Lagrangian Analysis of passive tracers dispersion in a confined convective flow

by

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ABSTRACT

Lagrangian motion in a quasi-two dimensional time dependent, convective flow is studied at different Rayleigh numbers (*Ra*). Convective motion is generated in a rectangular tank by a linear heat source placed 0.4 cm above the lower surface of the tank. Lagrangian trajectories (Fig.1) have been evaluated using PTV technique. An intermediate regime characterised by an almost periodic Eulerian flow is observed. The flow consists mainly of two counter-rotating rolls divided by an ascending thermal plume above the heat source. The upper end of the plume oscillates horizontally with a frequency which increases with *Ra*. It is known that Lagrangian motion of particles in a time-dependent flow can be very complex due to chaotic advection even if the Eulerian flow is laminar and regular in an Eulerian viewpoint (?). As a matter of fact, the non-autonomous one-degree of freedom Hamiltonian system of the equation of motion can not be trivially integrable. Dispersion phenomena occurring in the tank are investigated in a Lagrangian framework by using different statistics on particle displacements. One particle and two particle approaches have been considered. The classical way of looking at diffusive properties by computing average separation at fixed time is compared with an alternative measure known as *Finite Size Lyapunov Exponent* (FSLE) consisting in the evaluation of average separation time at fixed scale. The impact of an increasing forcing on the system and the corresponding dependence of Lagrangian statistics on *Ra* have been investigated varying this parameter by increasing the power supplied to the heat source (Table I).
The obtained results can be generalised to many geophysical problems allowing us to discuss about the right way of describing spreading of pollutants in closed basins characterised by a not sharp separation between the scale of the velocity field and the dimension of the domain.

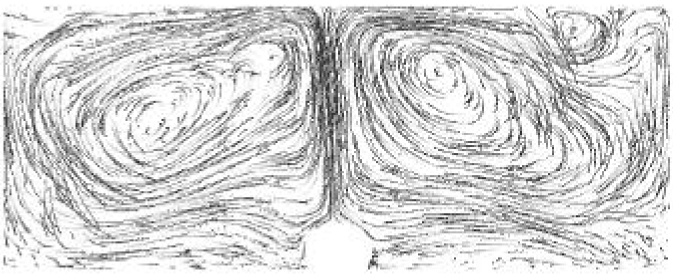

Figure1 Recognised trajectories during 100 frames of their evolution

Table I Parameters related to the performed runs.

$Q(W/m)$	$a(m^3/°C)$	$u(m^2/s)$	$k(m^2/s)$	$l(J/m\cdot s\cdot °C)$	Ra	$Ra\ Pr$
0.96×10^{1}	2.53×10^{-4}	8.85×10^{-1}	1.44×10^{-1}	$6,03\times10^{-1}$	6.87×10^{1}	6.14
1.44×10^{1}	2.60×10^{-4}	8.83×10^{-1}	1.44×10^{-1}	6.03×10^{-1}	1.03×10^{8}	6.13
3.36×10^{1}	2.58×10^{-4}	8.87×10^{-1}	1.44×10^{-1}	6.02×10^{-1}	2.39×10^{8}	6.16
7.93×10^{1}	2.65×10^{-4}	8.72×10^{-1}	1.44×10^{-1}	6.04×10^{-1}	5.96×10^{8}	6.04
1.17×10^{2}	2.32×10^{-4}	9.44×10^{-1}	1.42×10^{-1}	5.98×10^{-1}	7.19×10^{8}	6.61
2.15×10^{2}	2.81×10^{-4}	8.38×10^{-1}	1.42×10^{-1}	6.06×10^{-1}	1.74×10^{9}	5.77
2.80×10^{2}	2.72×10^{-4}	8.56×10^{-1}	1.44×10^{-1}	$6,05\times10^{-1}$	2.17×10^{8}	5.92

1. INTRODUCTION

The understanding of transport and mixing properties of passive impurities in fluid flows is of great concern in several fields of theoretical and applied research including oceanography, biophysics and chemical engineering.

