

Maximum likelihood approaches for sound field measurement using LDV

Cyril Mellet and Jean-Christophe Valière[†]

Laboratoire d'Acoustique de l'Université du Maine
UMR CNRS 6613
Avenue Olivier Messaïen
72085 LE MANS cedex 9, France
telephone: 33 (0)2 43 83 35 53
fax : 33 (0)2 43 83 35 20
email : cyril.mellet@univ-lemans.fr

ABSTRACT

Acoustic particle velocity can be estimated from the LDV measurement. The amplitude and phase of the velocity are inferred from adapted signal processing of the signal issue from the LDV set-up. Many investigations covering a wide range of signal processing method have been developed to analyze this signal, such as the adaptation to techniques developed in fluid mechanics based on the random sampling principle or photo-correlation estimation using the temporal amplitude variation. Time-frequency representations are also used to estimate velocity parameters using the estimation of the instantaneous frequency of the signal.

The best result obtained up to nowadays, in the configuration of low displacement of the particle, has been realized with the use of time-frequency estimators whose principle is related to parametric principle. So, to extend the estimation capability forward lower displacement, two parametric methods, the ML and LS estimators, using *a priori* information on signal, are adapted to allow the estimation of parameters velocity. Their estimation capabilities are evaluated on simulation signals and a preliminary confrontation to experimental signals is occurred.

[†] present address : LEA UMR CNRS 6609, 40 Av. du recteur Pineau 86022 Poitiers cedex, France
Email : jean-christophe.valiere@lea.univ-poitiers.fr , telephone : 33 (0)5 49 45 33 59 ; fax : 33 (0)5 49 45 36 63

1. INTRODUCTION

The measurement of acoustic particle velocity allows the description of complex acoustic fields and improves the determination of impedance and energy flux (Bailliet, 2000). The LDV set-up can be used to assess the instantaneous acoustic velocity with a good spatial resolution (Taylor, 1976 ; Valiere, 2000). This set-up is based on the measurement of the frequency shift of the light back-scattered by a seeding particle moving through the probe volume formed by the two incident laser beams. The velocity is inferred from the processing of the signal recorded by the photo-multiplier (PM) and filtered by a high-pass filter to eliminate the offset component introduced by the power difference between the two beams. The signal, so called Doppler signal, can be written as :

$$\left\{ \begin{array}{l} y(t) = s_q(t) + n(t) = A \cos[2\pi(F_p t + Dx_q(t)) + \psi] + n(t) \\ \text{with } x_q(t) = \frac{V}{2\pi f_{ac}} \sin(2\pi f_{ac} t + \varphi) \end{array} \right. , \quad (1)$$

where F_b is a carrier frequency imposed by a Bragg cell to avoid the ambiguity on the velocity direction. The Gaussian spatial distribution of the light intensity in the probe volume and the variable position of the particle in the probe volume induce the variation of amplitude in time. This amplitude is also determined by the sensitivity of the PM and scattering efficiency of the seeding particle. Previous work (Valeau, 1999) has shown that this amplitude can be considered almost constant over few periods of the acoustic signal. Therefore, the amplitude will be approximated as constant in the following.

Moreover, in equation (1), $x_q(t)$ represents the displacement versus time of a particle in the probe volume submitted to a frequency excitation f_{ac} , D is the sensibility of the set-up depending on the angle between the two beams and on the wavelength λ of the light.

The terms V and φ , in the expression of $x_q(t)$, represent respectively the amplitude and the phase of the acoustic particle velocity, the velocity being proportional to the derivative versus time of the displacement.

With respect to these assumptions and after a shift forward to the low frequency, the signal can be modeled by the following expression:

$$s_q(t) = A \cos(2\pi F_c t + 2\pi D x_q(t) + \psi), \quad (2)$$

F_c being linked to F_b ,

The velocity parameters V and φ must be deduced from the signal analysis of $y(t)$ to allow the characterization of the acoustic particle velocity. Previous work has been carried out in this area. For example, the classical Fourier transform of the signal allows the estimation of the velocity amplitude, but does not allow an estimation of the parameter φ . This technique was developed, for the first time by Taylor in 1976. A scientific team in the Edinburg university has worked out an estimator, so called photon correlation spectroscopy (Sharpe, 1987), based on the amplitude variation analysis, enables to estimate the acoustic velocity amplitude of an acoustic field superimposed on a fluid flow or only of acoustic fields. Recent papers (Valière, 2000; Valeau, 2000) have shown the possibility to use time-frequency representations to estimate simultaneously V and φ . These methods are based on the estimation of the instantaneous frequency of the signal $y(t)$, the velocity parameters are so deduced by a lock-in procedure on the instantaneous curve.

