LASER-SHEET CT-SCAN TYPE
2-D INSTANTANEOUS CONCENTRATION METER
with application of wavelet transform technique for high resolution

by

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ABSTRACT

A heap of knowledge accumulated in recent years on the coherent structure of turbulence made it possible to connect the structure of concentration field with that of turbulence, and the needs for advancement of physics of concentration structure are rapidly increasing.
This paper presents several improvements of both hardware and software for the concentration instrument reported at the 1998 Int. Symp. on Applications of Laser Techniques to Fluid Mechanics. The improvements increased remarkably the performance of CT-scan type 2-D instantaneous concentration meter.

Improvement in Laser Shedding and Scanning Mechanism : Laser shedding mechanism has been changed from mechanical scan by stepping motor, to shedding laser sheet expanded by cylindrical lens, avoiding rotating parts completely. Laser lights from an array of emitters are switched on and off, successively. Signals of laser light from emitters which passed through the concentration field have been received by an array of receiving units and recorded continuously, the sampling period being considerably shortened.
Improvement in Analysis System – Application of Wavelet Transform Technique : The conventional means to express an unknown field in the inverse problem is to recourse to the double Fourier expression. However, as we have described in the previous paper, this does not give a favorable result, because the information obtained from the one directional search of the field is too scarce. While the "Virtual Load Method" proposed by Hino(1975), an idea similar to the spline approximation, is successfully applied. A 2D concentration field, arranged to a vector form, is expressed, as the deflection of virtual load (vector w_p). The intensities of laser lights passing through the concentration field and received by receiving unit (arranged in a vectorial from R1) are expressed in terms of virtual load vector and the integrated Green's function matrix. The problem reduces to an inverse problem to solve for w_p. Substitution of w_p (virtual load thus obtained as a result of inverse problem) into the fundamental relation between the virtual load and the concentration field gives concentration field.
In general, signals received by a receiving unit are contaminated by random or pulse-like noise. Wavelet transform has been applied to suppress noise and recover the pure signals. The wavelet spectrum of the received light intensity vector is cleaned by cutting off the high frequency and/or higher level fluctuations. Finally, the estimated concentration field determined.

1. INTRODUCTION

1.1 Non-intrusive field Measurement as Inverse Problem

A variety of non-intrusive (non-invasive) methods of field measurement by the inverse detection are put to practical use; for instances, CT-scanner and MRI (Magnetic Resonance Imaging) in medicine, sonic Doppler radar in meteorology, sonic profiler in oceanography, and seismic tomography in geology and geophysics.
On the other hand, non-intrusive direct measurement means of concentration field, such as sediment-load and dye concentration, are rare to the writers' knowledge, except for the indirect measurement of photographing the concentration pattern illuminated by a sheet of strong light. Rather, a primitive method such as concentration measurement at one point by using decay of light passing through a small gap between light emitter and photodiode are prevailing.

1.2 Difficulty of Development of 2-D Instantaneous Concentration Meter

The reasons that hindered the development of 2-D instantaneous concentration meter and resulted in a fatal lack of 2-D concentration meter may be mentioned as follows;

- Contrast of concentration fields is vague and the pattern of concentration contour is complex compared to the fixed pattern of internal organs in human body,
- Concentration fields change every moment,
- Measurement of sediment is made in water,
- The investment efficiency of the concentration meter development is not so high. In other words, the needs for the instrument development with such high quality are probably low.
- Apparatuses such as CT scanner and MRI installed in hospitals are so large in size to be used in laboratories and in field. Moreover, they are too expensive.

1.3 Growing Needs for 2-D Instantaneous Concentration Meter

In the past, the needs for detailed information on concentration fields such as sediment-laden flow were relatively low, because for design purposes only the information on the mean concentrations were necessary. While, a heap of knowledge accumulated in recent years on the coherent structure of turbulence made it possible to connect the structure of concentration field with that of turbulence, and the needs for advancement of physics of concentration structure are rapidly increasing.

1.4 The Principles of Design of CT-scan type 2-D Instantaneous Concentration Meter

This research aims at the development of CT-scan type 2-D instantaneous concentration meter based on the following design principles.

- The instrument should be as small as possible to be used in laboratories and/or fields,
- Its cost should be cheap enough for a scientist or engineer to be able to buy within his budget,
- The measurement should be done almost instantaneously and 2-dimensionally,
- The device should not disturb the flow field.

