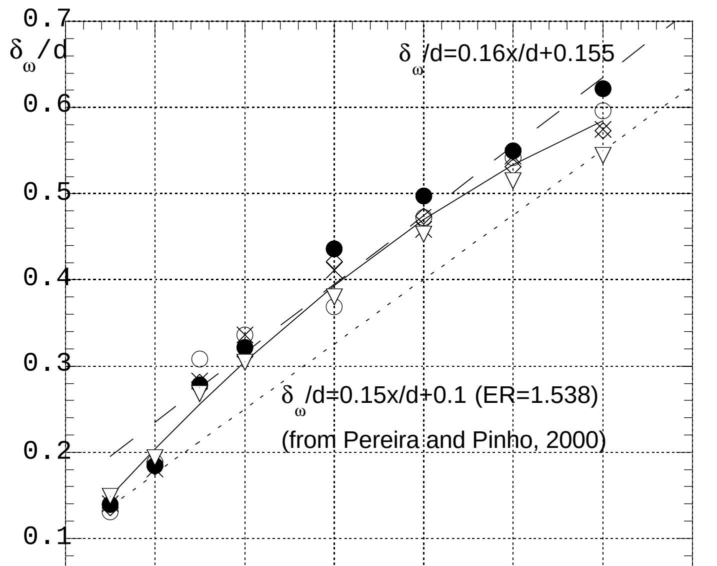

Fig. 4. Longitudinal variation of the vorticity thickness. X *water Re=50,400;* ∇ *water Re=134,000;* ◊ *0.1% XG Re=19,600;* O *0.2%XG Re=27,100;* • *0.2% XG Re= 19,400. Full line: Eq. (3).*