Most of these techniques are less effective for analyzing signal with low modulation index, defined as $\alpha = \frac{DV}{f_{ac}}$.

Yet, acoustic measurements frequently provide signals with low modulation indices, this situation occurring for acoustic fields with the following properties:

- the amplitude velocity is low (low acoustic level)
- high acoustic frequency f_{ac} (fast variation).

The paper aims at testing signal processing methods in order to extend the LDV measurement principle to a large acoustic scale, and mainly forward the low indices. Parametric methods are selected to their capabilities to use *a priori* information to model signal. An adaptation of these methods have been realized to allow the Doppler

signal analysis. Firstly, a brief review on the principle of parametric methods is presented. Secondly, a thorough study is exposed of two particular methods retained among the lot of parametric methods. Finally, results from simulation and experimental signals are carried out and discussed.

2. PARAMETRIC METHODS

The principle of parametric methods, illustrated by the figure 1, is based on the modeling of the system which generates the output signal $y[n]$. This model built from the *a priori* information of the signal depends on unknown parameters represented by the vector θ . The parameter estimation is satisfied by the minimization or maximization of a criteria e comparing the output signal $y[n]$ and the estimation $\hat{y}[n]$ obtained by the model $\hat{M}(\theta)$.

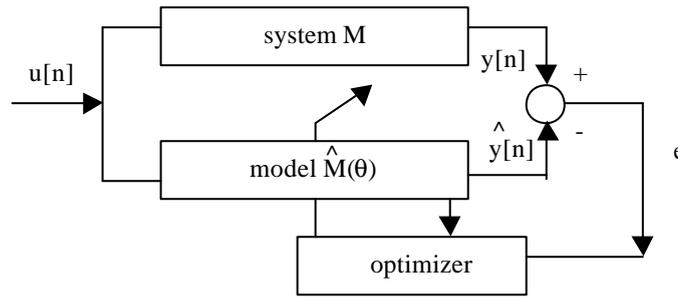


Figure 1: General principle of the parametric methods

In the problem, the signal model, \hat{y} , is described by the equation 2, so only the criteria, e , must be determined. The usual criteria is based on the measurement of the mean square error, defined as the difference between the output signal $y[n]$ and the estimation \hat{y} depending on the parameters θ :

$$e = \sum_{n=0}^{N-1} |y[n] - \hat{y}[n]|^2 . \quad (3)$$

Two techniques, respecting the previous criteria, have been suited to the given problem. The first one is the least squares estimator-based method and the second one is based on the principle of maximum likelihood. The first one processes the output data of the PM and the second acts on the estimation of the signal phase.

3. LEAST SQUARES ESTIMATOR

The technique based on least squares estimation consists in a direct identification of the unknown parameters of the signal (eq.3) by means of minimizing the function e_{θ} :

$$e_{\theta} = \sum_{n=0}^{N-1} |y[n] - \hat{y}_{\theta}[n]|^2 , \quad (4)$$

where θ is the vector of unknown parameters.

In the present problem, the unknown parameters of the signal \hat{y}_{θ} are the amplitude A , the random phase ψ depending on the date at which the particle is located inside the probe volume and the amplitude V and the phase ϕ of the acoustic particle velocity.