2. DESIGN OF APPARATUS AND IMPROVEMENTS

2.1 Arrangement of Emitters and Receivers of Laser Ray

An array of laser source units are placed on one side and photocells (laser ray receivers) on the opposite side of a rectangular frame. A laser light from an emitter at point (x_{1i}, y_{1i} ; i=1,2,···imax) is expanded by a cylindrical lens to a fan-shaped sheet and received simultaneously by an array of receivers (photocells). Laser source is numbered as N_{1i} where subscript 1i means that the ray is emitted from the i-th source on the source side of frame. j-th receiver photocell at point (x_{2j}, y_{2j} ; j=1,2,···jmax) is numbered as N_{2j} and the intensity of ray from i-th emitter and received by j-th receiver is denoted as I_{ij} (Figure 1).

The number of emitters and receivers was both 12 and 12, but afterwards they were augmented to 32×32 in order to apply the wavelet technique.

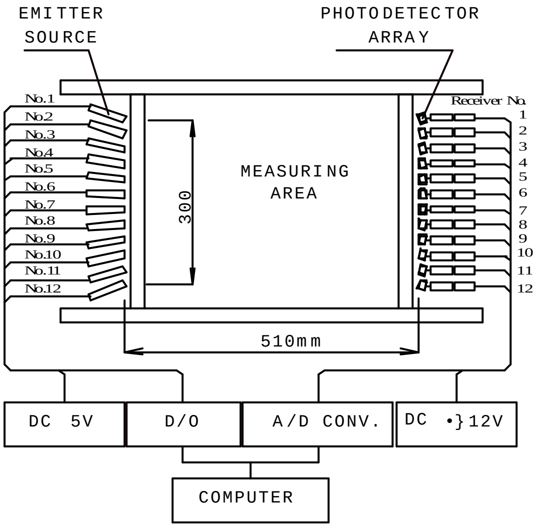


Fig. 1. Schematic diagram of the laser-sheet CT-scan type 2-D instantaneous concentration meter.
(In the case of the number of emitters and receivers: 12´12 channels)

2.2 Method of Ray Emitting – High Speed Scanning

At the stage of proposal of the instrument, two kinds of emitter and receiver systems were considered (Hino 1997, Hino, *et al*. 1998); *i.e.*, (i) The stepping motor control of shedding light beams, and (ii) the on- and off-switching of fan-shaped sheets of laser ray shed through cylindrical lens. At first, the former system of shedding laser beam from emitters to the direction of which was controlled by stepping motor had been adapted, considering the advantage of using strong ray intensity transmitted through the concentration field, with trade off of slow scanning speed.

After completion of the first type system, it was soon realized that the latter type of shedding and control system, is favorable to avoid the mechanical part and to increase the scanning period. In the fan-shaped sheet shedding system, every receiver is simultaneously lit from a source which is switched on and off at an interval with a lag between the neighboring sources. Figure 2 demonstrates the timing chart of emitters (laser diode) and receivers.

Figure 3 is an example from a record (time series) of receiver light intensity, Rij ; the distribution of laser intensities from an emitter received at the same instant by array of receivers.

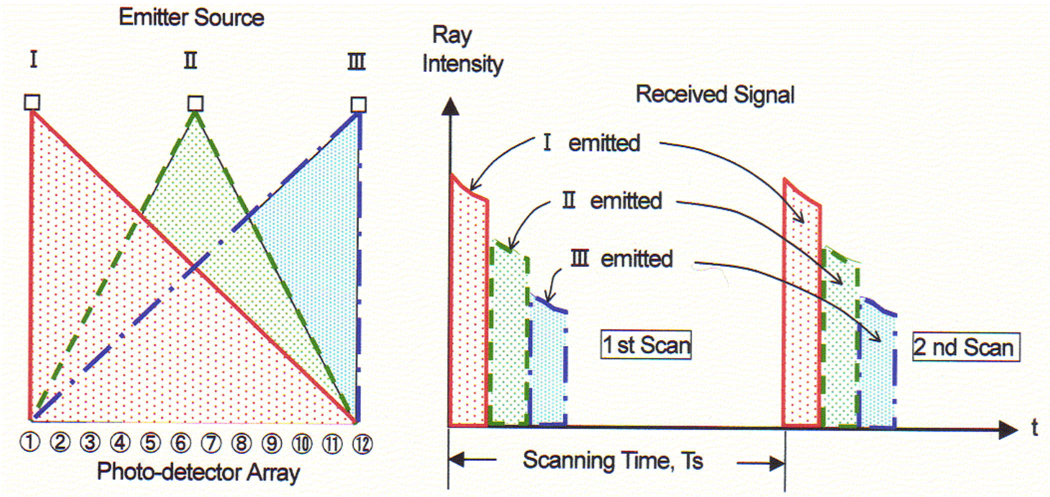


Fig. 2. Timing chart of emitters (laser diodes) and receivers (photo-detector array).
In the case of the number of emitters and receivers: 12´12 channels,
Ts = Te×Number of Emitters (12)
Te: Emitting time (=1.8 ms at minimum)
Ts = 21 ms at minimum