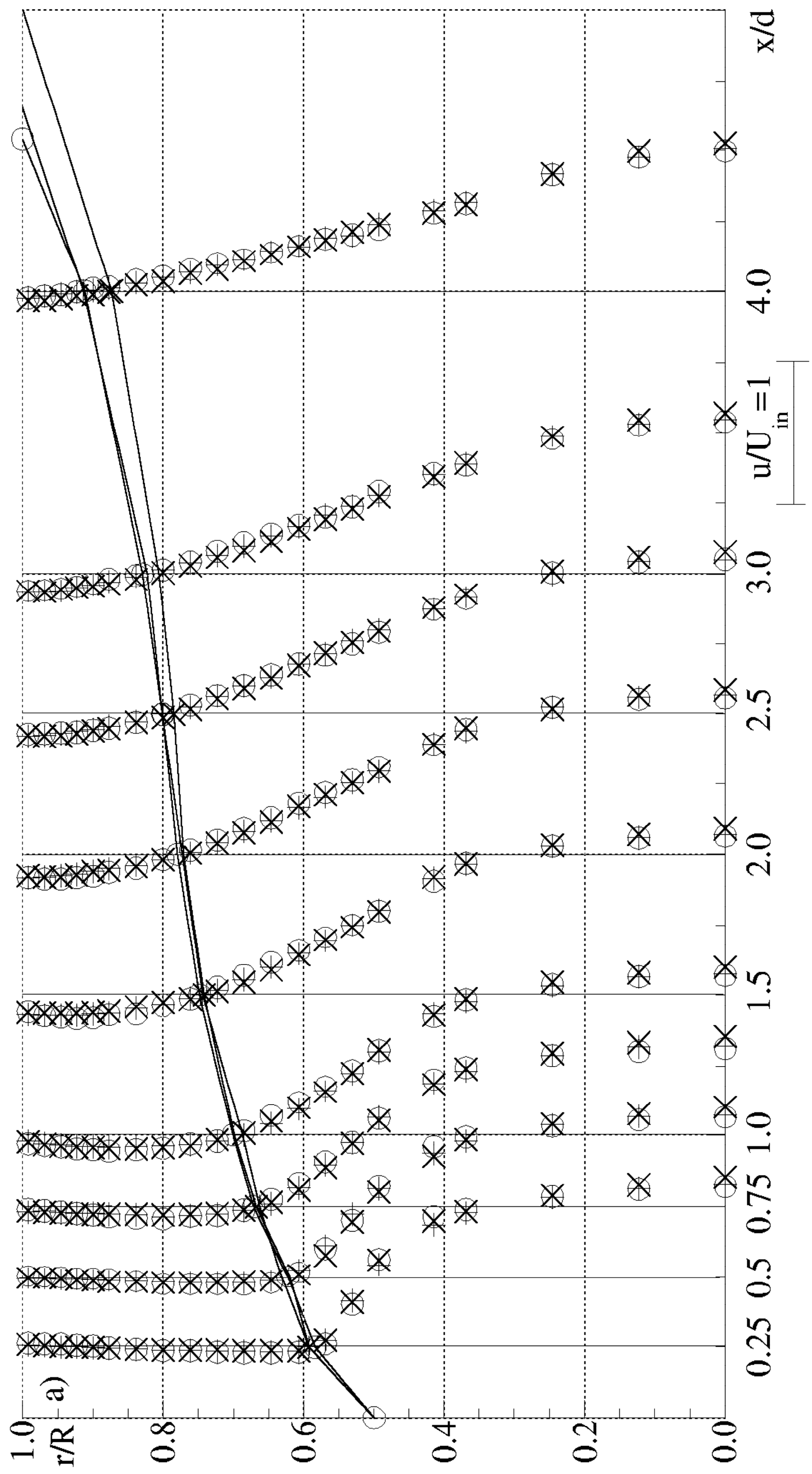

Fig. 5-a) Normalised mean velocity profiles: X water Re= 50,400; + water Re= 134,000 ; O 0.1% › gum Re_W= 19,600.

