An improved cross-correlation method for (digital) particle image velocimetry

by

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ABSTRACT

Cross-correlation (digital) particle image velocimetry (PIV) has become a well-known and widely used experimental method. It has been already documented that difficulties arise in accuracy and spatial resolution. An improved method that brings enhancement in accuracy and spatial resolution for the interrogation of (digital) PIV images is described. This method is based on cross-correlation with discrete window offset. It makes use of a translation of the interrogation window and rebuilds the second interrogation window considering rotation and shear. This paper also presents the limits of conventional cross-correlation method in accuracy and spatial resolution, and explains the expected enhancement in terms of them. The accuracy and spatial resolution are compared by interrogation of synthetic (digital) PIV images with the conventional cross-correlation method and the present, improved cross-correlation method. This improved cross-correlation with discrete window offset is applied to real PIV data and the results are discussed.

1. INTRODUCTION

It is no longer necessary to stress the importance of (digital) particle image velocimetry (PIV) that is an effective instantaneous and non-intrusive new technique for the measurement of velocity fields in modern fluid mechanics research. That the technique has developed rapidly over the last two decades is indicated by the large amount of published work, which has been reviewed by Adrian (1986, 1991) and Kobaybashi et al. (1988), etc.

Generally, all PIV methods can be divided into two steps: firstly, recording the images of a particle seeded flow, and secondly, extracting velocity vectors from the recorded images. The PIV techniques are often categorized according to the recording media used (Willert and Gharib 1991).

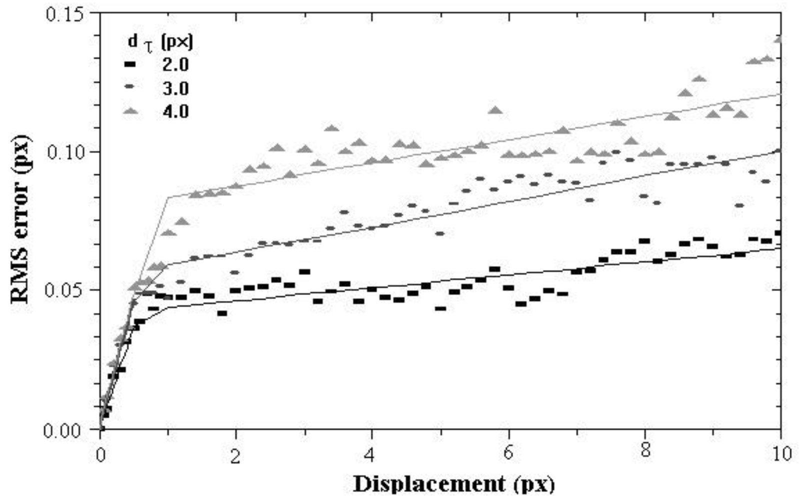
There are three major methods for PIV images processing: (1) the Young's fringe (Backer and Fourney 1977), (2) auto-correlation (Adrian 1989) and (3) cross-correlation (Utami and Blackwelder 1991, Willert and Gharib 1991). In the cross-correlation method, spatial cross-correlation between two sequentially exposed flow images are used to determined the velocities. An important advantage of the cross-correlation method is that this method removes the so-called directional ambiguity, which is inherent in both the Young's fringe and auto-correlation methods (Keane and Adrian 1993).

However, cross-correlation also has drawbacks. The deformation of particle image patterns by strong velocity gradients and out-of-pattern motions results in errors (Huang et al. 1993a), and spatial resolution is limited (Scarano and Riethmuller 1999). In order for the enhancement in accuracy and spatial resolution, a series of papers have been published. Reconstruction of the interrogation windows by considering the velocity gradient has been suggested by Ashforth-Frost et al. (1993) and Huang et al. (1993b). Rebuilt the second image taking into consideration the effects of three flow movement components, namely spatial uniform transform, rotation and shear (Jambunathan et al. 1995).

Adrian (1991) conjectured that the variation of measurement error (e) is directly proportional to the particle image diameter (d_t) which was actually confirmed in a study by Prasasd et al. (1992). The same behavior can be observed in Fig. 1, which shows the results from Monte Carlo simulations for the root-mean-square (RMS) error in pixels (px) of the displacement estimated with the so-called three point Gaussian peak fit (Willert and Gharib 1991, Westerweel 1993ab) as a function of the actual displacement (u), also in px, for three different particle image diameters: note that variation of the measurement error increases for increasing d_t. Also note that the variation of the measurement error is practically independent of the displacement, except for small displacements, i.e. $|u| < \frac{1}{2} px$; for this region, the variation of the measurement error appears to be directly proportional to u.