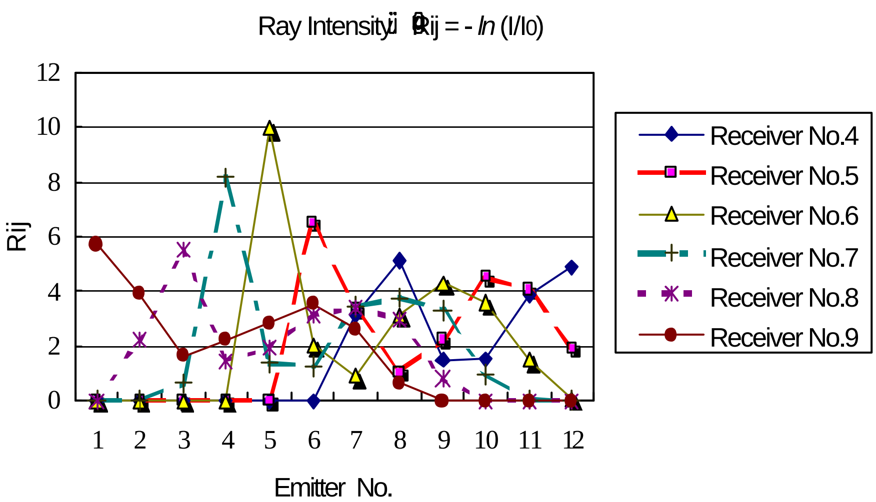


Fig. 3. Example of a time series record for light intensity data Rij, detected by receivers.

3. PRINCIPLE OF INVERSE ESTIMATION OF 2-D CONCENTRATION FIELD

3.1 Field Expression and Inverse Problem - Identification of 'Virtual Load'

Laser light is shed through a concentration field, C(x, y) . The extinction of light intensity (I) is expressed by

$$dI/ds = \lambda C(x, y) \cdot I \qquad (1)$$

where λ is the light extinction rate, which may be, strictly speaking, a function of particle size distribution. For the sake of simplicity, λ is presently assumed constant. The intensity of laser-light shed from an emitter (x_1, y_1) with intensity I_0 and received at a point (x_2, y_2) is given, by integration of Eq. (1), as

$$R(y_1, y_2) = \int C(x, y)\, ds$$

$$= \int C(x, y_1 + x \tan\theta)\, dx/\cos\theta \qquad (2)$$

where

$$R(y_1, y_2) = (\lambda)^{-1} \mathit{ln}(I_0/I) \qquad (3)$$

and

$$\tan\theta = (y_2 - y_1)/ x_2 \qquad (4)$$

$$x_2 = x_1 + s \cos\theta,\ \ y_2 = y_1 + x \tan\theta \qquad (5)$$

and, $x = x_2 - x_1$, s is the distance between the emitter and the receiver, θ being the angle of light emission.

The aim is to find the 2-D concentration field from the integrated information R. In order to increase the amount and quality of information, the sounding laser light is needed preferably to shed from various direction, as is done in the case of CT-scanner in medicine. However, the arrangement of the emitter and receiver system is restricted in order not to disturb the flow field on the one hand, and rotating the system is prohibited in order to obtain in an instant the information of the field which varies rapidly every moment, on the other hand.
This is the difficulty that we must resolve compared to the other finished devices of inverse and/or non-intrusive measurement techniques. If the distances to concentration particles are long, and the sonic wave emitter is available for the detection as in the case of meteorology (sonic Doppler radar) and of oceanography (sonic profiler), the information of the distance to a point may be obtained using sonic signals, and the inverse problem is loosened considerably.

3.2 Inverse Estimation by Double Fourier Series Expansion

The standard way of solving the inverse problem of this kind is to have recourse to the double Fourier expansion of an unknown field, C(x, y). However, as explained in the previous paper (Hino and Sato, 1998 ; Paper I), the method resulted in failure, the reason of which has been analyzed also in Paper I.