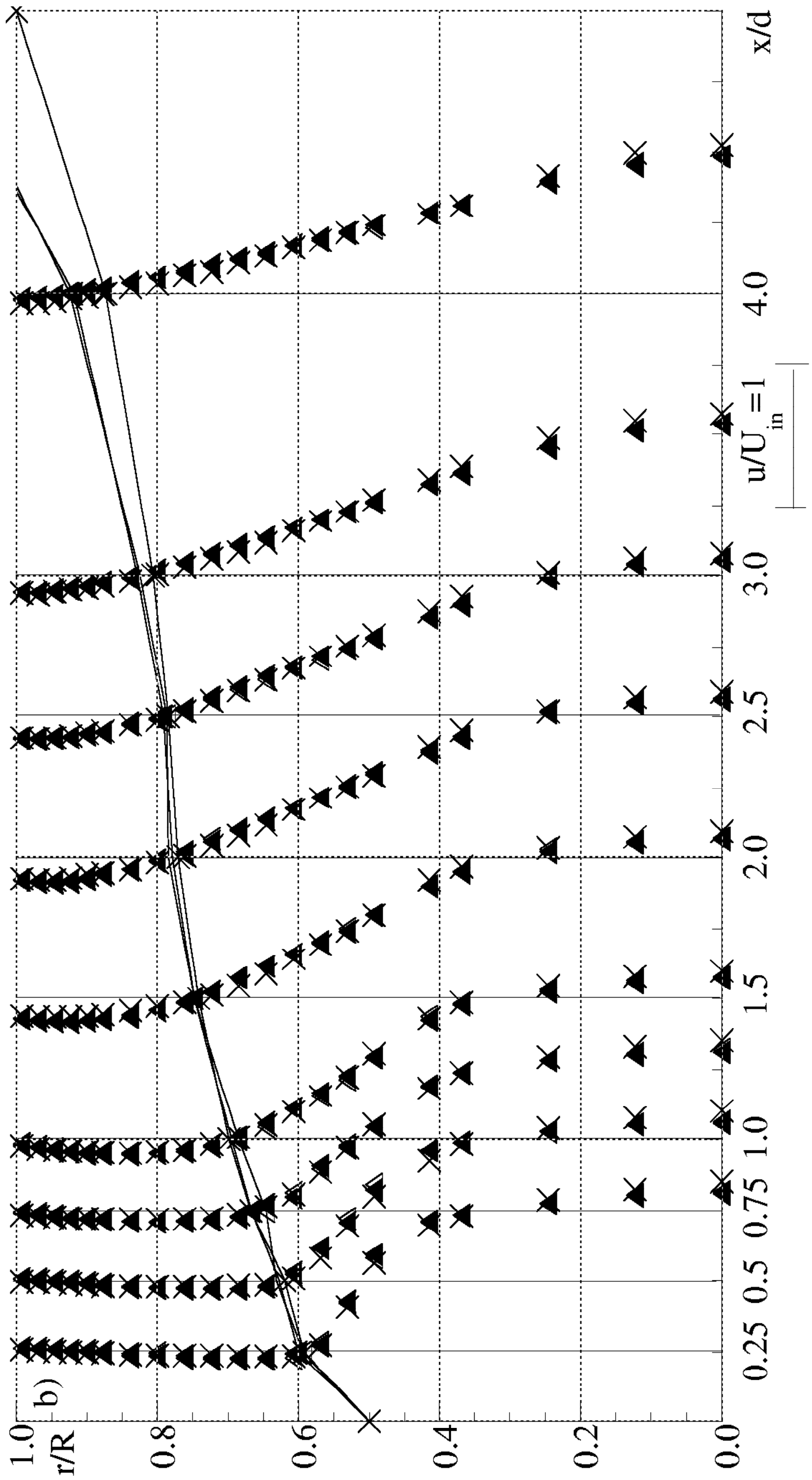

Fig. 5-b) Normalised mean velocity profiles: X water Re= 50,400; Δ 0.2% xanthan gum Re_W= 19,40
0.2% xanthan gum Re_W= 27,100.

Fig. 5-a) compares the downstream mean flow field of the 0.1% xanthan gum solution with that of the water flows, whereas Fig. 5-b) compares water and the two 0.2% xanthan gum flows. The lines in the figures represent the location of zero axial mean velocity. The mean flows are all similar, with differences only detected in detailed comparisons. Early within the recirculation region, the xanthan gum solutions have more negative velocities than the water flows, but that is reversed for $x/d \geq 2.5$, in agreement with the shorter lengths shown by the non-Newtonian flows. Also, in the early stages of the downstream flow, the xanthan gum mean axial profiles are flatter in the central core: note their lower velocities for $r/R < 0.3$ and higher velocities for $0.3 \leq r/R < 0.5$. Together with the more negative velocities in the recirculation region are higher mean velocity gradients in the shear layer which contributes to higher turbulence production there, as observed previously for the ER=1.538 case.

Further downstream, the difference in mean velocity in the central region remains as the flow tends to a fully-developed condition; there, the fully-developed normalised velocity on axis is lower than for the water flow, as already happened in the upstream pipe (Pereira and Pinho, 2000).

In terms of the turbulent flow field, the findings for the ER=2 geometry are qualitatively the same as those of Pereira and Pinho (2000) for the ER=1.538 expansion, namely:

- The addition of polymer results in drag reduction in the upstream fully-developed pipe flow, which is characterised by a flatter axial mean velocity and higher axial normal Reynolds stresses in the wall region, but lower radial and tangential stresses there;
- The higher axial turbulence at the inlet pipe wall is advected downstream by the mean flow and, together with the higher $\overline{u'^2}$ production in the initial stages of the downstream flow, resulted in the earlier ocurrence of the region of high axial turbulence in the shear layer downstream of the expansion plane than for Newtonian flows. This earlier development of turbulence downstream of the expansion is responsible for the shorter recirculation region.

Figures 6 to 8 show contours of the three normal Reynolds stresses for the ER=2.0 expansion and, from their comparison with the corresponding figures for the ER=1.538 geometry in Pereira and Pinho (2000), the following can be concluded:

i) The earlier development of $\overline{u'^2}$ for xanthan gum and the subsequent faster decay of this turbulent component relative to that of the Newtonian fluids is more pronounced in the ER=2 expansion;

ii) The maximum axial turbulence increases by about 20% for an increase of 30% in expansion ratio. This is consistent with the findings of Khezzar et al (1985) for Newtonian fluids who reported increases of 30% in axial turbulence for an increase of 35% in expansion ratio. Similarly, Pronchick and Kline (1983) found the same for plane expansions. With the exception of the 0.2% XG flow at Re_W= 27,100, the axial turbulence of the non-Newtonian fluids also increased with the expansion ratio.

iii) Similarly, for the radial normal Reynolds stress ($\overline{v'^2}$) the maximum turbulence occurs earlier with xanthan gum than with water, and this effect is again more pronounced in the ER=2 case. However, the maximum values of the stress for the two types of fluids do not differ significantly;

iv) For the tangential component of turbulence the effect of the expansion ratio is reversed: with the increase in the expansion ratio the difference between the Newtonian and non-Newtonian characteristics is reduced.

v) Downstream of the region of maximum turbulence the rate of dissipation of turbulent kinetic energy is faster with the non-Newtonian fluids and consequently their turbulence drops more than that of the Newtonian fluids.