The immediate thought that arises when one observed the result in Fig 1 is whether to make use of the behavior of the measurement error for small u to improve the position of PIV measurements. Actually this improvement can be obtained quite easily. For the cross-correlation analysis of a pair of single exposure digital PIV images, it is relatively simple to offset the interrogation windows by the integer part of the particle image displacement. Hence, the residual displacement is only the fractional amount of the particle image displacement (in pixel units), which is always smaller than 1/2px, and subsequently would yield a more accurate result compared to the original analysis without the window offset. If considering the effect of rotation and shear and rebuilding the second interrogation window, the much more accurate result will be yielded.

Fig. 1. The RMS estimation error for the displacement as a function of the displacement u in pixel units for a 32×32



pixels interrogation window using the conventional cross-correlation method, for particle-images with a diameter of 2, 3 and 4 pixels. The solid lines are fit results (approximately same as Westerweel 1993ab). The symbols are obtained from Monte Carlo simulations (the error bar represents the uncertainty of the simulation results).

The papers about cross-correlation method with window offset have also published. Keane and Adrian (1993) proposed to use a window offset equal to the in-plane displacement to optimize the detectability of the displacement-correlation peak with respect to the random correlation peak. Westerweel (1997) investigated in particular the noise reduction effect as a result of using a (discrete) window offset in the (digital) evaluation of PIV recordings. Scarano and Riethmuller (1999) described iterative multigrid approach in PIV image processing with discrete window offset. However, in this paper the effect of enhancement in accuracy and spatial resolution as a result of the improved cross-correlation method with discrete window offset considering flow movement components, namely spatial uniform rotation and shear will be investigated.

In section 2, the limits of the conventional cross-correlation method in accuracy and spatial resolution are introduced. Section 3 describes the improved cross-correlation method with discrete window offset and its enhancement in accuracy and spatial resolution. Section 4 describes the validation of this improved method using synthetic PIV images. The application of this improved method in real PIV data is described in section 5, and in section 6 the main conclusions from this study are presented.

2. LIMITS OF THE CONVENTIONAL CROSS-CORRELATION METHOD

2.1 The Conventional Cross-correlation Method

After having acquired two sequential images of the flow, the cross-correlation method is used to extract information concerning the displacement. As the method of Willert and Gharib (1991), firstly split the images into multiple windows. Secondly, for each pair of corresponding windows, calculate the 2D FFT of both windows, compute the cross product of the first window FFT and the second window FFT conjugate, determine the IFFT of the result, find the location of the maximum in the correlation plane (determine displacement) and compute the velocity of flow. The program is shown schematically in Fig. 2 (Willert and Gharib 1991).

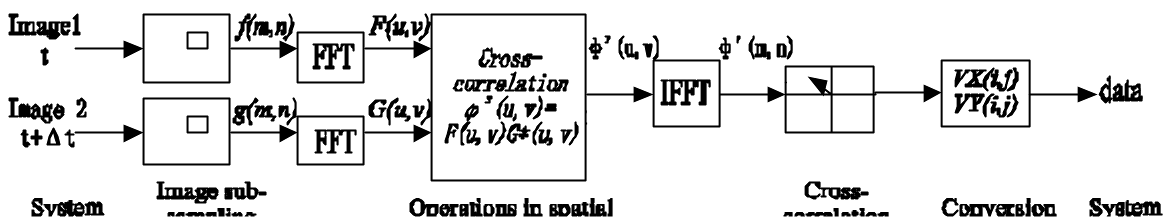


Fig. 2. Numerical processing flow-chart of (digital) PIV process: cross-correlation estimates are calculated by complex conjugate multiplication in the spatial frequency domain (Willert and Gharib 1991).

The features in this method are the use of FFT to simplify and significantly speed up the cross-correlation process and the determination of the offsets of the peak measured from the origin of the cross-correlation coefficient map to represent the average spatial displacement.

2.2 Limits in the Accuracy

The conventional cross-correlation method used in PIV to extract velocity vectors necessitates the assumption that the velocity gradients inside the interrogation windows are negligible. However, in real flow fields, it is invalid.