During the past decades, transport has been extensively studied using different approaches and methodologies. Recently, the classical approach based on the concepts of turbulent diffusion (Taylor, 1921), has been extended using dynamical system theory concepts and especially chaotic dynamics (Ottino, 1989). The most natural framework for investigating passive scalar dynamics is a Lagrangian viewpoint in which particles result advected by a given Eulerian velocity field $\boldsymbol{u}(\mathbf{x},t)$. However, in spite of the simple formal relation between Eulerian and Lagrangian approach, it can not be always a trivial task to obtain information on one description starting from the other. As a matter of fact it can be shown that totally impredictable Lagrangian motion (i.e. chaotic motion) can exist also in a laminar and totally predictable Eulerian velocity field (Aref, 1983-1984). This phenomena is referred to Lagrangian chaos which can be denoted as the *sensitivity to initial conditions* of Lagrangian trajectories (Crisanti *et al*., 1991). Moreover in closed domains, asymptotic mixing properties of the flow are strongly influenced by the presence of boundaries in particular when the characteristic length scale of the vortices is of the same order of the dimension domain (Artale *et al*., 1997).

Convective flows can be created ranging from time-independent, spatially periodic flow to turbulent flows by varying some control parameter e.g. Rayleigh number, *Ra*. As a consequence transport rates vary over a wide range (Solomon & Gollub, 1998). These systems represent then a good model system for a comprehensive investigation of transport: when the fluid is motionless, the transport is entirely due to molecular diffusion; at the opposite extreme (turbulent flow) transport is due to advection by the flow. There are two relevant laminar flow regimes in between these extremes: a time-independent and a time-periodic regime. In the time-independent regime, large-scale transport is limited by molecular diffusion between adjacent rolls while in the time-periodic regime transport is dominated by advection of tracers' particles across their boundaries. In order to analyze and to quantify the different mechanisms that lead to transport enhancement, alternative concepts and tools based on the dynamical systems theory have to (should) be considered and compared to the usual diffusion coefficients.

Following this idea, the study of an unsteady, quasi-two-dimensional, convective system has been experimentally carried out by analyzing the behavior of a thermal above a cross-sectional linear heat source located in the mid-line above the bottom surface of a rectangular tank filled up with water.

In these conditions, the buoyancy induces a flow characterized by two counter-rotating vortices separated by an ascending plume that, beyond a certain value of the Rayleigh number oscillates almost periodically in a plane perpendicular to the heat source. The dimension of these structures, is of the order of the length of the vessel. The symmetry of this field is broken by the formation of smaller structures that alternatively determine an oscillation of the upward current in the right and left half-field of the fluid domain. Such configuration remains unchanged in a wide interval of variation of the control parameter allowing the study of the dependency of Lagrangian quantities on it.

Lagrangian description of the fluid motion is obtained by means of Particle Tracking Velocimetry (PTV) technique. The characteristics of regularity, two-dimensionality and symmetry of the analyzed flow have allowed tracking a great number of particles for time intervals greater than the characteristic time-scales of the flow. A description of the fluid flow in an Eulerian framework can be obtained too from Lagrangian data. As a result one takes a spatio-temporal description of the velocity field with a higher frequency than those characteristics of the investigated phenomenon.

Different Lagrangian statistics on particles displacements have been evaluated in order to quantify dispersion phenomena. The approaches based on one particle statistics (absolute dispersion) and two particle statistics (relative dispersion, Finite Size Lyapunov Exponent) are then compared to discuss about the more suitable methodology for studying tracers dispersion in analogous problems.

2. EXPERIMENTAL METHOD

2.1 Experimental set-up

The experimental set-up consists of a rectangular tank filled with water. Horizontal dimensions of the tank are L=15.0 cm$\times D$=10.4 cm and its vertical dimension is H=6cm. Figure 2 shows the vessel and the optical configuration utilized in this experiment. The upper and lower surfaces are 0.8 cm aluminum plates kept at constant temperature by means of two heat exchangers where water flows at constant temperature. The side walls of the tank are 1 cm in thickness and made of perspex and can be considered adiabatic. The convection is generated by a linear heat source 0.8 cm in diameter placed near the bottom surface of the tank in correspondence of the mid line of it. An internal electrical resistance connected to a stabilized power supply heats the cylinder. It controls the heat flux furnished to the system with a precision of 2%. The choice of positioning the heating element externally of the bottom surface (Miozzi *et al*., 1998), reveals in a reduction in heat loss due to conduction phenomena allowing a better control of the heat flux supplied to the system.
In this configuration, the system is controlled by three non-dimensional parameters: Rayleigh number, Prandtl number and aspect ratio of the tank respectively:

$$\mathrm{Ra} = \frac{a \cdot g \cdot Q \cdot H^3}{u \cdot k \cdot l}; \qquad Pr = \frac{n}{k}; \qquad A = \frac{L}{H} = 0.4$$