The differences between the Newtonian and non-Newtonian features of $\overline{u'^2}$ and $\overline{v'^2}$ both contribute for the decrease of the recirculation length of the xantham gum solutions, but the relative behaviour of the tangential turbulence goes against this variation, and consequently, although the overall effect is still a reduction in recirculation length, the effect is less pronounced than in the smaller expansion.

The above findings were based on plots drawn in absolute coordinates, but more insight can be gained from the use of relative coordinates. The variation in the recirculation length is more adequately normalised by the step height than by any other length, and in high Reynolds number turbulent flow x_R/h usually becomes constant. Thus, it seems appropriate to compare the flow fields at identical values of x/h, and to normalise the radius downstream of the expansion wall as $y/h \equiv (r - R_1)/h$ so that, regardless of the expansion ratio, the value of y/h corresponding to locations downstream of the wall varies from 0 to 1. Since the measurements in both geometries were carried out at the same values of x/d, the data for ER=2 was linearly interpolated to obtain the profiles at the same x_R/h as in the ER=1.538 case. With this new normalisation the whole fields were redrawn and Fig. 9 shows some representative transverse profiles of the normalised $\overline{u'^2}$ and $\overline{w'^2}$. From the whole set it is possible to conclude:

i) For all fluids the axial turbulence reaches higher values in the ER=2 expansion than in the ER=1.538 case. The maximum $\overline{u'^2}$ occurs at around $x/h \approx 5.57$ downstream of the expansion plane, at $y/h \approx 0.4$ and 0.2 from the corner for the larger and smaller expansions, respectively.

ii) For the larger expansion, at $x/h \approx 5.57$ the maximum normalised $\overline{u'^2}$ for the water flows is around 10% higher than the corresponding xanthan gum values, but at lower values of x/h the levels of axial turbulence are similar for all fluids. However, at $x/h = 0.93$ the transverse profile of $\overline{u'^2}/U_{in}$ of all flows in the smaller expansion are more turbulent;

iii) On moving downstream the axial turbulence tends to peak at progressively higher values of y/h following the spread of the shear layer, but for $\overline{w'^2}$ the peak turbulence occurs always close to $y/h = 0$. In fact, the peak axial stress follows the outer edge of the shear-layer whereas the peak tangential stress occurs roughly along the mid-line of the shear layer (compare Figs. 6 and 8 and cross it with Fig. 5). The behaviour of the tangential turbulence in this relative coordinate normalisation is very similar to that seen with the absolute coordinates, ie, it is not much affected for Newtonian fluids;

iv) Within $x/h = 3.7$ of the expansion plane there is another difference in the locations of the peak values of the normal Reynolds stresses for the two geometries. For Newtonian fluids the peak in $\overline{u'^2}$ is always at $r > R_1$, but for the other two components of turbulence the peaks are at $r < R_1$ for ER=1.538 and at $r > R_1$ for ER=2. For the xanthan gum solutions this behaviour is pretty much the same with a few modifications: the peaks in $\overline{u'^2}$ tend to occur closer to $r \approx R_1$ than with the Newtonian fluids and for $\overline{w'^2}$ in the ER=1.538 expansion the region where the peak is at $r < R_1$ is limited to $x/h = 0.93$.

The differences between ER=1.538 and ER=2 reported in the previous paragraph for $\overline{u'^2}$ result from the role that advection plays in this flow and the effect of confinement by the downstream wall. The advection of the higher upstream wall region axial turbulence is responsible for the higher levels of axial turbulence immediately downstream of the expansion plane and its peak being closer to $r = R_1$ for the xanthan gum solutions. The increase in expansion ratio takes place on the side of the quasi-quiescent fluid and this enables the free-shear layer to spread faster to the outside of the jet, movig also outwards the zones of high turbulence.

The higher axial turbulence that develops on increasing the expansion ratio from 1.538 to 2 is not mirrored in increases of the other normal Reynolds stresses and especially of the radial turbulence which plays an important role in transferring momentum radially and hence the viscoelastic reduction in x_R/h relative to the Newtonian length becomes less pronounced with the larger expansion.

ACKNOWLEDGEMENTS

We would like to thank the financial support of the Stichting Fund of Schlumberger and project PBIC/C/CEG/2427/95 of Fundação para a Ciência e Tecnologia. The authors are also thankful to INEGI-Instituto de Engenharia Mecânica e Gestão Industrial- the institution that provided the laboratory facilities.