Considering a flow element of a general two-dimensional flow, Helmhotz velocity theory divides a movement of a flow element into three subsidiary terms:

$$\vec{V} = \vec{V}_o + \frac{1}{2}(\nabla \times \vec{V})_o \times \Delta \vec{r} + \Delta \vec{r} \bullet \vec{E} \tag{1}$$

Where $\vec{V}$ represents the velocity in the fluid element and $\vec{V}_o$ represents the velocity of the spatial transform of the fluid element. The three terms on the right-hand side of equation (1) are transform, rotation and element bi-axial shear respectively.

Westerweel (1993a) pointed out that, if the interrogation window is considered as a flow element, then the rigid shift results in only parts of interrogation window contributing to cross-correlation peak. The peak is shifted away from the origin and the rotation and deformation terms lead to broadening of the peak (Willert and Gharib 1991). Therefore, it is not very accurate to determine the displacement without considering the rotation and shear terms. Huang et al. (1993ab) described the limitation of the conventional cross-correlation method due to deformation of particle image patterns by strong velocity gradients and out-of-pattern motions and proposed an artificial distortion considering the linear velocity gradients. Jambunathan et al. (1995) rebuilt the second image taking into consideration transform, rotation and sheer. However, if the velocity of flow fields exceeds 1/2px, determining the velocity using these methods is not very accurate as in Fig. 1 though considering the deformation of particle image patterns.

Westerweel (1997) defined that the signal-to-noise ratio (the total signal power divided by the total noise power) SNR as:

$$\frac{S}{N} = \frac{\mathrm{var}\{u\}}{\int \mathrm{var}\{e \mid u\} f(u) du} = \frac{u'^2}{c^2} \tag{2}$$

where $|u| > \frac{1}{2}px$.

Fig. 1 defines: $$\mathrm{var}\{e \mid u\} = \begin{cases} 4c^2u^2, \ for\ |u| < \frac{1}{2} \\ c^2, \ elsewhere \end{cases} \tag{3}$$

Where u' is the variance of the fluctuating displacement (viz. velocity), c is a constant. It is shown that the SNR for PIV interrogation analysis without window offset is proportional to the variance of the fluctuating displacement.

2.3 Limits in the Spatial Resolution

The spatial resolution of the velocity fields that can be obtained from PIV images by correlation of high image density recordings is just the size of the interrogation window in the fluid, $d_I = D_I / M_o$ (lower case letters refer to quantities in the fluid and upper case letters denote quantities on the image recording plane) (Adrian 1997). Therefore, the smaller size of the interrogation windows, the higher spatial resolution of velocity fields.

However, the spatial resolution is inherently limited by the statistical nature of correlation based processing in dividing images into interrogation windows. A careful balance must be maintained between the statistical number of tracer particle images within the interrogation windows and the relative displacement of particle images between exposures from one area of a region to another. If a interrogation window is too small, two few tracer particle exit within the region and a statistically meaningful result for averaged flow velocity images can not be obtained through correlation. If the interrogation window is too large, however, the displacement of tracer particles between exposures from one area of the region to another due to local flow gradients can be greater than the image diameter of the tracer particles. This condition results in a decrease in signal-to-noise ration due to displacements in individual particle condition peaks too large to add to the net correlation signal (Keane and Adrian 1993, Hart 1996).

Adrian (1989) stated that the number of trace particle images doublets within interrogation window must exceed the value of 10. Because of motion of transform, rotation and shear, the trace particle images in the first interrogation window may exit from the second interrogation window, which is called out-of-pattern motion. It will broaden the interrogation window in order to extract the velocity, which will decrease the spatial resolution. This is described in detail in the paper of Huang et al. (1993b).

3. THE IMPROVED CROSS-CORRELATION METHOD WITH DISCRETE WINDOW OFFSET

A method based on cross-correlation with discrete window offset is now proposed that makes use of a translation of the interrogation window and rebuilds the second interrogation window considering rotation and shear. Vector validation and interpolation are also investigated.

3.1 The Improved Cross-correlation Method with Discrete Window Offset

The ration between the signal peak and the noise level in the correlation map is a crucial parameter in PIV measurements. It has already been found to be strongly dependent on the loss-of-pairs due to the in-plane motion (Keane and Adrian 1993). Nevertheless as referred by many authors (Adrian 1991, Huang et al. 1993ab, Keane et al. 1995, Jambunathan et al. 1995, Westerweel et al. 1997), also other parameters do contribute to the degradation of the signal like high velocity gradients, turbulent motion or loss-of-pairs due to out-of-plane motion.