where k is the thermal diffusivity, l the thermal conductivity, Q the linear heat flux, a the thermal expansion coefficient, g the gravity acceleration, and u the cinematic viscosity. The mean temperature of the fluid which is needed to evaluate its physical properties, is probed by a thermocouple. In this experiment, the fluid and the geometry of the tank have not been changed, consequently the complexity of the system increases with Ra. This parameter is varied in the range $6.87 \cdot 10^7 \div 2.17 \cdot 10^9$ by increasing the power supplied to the heat source. It has to be noted that Rayleigh number is evaluated by considering the linear heat flux instead of the temperature increment (Desryaud &Lauriat, 1993).
In each run, the heat exchangers on the horizontal surfaces and the electrical resistance of the heater are activated about 3 hours before the beginning of acquisitions to avoid transient regime. Acquisitions last for 2700s, during this period at least 22500 frames are digitized. Typically 900 particles are simultaneously tracked for each frame.
The scaling parameters of the flow are obtained following a typical non-dimensional formulation of the problem (Koschmider, 1993). Consequently we choose as unit length the height of the cell, H, and as unit time the diffusive time $t_k = H^2 / k$. The corresponding scale for the velocity can be obtained.

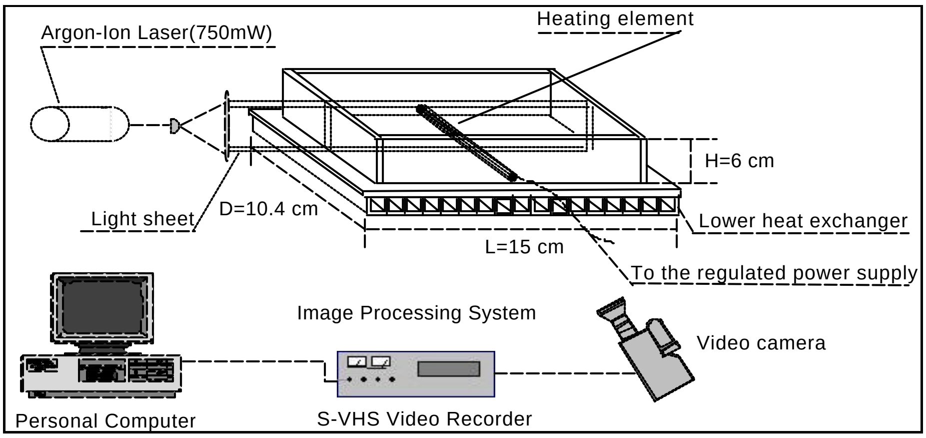

Figure 2 Experimental set-up.

2.2 Measuring technique

Particle Tracking Velocimetry technique, gives the Lagrangian description of the motion. Pollen particles of a mean size of 50 μm have been used to seed water. The test section is a vertical plane orthogonal to the wire placed in the middle of the tank and lighted by a 0.3 cm light sheet obtained by a 750 mW Argon-Ion laser beam passing through a cylindrical lens. Single-exposed images are acquired by a standard CCD camera placed orthogonally to the light sheet. After recording on S-VHS tape, these images are then digitized at 8.33 Hz at a 752×576 pixel resolution.

A particle tracking algorithm allows trajectories reconstruction by following particles displacements at different frames (Querzoli, 1996). The procedure for detecting particle locations is organized in three steps. The digitized frames are firstly reduced to Boolean images where the non-zero values represents the particle images while zeros are associated to the background. The Boolean image is then labeled for identifying sets of connected non-zero pixels which are good candidate particle images. Finally these sets are selected according a minimum and a maximum limit value on their areas and stored with a temporal information. Trajectories can now be identified as time-ordered series of particle locations matching two criteria indicating some threshold on the maximum velocity and acceleration related to the kinematic characteristics of the analyzed flow field. Velocity samples are then evaluated by dividing the particle displacements by the time interval between frames. To improve resolution, the velocities are actually evaluated using the displacements between 3 digitized frames. Eulerian velocity fields are obtained in each 4 consecutive frames by interpolating on a regular 15×35 grid. Although PTV allows for the evaluation of velocity vectors with high local accuracy and assures a statistical independence of data, this procedure of interpolation should be carefully carried out in order to avoid errors (Adrian, 1991).