3.3 Inverse Estimation by *'Virtual Load' Method*

Field Expression by 'Virtual Load' method: The unknown concentration field, C(x, y), may be rewritten in a double-series expansion based on the base functions G(x, y; i, j)

$$C(x, y) = \sum w_p(i, j)\, G(x,y ; i ,j) \qquad (6)$$

or, digitizing (x, y) into ($I\Delta x$, $J\Delta y$), as

$$C(I, J) = \sum w_p(i, j)\, G(I,J ; i ,j) \qquad (7)$$

In this paper, we apply the 'virtual load' method proposed by the first writer (Hino 1975, Hino and Miyanaga 1975) which uses as the base functions the Green's function of deflection of a rectangular elastic plate loaded by a 'virtual' unit load,

$$G(x,y ; i ,j) = 4/(\pi^2 ab)\cdot \sum_{m=1}^{m_{max}} \sum_{n=1}^{n_{max}} [\sin(m\pi x/a) \cdot\sin(n\pi y/b) \cdot\sin(m\pi i/a) \cdot\sin(n\pi j/b)/(m^2/a^2 + n^2/b^2)] \qquad (8)$$

where a and b are the length and width of a virtual elastic plate, respectively.

The 'virtual-load' method is a kind of 2-D interpolation technique. A 2-dimensional field C(x, y) is assumed to correspond to the deflection of an elastic plate supported at four edges covering a wider region than the 2-D field considered.

The Green's functions lack the orthonormality contrary to the Fourier functions. At a first glance, it may sound curious that the former method gives the successful result, while the latter fails. The reason has been analyzed in Paper I; *i.e.*, since the emitter-receiver system is fixed, the given data R, does not contain the information with high independency. In other words, the direction and path of laser light sounding the field are almost the same. Consequently, low independency results in the exaggeration of errors.

Substitution of Eq. (7) into Eq. (2) yields the following matrix expression,

$$\begin{bmatrix} GG_{1,1} & GG_{1,2} & \cdots & GG_{1,pmax} \\ GG_{2,1} & GG_{2,2} & \cdots & GG_{2,pmax} \\ \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \cdots & \bullet \\ GG_{N1max,1} & GG_{N1max,2} & \cdots & GG_{N1max,pmax} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \bullet \\ \bullet \\ \bullet \\ w_{pmax} \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ R_{NNmax} \end{bmatrix} \qquad (9)$$

or, in a matrix from, as

$$[\ GG\ ][\ w_p\ ] = [\ R\ ] \qquad (10)$$

where the vector [w] means the 'virtual - load' on points (ξ,η) or $(i\Delta\xi, j\Delta\eta)$,

$$w_p = [\ w_1,\ w_2,\ \cdots,\ w_{pmax}\ ]^T \qquad (11)$$

and the matrix element GG(NN, p) is given by

$$GG = \int G(x,\ y_1 + x\tan\theta;\ \xi,\eta)\ dx/\cos\theta \qquad (12)$$

where

$$NN = (N_1 - N)\cdot N_{2max} + N_2 \qquad (13)$$

and

$N_1 = 1, 2, \cdots, N_{1max}$ (emitter number)

$N_2 = 1, 2, \cdots, N_{2max}$ (receiver number)

$NN_{max} = N_{1max}\cdot N_{2max}$

[R] represents the light extinction vector,

$$[\ R\ ] = [\ R_{1,1}, R_{1,2}, \cdots, R_{1,N2max}, R_{2,1}, \cdots, R_{N1max,1}, \cdots, R_{N1max,N2max}\ ]^T \qquad (14)$$

where $R_{N1max,N2max} = R_{NNmax}$.