The present method has been conceived with the purpose of compensating for the loss-of-pairs due to in-plane motion and velocity gradients due to rotation and sheer motions. The correlating windows are provided with a degree of freedom in terms of pure translation one with respect to the other. The local displacement of each interrogation window is made on the basis of a flow pattern prediction. The predicted displacement is obtained by a previous interrogation using the conventional cross-correlation method of the set of two images.

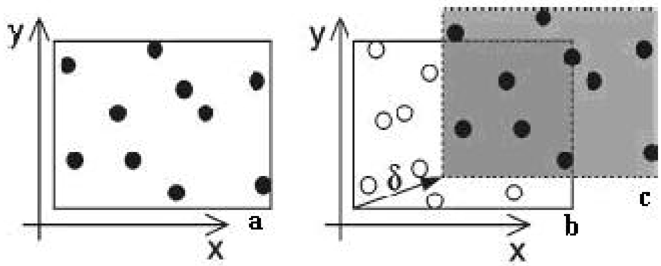


Fig. 3. Principle of the interrogation window displacement.

In Fig. 3 are shown two interrogation windows a and b extracted out of two subsequent images. If the displacement vector $d = (d_x, d_y)$ that occurred during the time separation Δt is estimated, then the interrogation window from the second image can be selected with a relation offset (window c, grey area containing all the blank filled particles) in order to maximize the number of particle doublets.

In order to compensate for velocity gradient due to rotation and sheer, the second interrogation window c should be rebuilt. The pixel intensity of interrogation window c is obtained using bilinear interpolation:

$$I(x_c, y_c) = \sum_{i=1}^{4} N_i I_i \tag{4}$$

$I_1 = I(\text{int}(d_x), \text{int}(d_y))$, $I_2 = I(\text{int}(d_x), \text{int}(d_y)+1)$, $I_3 = I(\text{int}(d_x)+1, \text{int}(d_y))$,

$I_4 = I(\text{int}(d_x)+1, \text{int}(d_y)+1)$, $e_x = d_x - \text{int}(d_x)$, $e_y = d_y - \text{int}(d_y)$,

$N_1 = (1-e_x)(1-e_y)$, $N_2 = (1-e_x)e_y$, $N_3 = e_x(1-e_y)$, $N_4 = e_x e_y$.

And then extract the minor displacement $d' = (d_x', d_y')$ computing the interrogation window a and c with the cross-correlation method. Therefore, the true velocity vector $V = (V_x, V_y) = \text{int}(d) + d'$, which is more accurate than d.

Therefore, the main procedures of this present, improved method are:
· Use the conventional cross-correlation method to extract initial full field displacement vectors. Eliminate error vectors and interpolate to achieve the true vectors using the method proposed in Sect. 3.2.
· Take the window offset of the second interrogation window b to c and reconstruct window c.
· Recompute the interrogation window a and c with the cross-correlation method to extract minor displacement vectors.
· Continue until all full field displacement vectors.
· Again eliminate the error vectors and interpolate to achieve the true errors using the method proposed in Sect. 3.2.

3.2 Vector Validation and Interpolation

Statistically, there is no dull probability of obtaining a false measurement at a certain grid node. If the deviation from the true value is large enough, future steps may be unable to correct the error. One method for dealing with these errors (already used by Jambunathan et al. 1995, Huang et al. 1993b) is to employ validation and interpolation algorithms in the intermediate steps. Many relevant papers on these important subjects have been written. Westerweel (1994) are Fujita and Kaizu (1995) were good examples for validation algorithms. Malik and Dracos (1995) and Agüi and Jiménez (1987), among others, dealt with interpolation in PIV.

Recently, additional insight on both topics has been reported in Nogueira et al. (1997b). The main objective for validation is to find an effective algorithm for false vector detection with low sensitivity to the parameters selected by the user. While for interpolation, in correlation based PIV, advantage can be taken on the square grid location of the values to interpolate. It allows the development of more accurate interpolating FIR filters. There, a detailed description shows that, even with low quality images, both steps can be taken with reliability. This paper adopts this method to eliminate the error vectors and interpolate to achieve the true vectors.