2.3 Experimental observations

It has been experimentally observed that when convection starts, an ascending plume rises above the heat source and a descending flow is noted in correspondence of the side walls of the tank. This circulation organized in two counter rotating rolls divided by a buoyant plume exhibiting a natural swaying motion almost periodical in a plane perpendicular to the wire. No meandering instabilities in the transversal direction has been noted in the examined range of Rayleigh number.

As already discussed, previous experimental and numerical evidences (Miozzi et *al,* 1998; Desrayaud & Lauriat, 1993), show that this flow, is characterized by an alternation of different vortical structures depending on the presence of the plume on the left or on the right of the tank. In particular two dominant structures form on the same side of the tank where the plume bends while, on the other side, only one circulation exists. Previous studies (Miozzi and Querzoli., 1996) have shown the Eulerian features of this flow, we recover here the same flow configuration. Figure 3 shows the mean velocity field corresponding to the run $Ra = 7.2 \cdot 10^8$ obtained by averaging over the whole acquisition (~5620 frames). In Figure 4 the horizontal component of this velocity field in a point located 0.8*H* above the heat source is plotted. The time periodicity corresponding to the thermal plume can be easily observed.

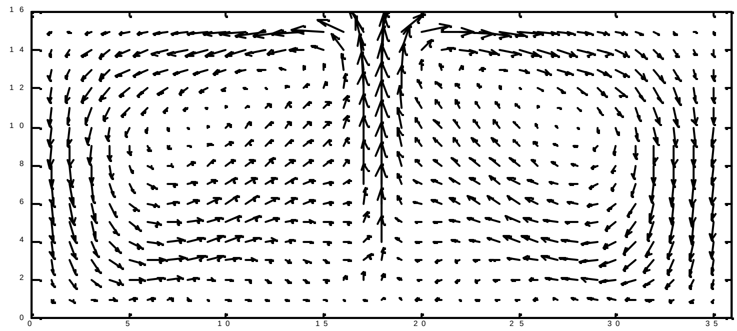

Figure 3 Mean velocity field at $Ra = 7.2 \cdot 10^8$.

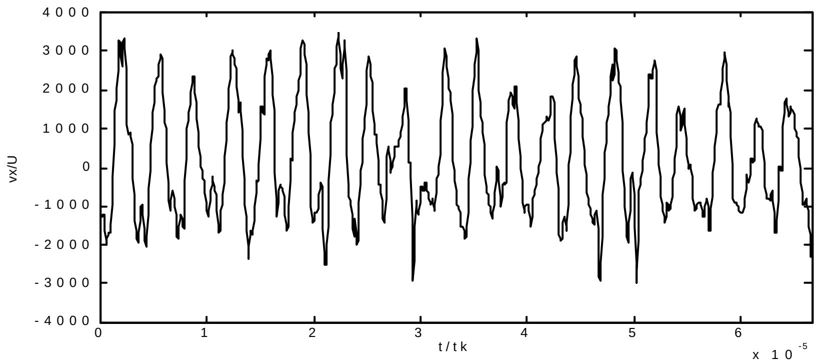

Figure 4 Time series of the Eulerian horizontal velocity in a point located 0.8*H* above the heat source.

3. DISPERSION PROPERTIES OF LAGRANGIAN TRACERS

The most natural framework for investigating transport phenomena is to adopt a Lagrangian viewpoint in which the particles are advected by a given Eulerian velocity field, $\boldsymbol{u}(\boldsymbol{x}, t)$ according to the equation:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{v}(t) \tag{1}$$

where $\boldsymbol{v}(t)$ is the Lagrangian particle velocity, molecular diffusion is neglected and the incompressibility constraint is assumed. Despite the apparent simplicity of this equation, the problem of relating the Eulerian properties of $\boldsymbol{u}$ with the Lagrangian properties of the trajectories $\boldsymbol{x}(t)$ can not be considered a trivial task. In the last decades, the discovering of the ubiquity of Lagrangian chaos has tuned this scenario even more complex (Ottino, 1989). As a matter of fact very simple Eulerian fields can generate very complex Lagrangian trajectories practically indistinguishable from those obtained in a turbulent flow. In a two dimensional incompressible flow the emergence of Lagrangian chaos can be explained in terms of dynamical system approach to the solution of (1). The equations describing these flows formally represent a one degree of freedom Hamiltonian system

$$\frac{dx(t)}{dt} = \frac{\partial y(x,y,t)}{\partial y}; \qquad \frac{dy(t)}{dt} = -\frac{\partial y(x,y,t)}{\partial x} \tag{2}$$

where the Hamiltonian is the stream function y. If the flow is steady, the system (2) is trivially integrable and all the solutions are regular (i.e. non-chaotic). For unsteady flows equations (2) are generally nonintegrable and at least some of the solutions are expected to be chaotic.