REFERENCES

Castro, O. S. and Pinho, F. T. (1995) Turbulent expansion flow of low molecular weight shear-thinning solutions. *Exp. Fluids*, *20*, pp 42-55.

Coelho, P. M. and Pinho, F. T. (1998) Rheological characteristics of some diluted aqueous polymer solutions (in Portuguese). *Mecânica Experimental*., 3, pp 51-60.

Escudier, M. P. and Smith S. (1999) Turbulent flow of Newtonian and shear-thinning liquids through a sudden axisymmetric expansion. *Exp. in Fluids*, 27, pp 427-434.

Escudier, M.P., Gouldson, I. W., Pereira, A. S. and Pinho, F.T. (1998) On the consistency of the rheology of dilute shear-thinning elastic fluids. Submitted to J. Non-Newt. Fluid Mech.

Khezzar, L., Whitelaw, J. H. and Yianneskis, M. (1985) An experimental study of round sudden expansion flows. Proc. 5th Symp. on Turbulent Shear Flows, Cornell University, pp 5-25.

Pak, B., Cho, Y. I. and Choi, S. U. (1990) Separation and reattachment of non-Newtonian fluid flows in a sudden expansion pipe. *J. Non-Newt. Fluid Mechanics*, 37, pp 175-179.

Pak, B., Cho, Y. I. and Choi, S. U. (1991) Turbulent hydrodynamic behavior of a drag-reducing viscoelastic fluid in a sudden expansion pipe. *J. Non-Newt. Fluid Mechanics*, 39, pp 353-373.

Pereira, A. S. and Pinho, F. T. (1999) Bulk characteristics of some variable viscosity polymer solutions in turbulent pipe flow. Paper AAAAHF in CD-ROM, Technical Sessions on Rheology (SR26), COBEM99 (XV Brasilian Congress of Mechanical Engineering), Águas de Lindóia SP, Brasil, 22-26 November 1999.

Pereira, A. S. and Pinho, F. T. (2000) Turbulent characteristics of shear-thinning fluids in recirculating flows. *Exp. in Fluids*, 28, pp 266-278.

Pronchick, S. W. and Kline, S. J. (1983) An experimental investigation of the structure of turbulent reattachment flow behind a backward-facing step. Report MD-42, Thermosciences Div., Mech. Eng. Dept., Stanford University.

Stieglmeier, M. and Tropea, C. (1992) A miniaturized, mobile laser-doppler anemometer, *Applied Optics*, 111, pp 4096-4099.