The minimization of the non-linear function e_{θ} yields the estimation of θ . Among the non-linear parameter estimation techniques the simplex method (Nedler,1965) has been selected for its convergence robustness, its easy computing and for its result independence to initial condition of the recursive algorithm. Moreover, this method is less prone to divergence and can even be made to converge to a global minimum and not forward a

local minimum. To converge to the minimum of the function, the simplex method begins to generate (m+1) points from an initial point P_0 (initial conditions) in the Euclidean space of m dimensional formed by the m parameters of \mathbf{q} . The method associates at each point the value of the function to minimize. The worst point, the one with highest value, is iteratively rejected and replaced by a new point. Four mechanisms are used for the calculation of the new point: reflection, expansion and contraction along one dimension and along all dimensions. These operations use only geometrical operation in the Euclidean space (Nedler, 1965). The recursive algorithm is stopped when two successive values are less than a given threshold.

4. MAXIMUM LIKELIHOOD ESTIMATOR

The maximum likelihood estimator (ML) is based on the estimation of the phase of the signal $y[n]$. So, this method needs the preliminary transformation of the signal $y[n]$ in an associated analytic signal $y_a[n]$, obtained by the numerical Hilbert transform of $y[n]$ (Ville, 1948). The analytical definition of the Hilbert transform is the following:

$$y_a(t) = y(t) + j \frac{1}{\pi} \text{v.p.} \left[\int \frac{y(\tau)}{t-\tau} d\tau \right], \quad (5)$$

where “v.p.” stands for Cauchy principal value. The numerical computing of this definition is realized thanks to the fast Fourier transform.

The ML estimator needs the generation of a reference signal built from an approximation the model described in the equation 2 and depending on the unknown parameters A , ψ , V and φ . In this problem, the reference signal can be written:

$$y_q[n] = A \exp \{ j(\mathbf{a} \sin(2\mathbf{p}f_{ac} nTe + \mathbf{j})) + \mathbf{y} \}, \quad (6)$$

The unknown parameter are then estimated to maximize the prior density of probability defined as:

$$p(y_b|\mathbf{q}) = \left(\frac{1}{2\mathbf{p}s^2} \right)^N \exp \left[-\frac{1}{2\mathbf{s}^2} \sum_{n=0}^{N-1} (\Re(y_b[n]) - \Re(y_q[n]))^2 + (\Im(y_b[n]) - \Im(y_q[n]))^2 \right] \quad (7)$$

where $\Re(\cdot)$ and $\Im(\cdot)$ respectively represent the real and the imaginary part of the signal. Furthermore, $y_b[n]$ is fitted from the signal $y[n]$ recorded by the PM by, in first time, computing the analytic signal $y_a[n]$ associated to $y[n]$ (eq.5) and in a second time, shifting its spectrum around the zero frequency. This last operation can easily be performed thanks to the knowledge of the carrier frequency F_c .

The expression of $p(y_b|\theta)$ is based on the assumptions that the additive noise $n[n]$ corrupted $y[n]$ (eq.2) is normal and on the independence of the noise samples.

The estimation of the unknown parameters can be obtained by the maximization of the likelihood function, defined as the logarithm of $p(y_b|\theta)$:

$$f_q(y|\mathbf{q}) \propto \frac{1}{N} \sum_{n=0}^{N-1} \left\{ (\Re(y_b[n]) - \Re(\hat{y}_q[n]))^2 + (\Im(y_b[n]) - \Im(\hat{y}_q[n]))^2 \right\}, \quad (8)$$

where \propto means “proportional to”.

From the contributions of D.C. Rife (1974) and B. Boashash (1990), it can be demonstrated that, in case of Doppler signal analysis, the maximization of the likelihood function is equivalent to modulus maximization of the function D defined as:

$$D(V, \mathbf{j}) = \frac{1}{N} \sum y_b[n] \exp \{ -j[\mathbf{a} \sin(2\mathbf{p}f_{ac} nTe + \mathbf{j})] \} \quad (9)$$

in relation to V and φ .

The parameters A and \mathbf{y} are then deduced from this maximization as:

$$\begin{cases} A = |D(\hat{V}, \mathbf{j})| \\ \mathbf{y} = \arg(D(\hat{V}, \mathbf{j})) \end{cases} \quad (10)$$

where $D(\hat{V}, \mathbf{j})$ represents the maximum value of the function D in relation to V and φ .

5. RESULTS

In this chapter the statistical performances (convergence value and variance) of the estimators are evaluated thanks to simulated data generate from Monte-Carlo process. Then, the results on experimental data analysis issue from the experimental set-up are compared with the measurement realized from a laser vibrometer measurement (Poggi, 2000) which serves as a reference measurement.