The possibility of understanding the different nature of particle trajectories gives an insight into the efficiency with which passive tracers are mixed by the flow. If we refer to advective and diffusive transport of tracers as stirring and mixing respectively (Brown and Smith, 1991) we can observe that under chaotic conditions the tracer is efficiently stirred which in turn causes an efficient mixing.

3.1 Lagrangian statistics

PTV technique measures the position and the velocity (2 component on a plane or on a volume) of the seeding particles in each acquired frame i.e. in a Lagrangian framework. Dispersion of a passive tracer in a given flow is consequently quantified by evaluating the statistical moments of these quantities. In particular different mechanisms driving dispersion phenomena can be analyzed by looking at one particle-

two times statistics (absolute dispersion), two particles-one time statistics (relative dispersion), two particle statistics at fixed scale (Finite Size Lyapunov Exponent).
If we consider the motion of a cloud of N particles x_i $i=1,\ldots..N$ advected in a domain of size L by a smooth velocity field with characteristic length l_u we respectively obtain:

i) the mean square particle displacement or absolute dispersion:

$$D(t)=\left\langle \left|\mathbf{x}(t)-\mathbf{x}(0)\right|^2 \right\rangle \tag{3}$$

where the averaged is taken over the set N;
ii) the mean square separation of particle pairs or relative dispersion:

$$R^2(t)=\left\langle \left|\mathbf{x}_i(t)-\mathbf{x}_j(t)\right|^2 \right\rangle \tag{4}$$

where the average is taken over the whole considered couples {i, j};
iii) the mean separation growth rate of initially close particles at scale d or Finite Size Lyapunov Exponent (FSLE):

$$l(d)=\frac{1}{\langle t(d)\rangle} \ln r \tag{5}$$

where $\langle t(d)\rangle$ represents the average *doubling time* at scale d. It is introduced by defining a series of thresholds $d^{(n)} = r^n d^{(0)}$ where $d^{(0)}$ is the initial pair separation and r=cost>1. By measuring the time T_i it takes for the separation R_i to grow from $d^{(0)}$ to $d^{(i)}$ and by performing the doubling time experiments over N_p particle pairs one gets the doubling time $< t(d) >$ by averaging (Artale et al., 1997).
It has to be noted that the term d*oubling time* strictly refers to the threshold rate r=2.
By comparing the quantities introduced in i) and ii) it can be observed that, especially when flows in bounded domains are considered, the relative dispersion can be more interesting than the absolute one which results dominated by large scale flows. As a matter of fact, if we consider two trajectories $\boldsymbol{x_1(t)}$ and $\boldsymbol{x_2(t)}$, the evolution of the separation $\boldsymbol{R(t)} = \boldsymbol{x_2(t)}$- $\boldsymbol{x_1(t)}$ is given by:

$$\frac{d\mathbf{R}}{dt}=\mathbf{v}_2(t)-\mathbf{v}_1(t)=\mathbf{u}(\mathbf{x}_1(t)+\mathbf{R}(t),t)-\mathbf{u}(\mathbf{x}_1(t),t) \tag{6}$$

and thus depends on the velocity difference at the scale $R = |\boldsymbol{R}|$. It follows that the Eulerian velocity components of typical scale much larger than $\boldsymbol{R}$ will not contribute to the evolution of $\boldsymbol{R}$.
If we analyze the asymptotic behavior of (4) in a bounded domain of size L when the typical lengths of the Eulerian velocity field are l_u (smallest structures) and L_0 (largest) we gets the following scenario:

- at very small separation ($<<l_u$) the velocity difference in (6) can be linearly expanded in R. In most time-dependents flow this expansion revels in exponential growth of initially close trajectories (i.e. to Lagrangian chaos):

$$\langle \ln R(t)\rangle \approx \ln R(0)+ lt \tag{7}$$

where λ represents the Lagrangian Lyapunov exponent of the system (Ottino, 1989).
- for very long times and separation ($>>L_0$) the two considered trajectories can be considered decorrelated and a diffusive regime

$$R^2(t)\approx 2Dt \tag{8}$$

is attained.
It has to be observed that in several realistic situations this behavior can not be observed, mostly when L is of the same order of L_0.