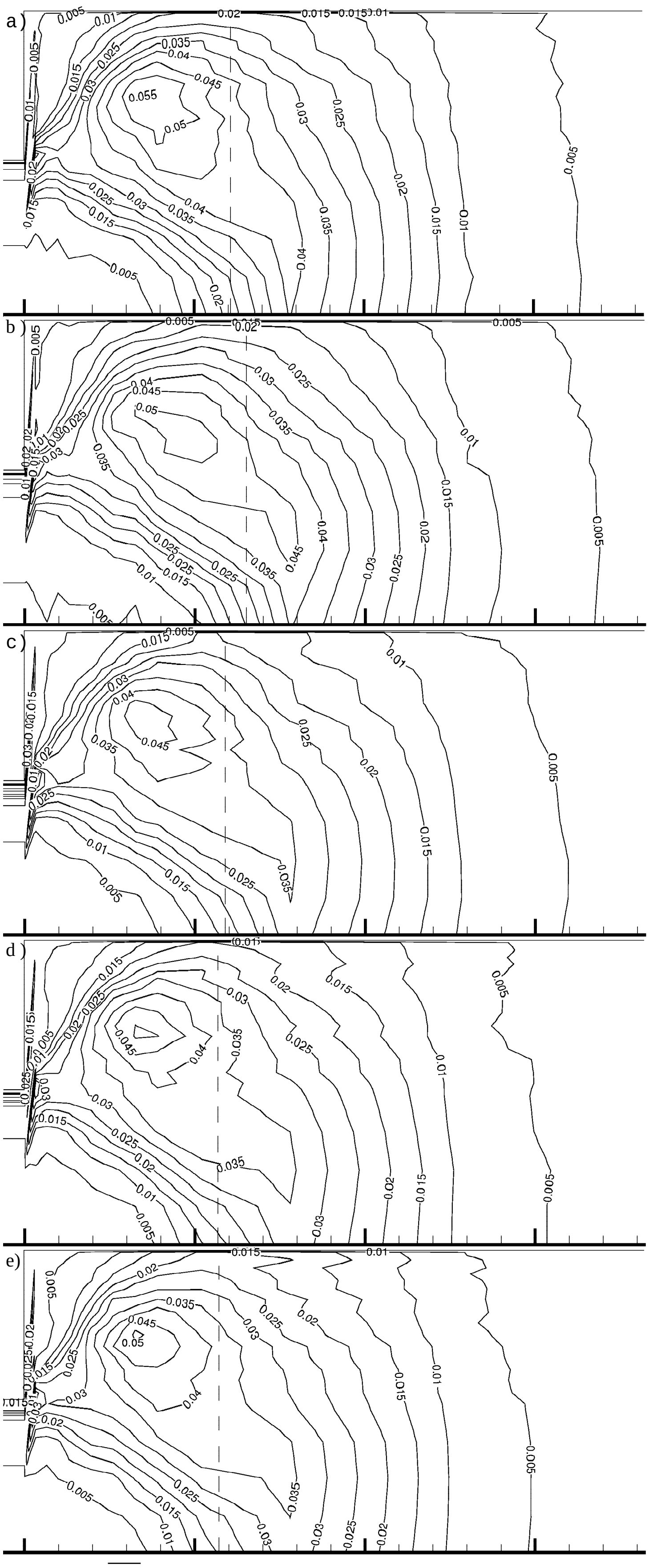

Fig. 6. Contours of normalised $\overline{u'^2}$ *for ER=2 a) water Re=134,000; b) water Re=50,400; c) 0.1% XG Re= 19,600; d) 0.2% XG Re=27,100; e) 0.2% XG Re=19,400.*