5.1 Statistical Performances of estimators

The statistical performances of the estimators are evaluated from the analysis of 100 realizations of simulated signals. All the data are built from equation 2 with a sampling frequency, a carrier frequency, a excitation frequency and a duration respectively set to 800kHz, 47kHz, 5kHz and 1 ms.

5.1.1. Convergence value

For a 15 dB signal-to-noise ratio (SNR), the convergence value of the estimation is calculated by averaging the estimations over 100 realizations. The simulations have been achieved with the following parameters:

$A = 1$, $\mathbf{y} = 0$, $\mathbf{j} = \frac{\mathbf{p}}{4}$, and several values of the velocity corresponding to the α values. In this paper, only the results relative to the amplitude velocity are discussed (phase behavior is the same).

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(a)

(b)

Figure 2 : Relative error for the estimation of the modulation amplitude V , as a function of \mathbf{a} for :

(a) a 15 dB signal-to-noise ratio (**o ML**, **à LS**),

(b) a 10 dB signal-to-noise ratio (**o ML**, **à LS**).

As seen by figure 2(a), the estimation of V is less than 10% the real value, down to a modulation index threshold α_t . Below this threshold, \hat{V} is over estimated, and error increases when α decreases. Furthermore, the error increases more rapidly for the LS estimator than for the ML estimator. The ML estimator seems to be more adapted to the signal analysis with low modulation indices. The threshold is around $2 \cdot 10^{-2}$ for both methods. Note also that beyond α_t , both estimators are unbiased.

Figure 2(b) illustrates The robustness of estimators to SNR. The results are achieved with the same parameter set excepted for SNR which value is 10 dB. Both methods approximately exhibit the same behavior than on the figure 2(a), but the threshold α_t is higher and around $4 \cdot 10^{-2}$. The threshold value depends on the SNR value and the lower the SNR is, the higher the α_t value is. The figure 3 sheds light on this behavior. Indeed, each curve of this figure is a representation of the D function in relation to V and ϕ obtained from the analysis of two different simulations of a signal issue from the same process. In these figures “(a)and (b), the simulated signal has been built with a modulation index of 10^{-2} , a SNR of 15 dB and the same value for the others parameters than for the previous simulations. These two curves get a minimum for a value of V and ϕ which represents respectively the parameter maximizing the likelihood function. But, the simultaneous analysis of these curves shows that the minimum of each curve does not get for the same value of V and ϕ , even if both simulated signal are issue from the same process.

The analysis of the figure 3(a) and 3(b) induces the conclusion that for low modulation indices the noise becomes preponderant behind the signal component and imposes the minimum position as a random variable. The estimation of the parameters V and \mathbf{j} can not be achieved with these estimators in these conditions or with a preprocessing to decrease the effect of the noise. This process may be identical for the least squares estimator, but a representation is impossible due to the larger dimension of the vector \mathbf{q} . An identical study has begun on the LS estimator and the first result seems to present identical behavior.