3.2 Finite Size Lyapunov Exponents

The Finite Size Lyapunov Exponents (FSLE) introduced in the previous section, represent a generalization of the Lyapunov exponent to finite separations (Aurell et al., 1997) developed by realizing that the separation R between trajectories is nothing but the scale at which the system is observed. This means that by keeping R finite, the FSLE allows to quantify the dispersion properties of the flow at different length-scales. The evaluation of $l(R)$ in a bounded domain of size L achieved by the evaluation of the separation doubling time (5), shows the following scale-dependant scenario:

$$l(R) \sim \begin{cases} l & \text{if } R << l_u \\ D/R^2 & \text{if } R >> l_u \end{cases} \tag{9}$$

We can thus remark that:

(i) for small separation $(l_u << L)$, if the Lagrangian dynamics is chaotic i.e. we recover the standard Lagrangian Lyapunov exponent;
(ii) in the intermediate regime $l< R <L$, two particles are advected by almost uncorrelated velocities and therefore a diffusive behaviour is expected. This implies that the scaling behavior of the FSLE is $l(R) \sim D/R^2$ where D is the diffusion coefficient;
(iii) at very large separation, $R \sim L$, $l(R)$ should go to zero because particles cannot separate more than the domain size. For a large class of systems a universal behavior has been found (Artale et al., 1997)

$$l(R) = \frac{D(R)}{R^2} \approx \frac{1}{t_r} \frac{(R_{max} - R)}{R} \tag{10}$$

where $R_{max} = O(L)$ is the saturation value of R at the boundary which corresponds to having reached an uniform distribution of the tracer in the basin, being t_r dimensionally a time. Equation (10) can be obtained by assuming that particle distribution relaxes exponentially (being t_r the relaxation time) to the uniform distribution (Artale et. al. 1997).

4. RESULTS AND DISCUSSION

As discussed, the convective flow analyzed in this experiment maintains essentially two-dimensional and time dependent in the investigated range of Rayleigh number values. Therefore we expect chaotic Lagrangian motion (Ottino, 1989). We have used the FSLE analysis on the experimental trajectories in order to investigate the separation growth at different scales from the Lyapunov exponential regime up to the saturation regime.
The FSLE analysis on the experimental data has been done as follows. After having fixed a set of thresholds $R_n = R_0 r^n (n = 0, \ldots N)$, for each time t_0 a new couple is considered whenever we find two particles (not yet forming a couple) at distance $R(t_0) \leq R_0$. The separation growth between these particles is then followed for times $t>t_0$ and the *doubling times* $T_r(R_n)$, at scale R_n, are evaluated by measuring the times the separation takes to grow from R_n up to $R_{n+1} = rR_n$. Since trajectories are sampled at discrete times, the average over for computing $l(R)$ is done by extending (5) to the time-discrete case using:

$$l\,(d) = \frac{1}{\langle T(R_n)\rangle} ln\left(\frac{R(T_r)}{R_n}\right) \qquad (11)$$

In order to increase the statistics at large separations R, we computed the FSLE for different values of the smallest scale $R_0 = 0.067H$, $0.1H$, $0.13H$. The threshold rate r is equal to 1.2 in each computation.

Figure 5 and 6 show the FSLE computed for two different Rayleigh numbers (Ra=2.39×10^8, Ra=96×10^8). At small values of R, we observe a collapse of $l(R)$ to the plateau indicating the Lyapunov exponent which is found to be positive ($l > 0$). This gives a direct evidence of Lagrangian chaos in the investigated flow (Boffetta et al., 1999). For larger separation, $l(R)$ drops to smaller values, indicating a decreasing in the separation growth well described by the saturation regime. The collapse of the curves at different R_0 confirms that high statistics are reached even at large scales. The characteristic Eulerian scale of smallest structures l_u=$0.5H$ can be estimated by the end of the exponential regime (i.e. the plateau $l(R) \approx l$) after which the non linear effects start to dominate. From the discussion of the previous section, we do not expect to observe the intermediate diffusive regime since in this case $L_0 \sim L_u$. The saturation scale $R_{max} \approx 1.9H$, and the relaxation time $t_r \cong 8.0 \cdot 10^{-4} t_k$ have been estimated too. We observe that both $1/l$ which gives an estimation of the small scale mixing time, and t_r are smaller than the diffusive time t_k. This indicates that dispersion of Lagrangian tracers in the analyzed flow is not driven by diffusion.