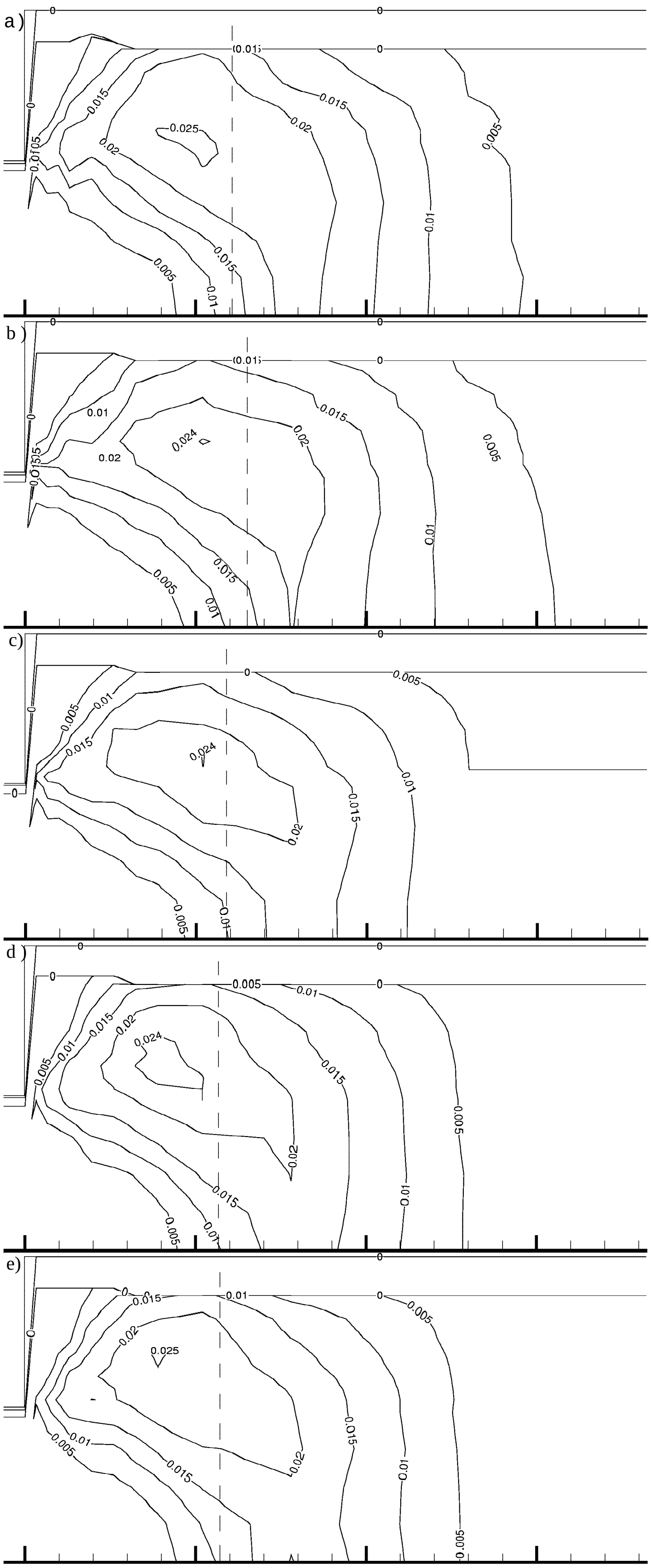

Fig. 7. Contours of normalised $\overline{v'^2}$ *for ER=2 a) water Re=134,000; b) water Re=50,400; c) 0.1% XG Re= 19,600; d) 0.2% XG Re=27,100; e) 0.2% XG Re=19,400.*

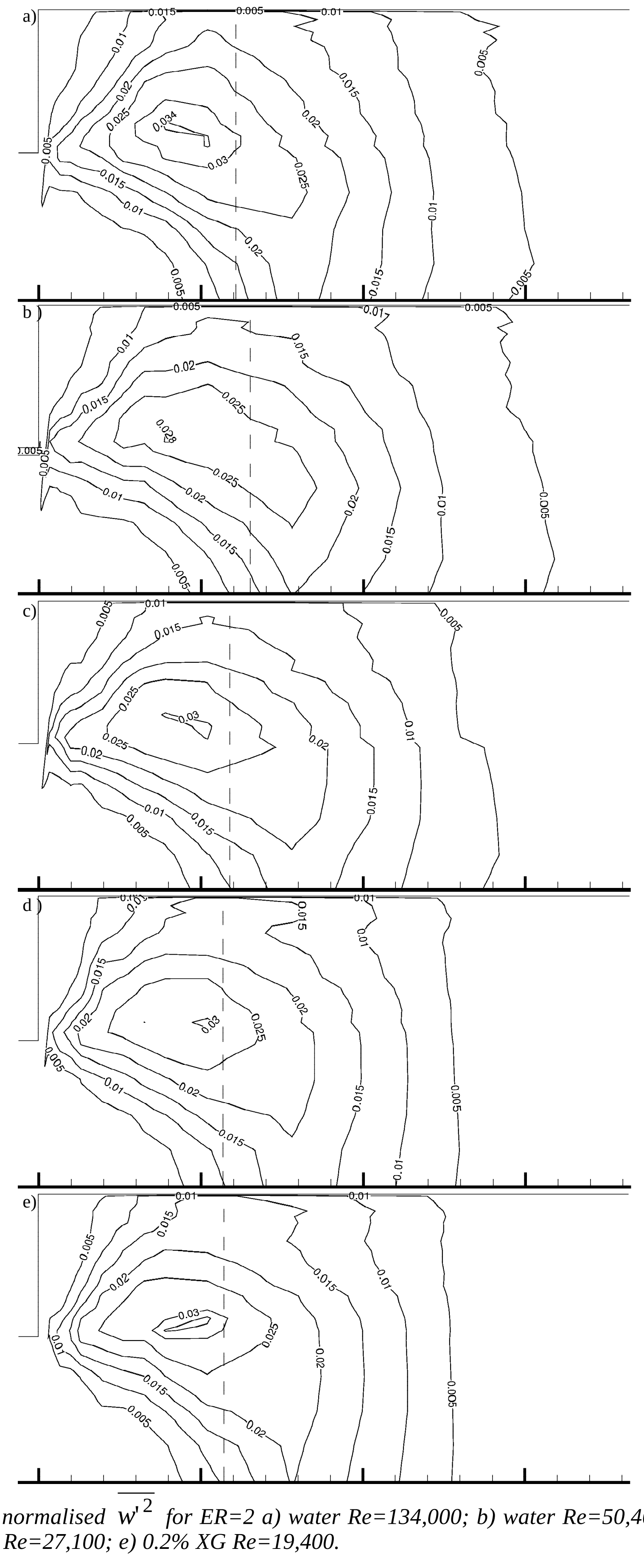

Fig. 8. Contours of normalised $\overline{w'^2}$ for ER=2 a) water Re=134,000; b) water Re=50,400; c) 0.1% XG Re= 19,600; d) 0.2% XG Re=27,100; e) 0.2% XG Re=19,400.