3.3 Enhancement in accuracy

The main characters of the improved cross-correlation method are to use cross-correlation with discrete window offset and to reconstruct the second interrogation window considering rotation and shear.

Westerweel (1997) defined the noise reduction (NR) that is achieved by applying the window offset as the ratio of the SNR for the measurement without a window offset and the SNR for the measurement with the window offset:

$$NR = [\int \frac{\text{var}\{e \mid u\}}{c^2} f(u) du]^{-1} \tag{5}$$

From the equation it is easily verified that if: $\text{var}\{e \mid u\} < c^2$ over any interval of u, then $NR > 1$. Thus, interrogation method with a window offset will always yield a more precise result compared to the conventional cross-correlation method.

Reconstruction of the second interrogation window considering rotation and sheer leads to the correlation map more spiculate, which determines the peak position more accurate. The whole field displacement is currently unknown. The displacement data (d_x, d_y) at each point can be calculated using the conventional cross-correlation method. Supposing that the actual displacement is $(d_x + d_x'', d_y + d_y'')$, where (d_x'', d_y'') is a measurement of the non-uniformity of the displacement around the points, then if the second interrogation window c is built based on (d_x, d_y), the peak position of the cross-correlation between the interrogation window a and c will be offset from the origin by minor value (d_x', d_y'), which the offsets of non-uniformity are reduced. A true displacement vector $(\text{int}(d_x) + d_x', \text{int}(d_y) + d_y')$ is found, which is much more accurate.

The enhancement in this improved cross-correlation method will also be validated with synthetic PIV images in Sect. 4.

3.4 Enhancement in Spatial Resolution

The use of this improved cross-correlation method improves the spatial resolution of the conventional cross-correlation method.

In the conventional cross-correlation method, the spatial resolution of (digital) PIV depends on the size of interrogation window since the detected velocity is the statistical mean velocity of each particle within the interrogation. The smaller the size of the interrogation window, the higher the spatial resolution obtained. Usually the size of interrogation window must not be too small because sufficient particle images must be available within it. Furthermore, the second interrogation window is reconstructed as if there are no velocity gradients in the flow field. As a result, reduction of the size of the interrogation window will not influence the accuracy of the determination of displacement if the interrogation window contains enough particle images, which is validated with synthetic PIV images in Sect. 4. The spatial resolution hence depends on the accuracy of the pixel location, which in turn depends on the accuracy of the velocity gradients calculated.

4. VALIDATION OF THE IMPROVED METHOD

To verify the enhancement in accuracy and spatial resolution using this improved cross-correlation method, a number of simulations were carried out in which synthetic digital PIV images were generated. The interrogation results of these synthetic images using the conventional cross-correlation method and this improved cross-correlation method will be compared. In this section, First describe the generation of the synthetic PIV images, which is followed by a discussion of the simulation results.

4.1 Generation of Synthetic PIV Images

Several runs on 256 by 256 pixels synthetic images were performed. These experiments were designed to compare the characteristics of this improved method with the conventional one.

In these synthetic images, particles were convolved with the kernel matrix having a Gaussian intensity distribution that has the following form (Willert and Gharib 1991):

$$I(x) = I_0 \exp(-\frac{x - x_c}{2s^{2}}) \qquad (6)$$

Where x_c denotes the gravity center of a particle and s stands for the standard derivation of a Gaussian intensity distribution. The diameter of particles was 3 pixels, and the number of particles in the interrogation window 32 by 32 pixels was about 10. Particles with the coordinates *(x, y)* were located randomly, The coordinates *(x, y)* were calculated with adding a ±1% error randomly to simulate a realistic range. Furthermore, ±3% of the particles were removed or added randomly in the second image because several particles often disappeared when photographing a real image.

Two sets of synthetic PIV images were generated. In the first ones, the displacement components *?x* and *?y* were fixed values, and so there was no velocity gradients in the interrogation window. In the second ones, the displacement *?x* and *?y* were calculated according to Eq. (7)

$$\begin{cases} \Delta x = -3(\sin(px/100)\cos(py/100) + \sin(px/120)\cos(py/120) + \sin(px/80)) \\ \Delta y = 3(\cos(px/100)\sin(py/100) + \cos(px/120)\sin(py/120) + \cos(py/80)) \end{cases} \qquad (7)$$

Where, $x \in [0,256], y \in [0,256]$.

Eq. (7) did not represent a real physical flow field, but incorporated representative vortical structures, which could also verify the enhancement in accuracy and spatial resolution using this improved method.