By performing the FSLE analysis for different Rayleigh numbers (Table I) we investigate the dependence of the Lagrangian Lyapunov exponent on Ra. Figure 7 shows a clear scaling which indicaties a power law dependence:

$$l \sim Ra^g \qquad (12)$$

with $g = 0.51 \pm 0.02$. An analogous scaling has been observed for the Eulerian characteristic times in a similar flow (Miozzi et al., 1998).

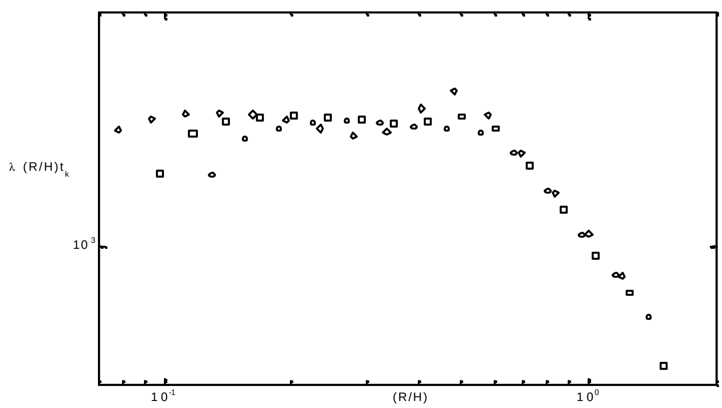

Figure 5 FSLE vs R (Ra= $2.39 \cdot 10^8$) for different initial thresholds.

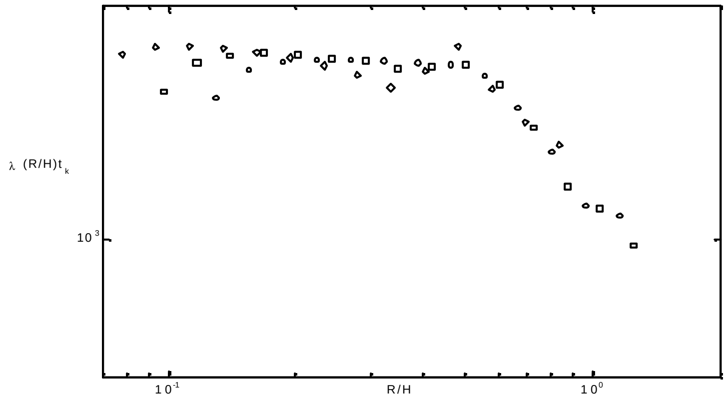

Figure 6 FSLE vs R ($Ra= 5.96x10^8$) for different initial thresholds.

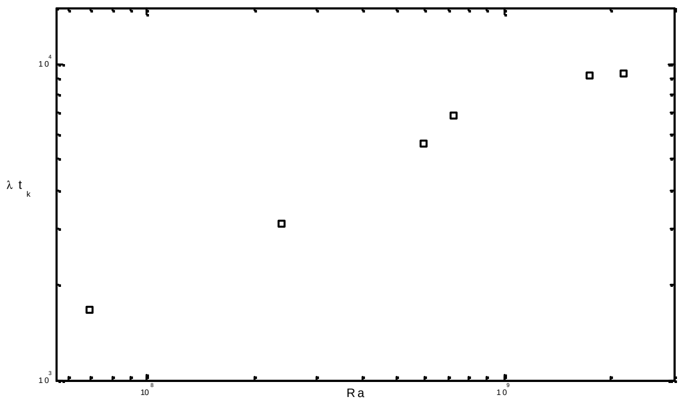

Figure 7 Lagrangian Lyapunov exponents vs Ra.

In order to compare the FSLE and the relative dispersion analysis, we have computed the moments of relative dispersion for $Ra=2.39x10^8$. The result, plotted in Figure 8, indicates that at small times (i.e. small separation) an exponential regime characterised by a slope which increases with the moment order, p. The reason for this behavior is that many pairs remain very close for very long times before being exponentially separated.

As a further analysis we evaluate the probability distribution function of the pair distance for different times plotted in Figure 9. As it is clearly shown in the Figure, most of the couples, representing the peak of the distribution, remain close for long time. Moreover, for times longer that the circulation time, some of the separated particles come close again causing strong fluctuations in the relative dispersion at large times (see Figure 8). As a consequence, the saturation regime is not observable with this kind of analysis.