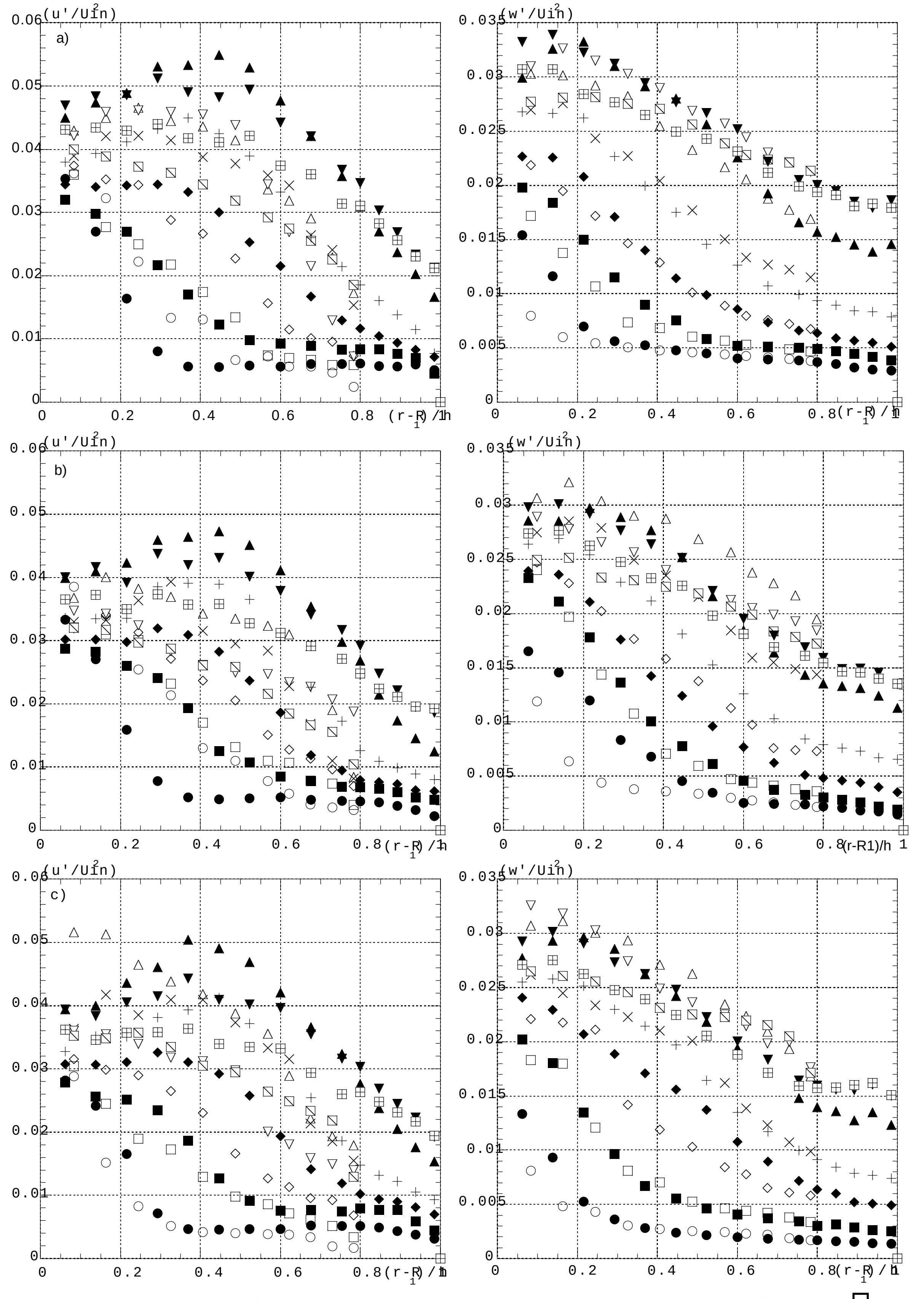

Fig. 9- a) Water Re=134,000; b) 0.1% XG Re=19,600; c) 0.2% XG Re=27,000. Location x/h: O *0.93;* □ *1.86;* ◊ *2.79;* X,+ *3.71;* Δ *5.57;* ∇ *7.43;* ⧅,⊞ *9.3. Open/first symbol: ER=1.538, Closed/ second symbol: ER=2.*