4.2 Interrogation Results and Comparisons

One set of runs dealt with a displacement field of a single displacement in the x direction and no displacement in the y direction. When extracting the displacement fields with this improved cross-correlation method, the second interrogation wi[...] e interrogation results are compared in Fi[...] igher than that using the conventional one[...] terrogation accuracy using this improved m[...] accuracy is not influenced by the size of t[...] however, the conventional method is. There[...] roved method can also be reached.

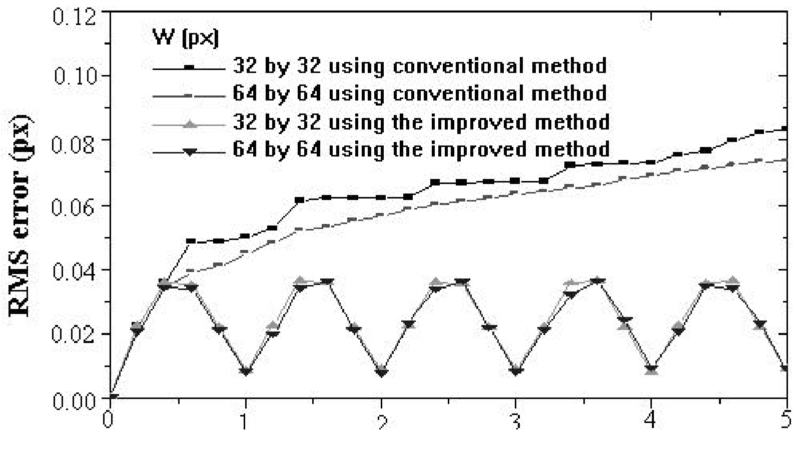
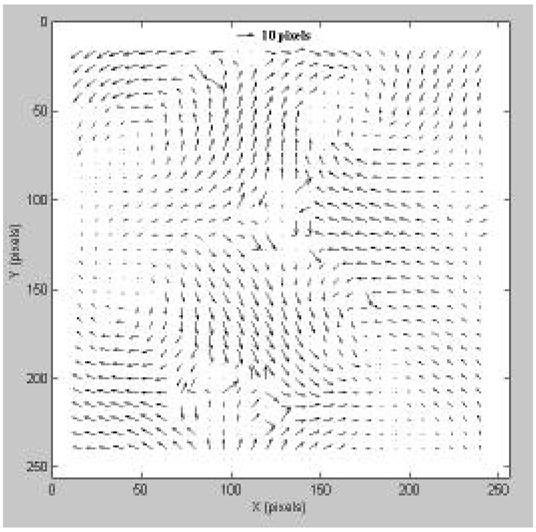
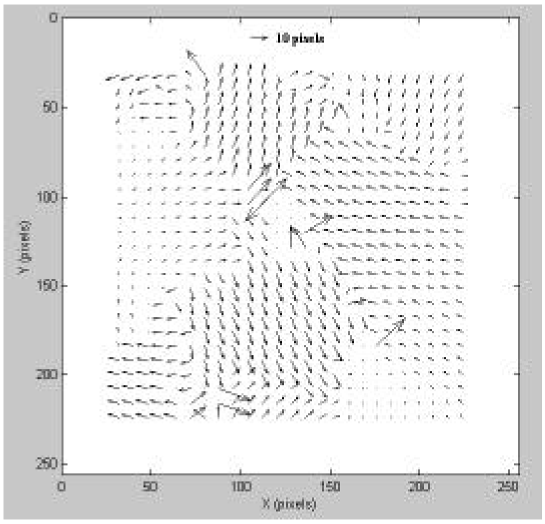


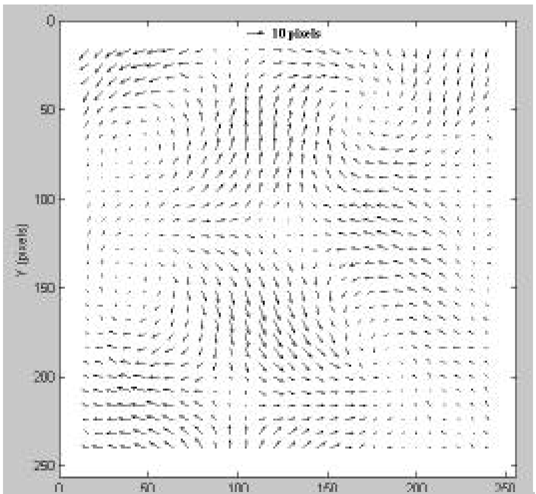
Fig. 4. Performance comparison for this improved cross-correlation method and the conventional one with different sizes of interrogation window.