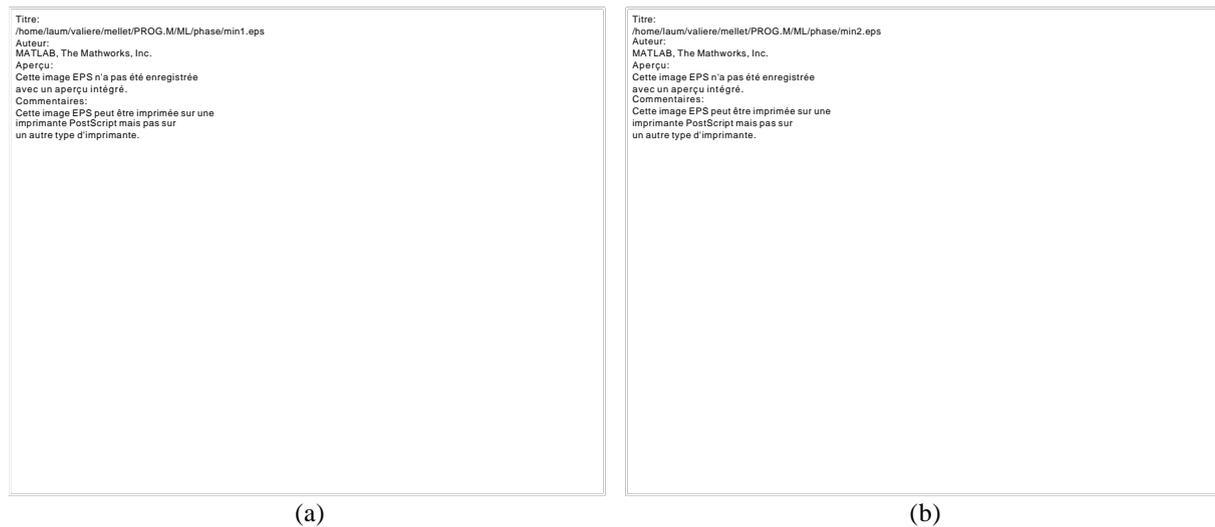
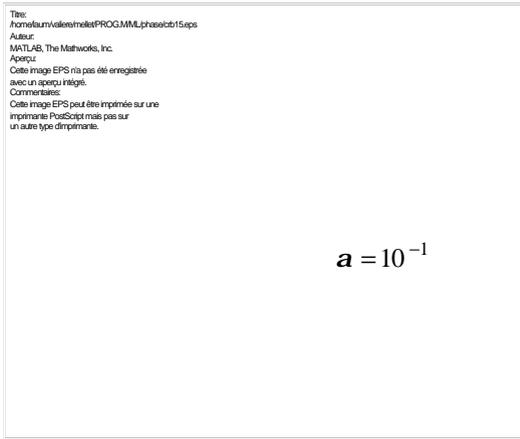


Figure 3 : Representation of the function D in relation to V and \mathbf{j} for the analysis of a two iterations of a signal issue from the same process:
 (a) first simulation
 (b) second simulation

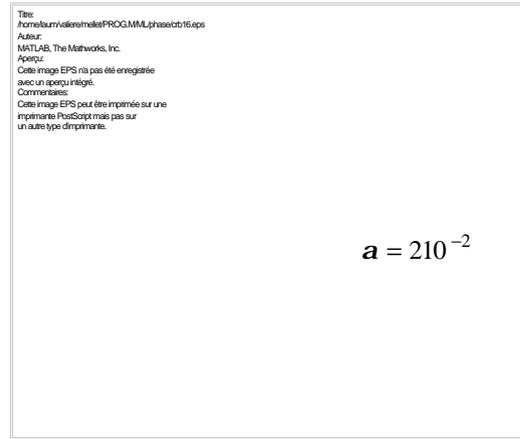
Like for the previous studies, a limitation happens upon low modulation indices, especially for low SNR.

5.1.2. The variance

In order to study the statistical efficiency of the estimator, the inverse of the estimation variance in dB is compared to the Cramer-Rao bound (CRB), theoretically derived by Valeau (Valeau, 2000), representing the minimum variance estimator. The variance is evaluated as a function of SNR over 100 realizations. For example, for a modulation index of 0.1, the variances of \mathbf{a} estimations are displayed on the figure 4(a).



(a)



(b)

Figure 4 : Comparison of the \hat{a} estimation variance in dB to the Cramer-Rao bound (solid line):
 o o o ML variance, \hat{a} \hat{a} LS variance

For a modulation index of 0.1, the ML estimator variance equals the CRB in the scale of SNR [0,30] dB. This estimator is then efficient, while the LS estimator follows asymptotically the CRB without reaching it. The same observations can be made from figure 4(b) in which the behavior of the estimator variance, obtained for $\alpha=2 \cdot 10^{-2}$ is the same than on figure 4(a) up to a 10 dB SNR. Below to this bound the variances rise when SNR decreases. The ML estimator seems to be more sensitive to the SNR than the LS estimator.