Both these problems do not affect the FSLE analysis because in this case statistics are performed only on the set of pairs which effectively separate.

Let us end this section with some comments on the evaluation of the absolute dispersion (3). In Figure 10, the mean square particle displacement versus time is plotted for different value of Rayleigh number. It is possible to observe that, as expected, dispersion increase with Rayleigh number following a parabolic law and that after a characteristic time (length) a saturation regime is attained.

As a matter of fact in this case this quantity only gives an indication of the mean spreading of the particles from their initial condition without allowing neither interesting information about the spreading mechanisms nor a suitable quantification of dispersion phenomena since contribution from different structures at different times are included in the same average. This consideration can be extended to all systems in which the characteristic length-scales are not sharply separated and consequently dispersion can not be well described by using asymptotic quantities (Artale et al., 1997).

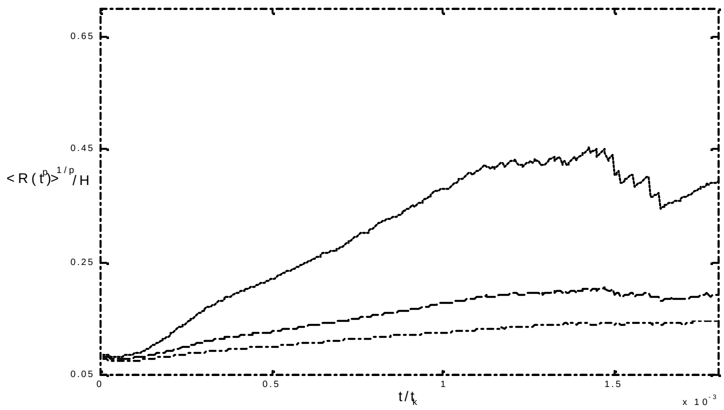

Figure 8 Rescaled relative dispersion $<R(t)^p>^{1/p}$ for $p=1,2,4$ (from bottom to top) in lin-log plot.

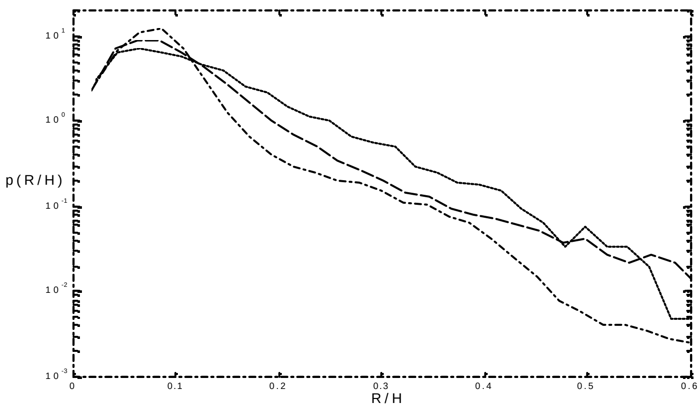

Figure 9 Probability distribution function of pair separation at times $t=2.510^{-4}t_k$ (solid line) $t=5.010^{-4}t_k$ (dashed line) $t=1.010^{-3}t_k$ (dotted line) in lin-log plot.

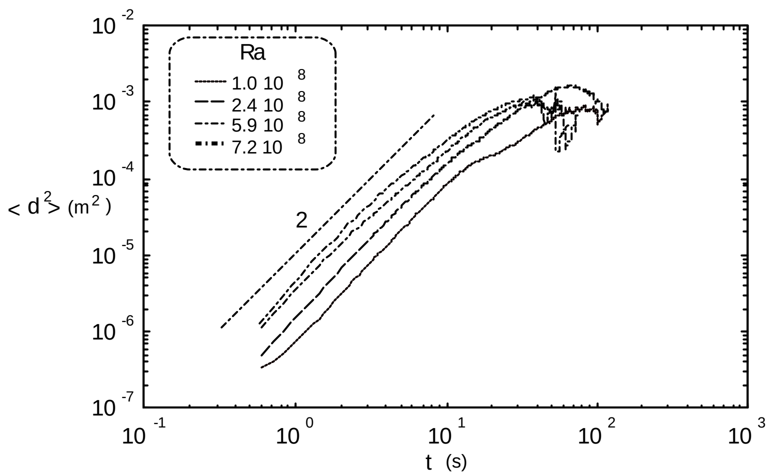

Figure 10 Mean square particle displacements vs time at different Ra.

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