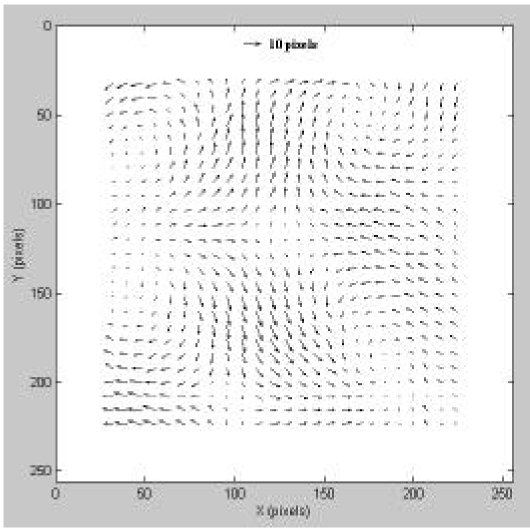
Other runs were performed for 2D displacement fields. The interrogation results are shown in Fig. 5, in which more correct displacement vectors with this improved method than with the conventional method can be also determined. Comparing case (a) and (b), that it is not accurate to analysis the flow fields with high velocity gradient using the conventional cross-correlation method can be seen. However, there is almost no difference between in case (c), (d) and case (e), the extract displacement field. Therefore, the conclusion that the accuracy and spatial resolution are enhanced using the improved method can be reached.

(a) (b)









(c) (d)

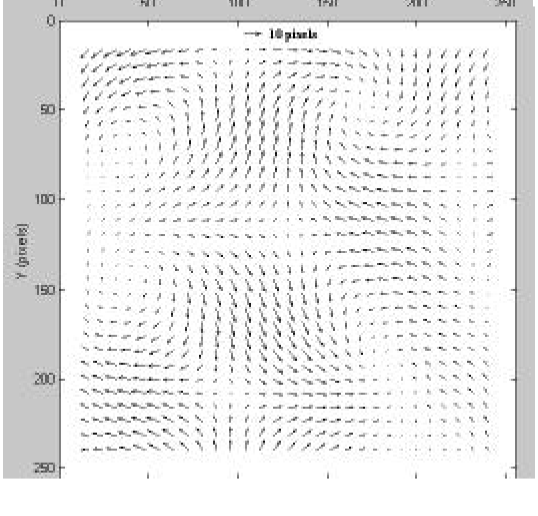


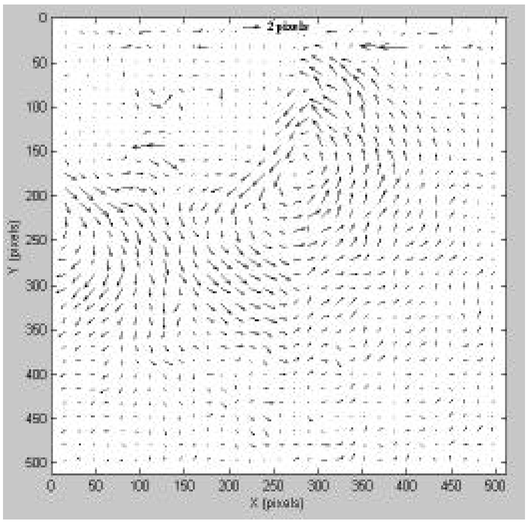
Fig. 5. Displacement fields obtained from synthetic images. (a) Displacement field extracted using the conventional cross-correlation method with 32 by 32 pixels interrogation window.