5.1.3. Conclusion

The new signal processing approaches which consist on using parametric methods to estimate amplitude and phase acoustic velocity measure by LDV set-up improves phase and amplitude velocity estimations up to a modulation index of $2 \cdot 10^{-2}$. For decreasing this limitation the signal-based model has to include *a priori* information on the additive noise or a preprocessing must be achieved to increase SNR. Finally, these both estimators were used to analyze experimental signals from the measurement of vibratory structure (Poggi, 2000).

5.2 Experimental analysis

The two estimators have been used in order to estimate the velocity of vibratory structure to avoid the problem relied to media seeding. Moreover, the amplitude fluctuation are lower than in case of study in fluid. The study of vibratory structure allows to insure the continuity of the signal for low modulation indices and so to be in conditions close to the previous assumptions.

The estimations obtained by the analysis of the signal Doppler must be compared to the measurement obtained by laser vibrometer which allows a precise reference. More details on experimental set-up are displayed in Poggi, 2000.

The first results are presented in the table 1, where the estimation values by the two estimators is compared to the reference value obtained by laser vibrometer. These tests are realized for several excitation frequency f_{ac} and amplitude velocity value controlled by the vibrometer.

tab 1: Velocity parameters of vibratory measurements

signal characteristics	ML estimator	LS estimator	Laser vibrometer
Fe=1Mhz, Fc=93750Hz, $f_{ac} = 1\text{kHz}$.	$\hat{V} = 1,11 \text{ mm/s}$ $\hat{f} = 0,98^\circ$	$\hat{V} = 1.33 \text{ mm/s}$ $\hat{f} = 1.26^\circ$	$\hat{V} = 1.41 \text{ mm/s}$ $\hat{f} = -139.7^\circ$
Fe=1Mhz, Fc=93750Hz, $f_{ac} = 5\text{kHz}$.	$\hat{V} = 1.85 \text{ mm/s}$ $\hat{f} = -39.5^\circ$	$\hat{V} = 1.84 \text{ mm/s}$ $\hat{f} = -39.1^\circ$	$\hat{V} = 1.41 \text{ mm/s}$ $\hat{f} = -74.1^\circ$

Both velocity estimations from experimental data allow to conclude that the two estimators give more or less the same values. But these values are not the same ones than obtained with the laser vibrometer, even for a high modulation index ($\mathbf{a} \cong 1.41$, first line of the table).

This difference must be due to variations in the characteristic of the signal (speckle noise for example). This phenomena can explain that the estimators do not give the same result following the analyzed signal part. Indeed, the speckle is known to introduce a multiplicative random noise which decreases the SNR and can explain the limitation of the estimation precision (e.g. section 5.1).

An other possibility could be the precision for the Doppler signal model of equation 3. For example, in order to use the ML estimator a modification of the model has to be achieved. The model as presented in the section 4, needs the precise knowledge of the carrier frequency F_c , which is supposed perfectly controlled by the experimental set-up when the media is not seeded. But the demodulation by the carrier frequency given by the experimental set-up could be not satisfied and could provide a supplementary estimation error. So, the carrier frequency must be included in the vector \mathbf{q} as a parameter to estimate and the function D, then, becomes :

$$D(\mathbf{V}, \mathbf{j}, F_c) = \frac{1}{N} \sum y_a[n] \exp \left\{ -j \left[2pF_c nTe + \mathbf{a} \sin(2pf_{ac} nTe + \mathbf{j}) \right] \right\}. \quad (11)$$

Moreover, others errors may be introduced and not controlled.

6. CONCLUSION

Parametric methods have been used to estimate parameters of the acoustic particle velocity issue from LDV set-up. Two estimators have been selected, the LS and ML estimators, and their statistical properties have been deduced from simulations realized with Monte-Carlo processes. These studies have shown the possibility to estimate the velocity parameters up to a modulation index of $2 \cdot 10^{-2}$ with an estimation error less than 10%. At a frequency excitation $f_{ac}=5 \text{ kHz}$, this index induces a particle displacement around $3 \cdot 10^{-9} \text{ m}$. Moreover, for the index the efficiency of estimators has been shown until a SNR of 10 dB.

The analysis of experimental data issue from velocity vibratory structure measurement has been carried out. The results obtained by both estimators give similar values, but are different of the reference value given by the vibrometer.

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