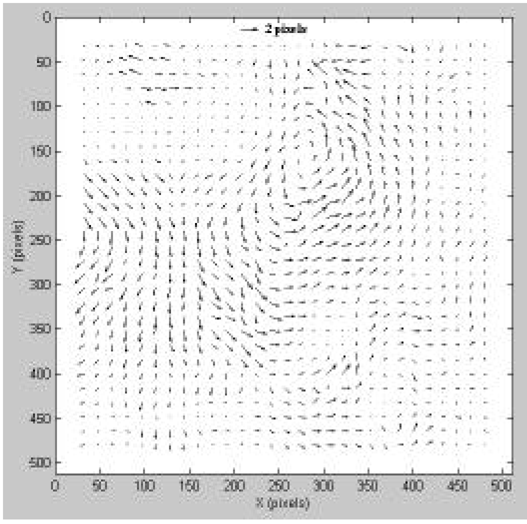
(b) Same as case (a), but with 64 by 64 pixels interrogation window. (c) Displacement field extracted using this improved cross-correlation method with 32 by 32 pixels interrogation window. (d) Same as case (c), but with 64 by 64 pixels interrogation window. (e) Extract displacement field.
(e)

5 APPLICATION TO REAL PIV DATA

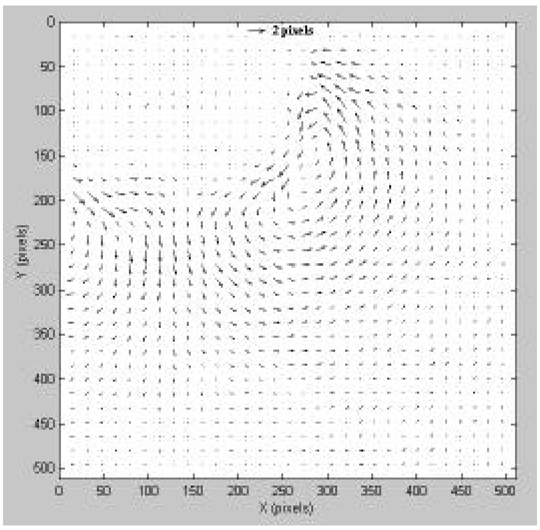
In order to validate the enhancement in accuracy and spatial resolution using this improved method with discrete window offset, an example of real PIV data was taken. The real PIV images represented the flow field of driving flow by surface tension in interface of gas and liquid when the temperature gradient exists in it. The flow was seeded with 50μm liquid crystal (BM/R294W/S-33). A Sony TR750E 512×512 pixels CCD camera recorded PIV images at 25Hz.

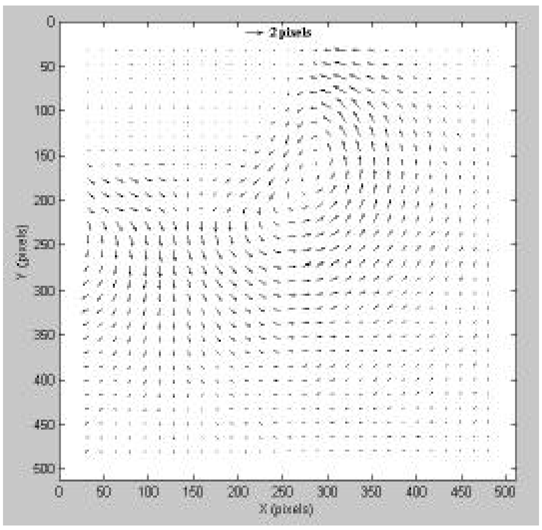
The interrogation results using the conventional method and this improved method are shown in Fig. 6. In case (a) and (b), whose displacement fields extracted using the conventional cross-correlation method, there are a few spurious vectors in spite of vectors validation and interpolation. Case (c) and (d), extracted using this improved cross-correlation method, are more accurate and same as the extract fluid motions. Therefore, the results also show the enhancement in accuracy and spatial resolution using this improved cross-correlation method.





(a) (b)





(c) (d)

Fig. 6. Displacement fields obtained from real PIV images. (a) Displacement field extracted using the conventional cross-correlation method with 32 by 32 pixels interrogation window. (b) Same as case (a), but with 64 by 64 pixels

interrogation window. (c) Displacement field extracted using this improved cross-correlation method with 32 by 32 pixels interrogation window. (d) Same as case (c), but with 64 by 64 pixels interrogation window.

6 CONCLUSIONS

This paper describes the principle of this improved cross-correlation method: making use of a translation of the interrogation window and rebuilding the second interrogation window considering rotation and shear, and compares with the conventional cross-correlation method using the synthetic and real PIV images. Conclusions can be reached. This improved cross-correlation method with discrete window offset is a method by which PIV images can be more accurately proceeded than the conventional cross-correlation one. This method based on cross-correlation with discrete window offset considering the effects of rotation and shear. It brings enhancement in accuracy and spatial resolution of the cross-correlation method.

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