<u>Identification of 'Virtual Load'</u>: Our problem of determining the concentration field from an integrated information, R ; *i.e.*, the extinction of laser light, reduces to the identification of weighing [w_p] by solving the simultaneous eqs. (10). If $NN_{max} > p_{max}$, we can solve Eqs. (10), by applying the transpose matrix $[\ GG\ ]^T$ on both sides, to obtain the 'virtual loads' [w_p] as

$$[\ w_p\ ] = \{\ [\ GG\ ]^T [\ GG\ ]\ \}^{-1}\cdot[\ GG\ ]^T\cdot[\ R\ ] \qquad (15)$$

<u>Inversely Estimated Concentration Field</u>: The final step is to reconstruct the concentration field by substituting the solution [w_p] into Eq. (7). Substitution of the 'virtual-Load' vector [w_p] obtained from Eq. (15), gives the concentrations as

$$\mathbf{[\ C\ ] = [\ G_c\ ]\cdot[\ W_p\ ]} \qquad (16)$$

In Eq. (15), [C] is the concentration field expressed in a vectorial form as

$$[\ C\ ] = \begin{bmatrix} C_{1,1} \\ C_{1,2} \\ \bullet \\ \bullet \\ C_{1,Jmax} \\ C_{2,1} \\ C_{2,2} \\ \bullet \end{bmatrix} \qquad (17)$$

•
$C_{2,Jmax}$
•
•
$C_{Imax,1}$
•
$C_{Imax,Jmax}$

and $[\,G_c\,]$ is the Green's function matrix

$$[\,G_c\,] = \begin{bmatrix} G(i_1,j_1\,;\,\xi_1,\eta_1) & G(i_1,j_1\,;\,\xi_2,\eta_2) & \cdots & G(i_1,j_1\,;\,\xi_{pmax},\eta_{pmax}) \\ \vdots & & & \\ G(i_{max},j_{max}\,;\,\xi_1,\eta_1) & G(i_{max},j_{max}\,;\,\xi_2,\eta_2) & \cdots & G(i_{max},j_{max}\,;\,\xi_{pmax},\eta_{pmax}) \end{bmatrix} \qquad (18)$$

4. INCREASE OF RESOLUTION BY WAVELET ANALYSIS

4.1 Wavelet Analysis and 1-Dimensional Wavelet Transform

Fourier transform is the frequency domain analysis of random signals. While the wavelet analysis is the decomposition of a random signal both in time and frequency domain. At first, the wavelet analysis has been applied as an extension of Fourier analysis. In recent years, the method finds various applications, such as signal and graphics data processing, graphics recovery, graphical data compression, and securing information.

A variety of the wavelet analysis techniques may be applied to the analysis of CT-type concentration meter. In this paper, only two and the simplest among them are explained; *i.e.*, 1 and 2 –dimensional wavelet transform method. (The applicability of other wavelet analysis techniques are discussed in Hino (2000).)
Column vector of receiver, R (= $ln(\,I_0(x)/I(x)\,)$, is expressed in terms of discrete wavelet, W_N, as

$$R = W_N \cdot S_1 \qquad (19)$$

where S_1 is the one dimensional wavelet spectrum, and $N = n \times m$, n = number of emitter, m = number of receiver (Saito 1996). The inverse wavelet transform of Eq. (19) is written as

$$S_1 = W_N^T \qquad (20)$$

where the property that the inverse of W_N is equal to its transpose expressed by the following Eq. (21) is applied,

$$W_N^{-1} = W_N^T \qquad (21)$$

In this paper, we applied the simplest wavelet (Haar function), in order to be able to follow the process data treatment.

4.2 Two-Dimensional Wavelet Transform

Information of laser-light extinction may be expressed as a matrix, $[RR_{n,m}]$, where the row number (n) corresponds to the number of emitter and the column number (m) means the number of receiver. The matrix [RR] is rewritten by the two-dimensional wavelet transform as

$$[\,RR_{n,m}\,] = W_n^T \cdot S_2 \cdot W_m \qquad (22)$$

Inverse expression of the above equation yields the wavelet spectrum S_2 as

$$S_2 = W_n \cdot [\,RR_{n,m}\,] \cdot W_m^T \qquad (23)$$

4.3 Cleaning of Wavelet Spectrum and Recovery of Reliable Data

The signals received through the concentration field are contaminated by noise, such as high frequency random noise or, pulse or spike-like noise. These high frequency noises may be eliminated in the frequency domain of wavelet spectrum to obtain S_{1clean} or S_{2clean}.
The inverse wavelet transform of S_{1clean} and S_{2clean} gives the cleaned receiver information,

$$R_{clean} = W_N \cdot S_{1clean} \tag{24}$$

and

$$[\ RR_{n,m}\ ]_{clean} = W_n^T \cdot S_{2clean} \cdot W_m \tag{25}$$

5. EXPERIMENTAL RESULTS AND CONCLUSION

5.1 Laboratory Experiment of a Jet Concentration

Experiment on the concentration measurement has been performed for a jet of spray from humidifier. Figure 3, as already explained, shows the record of signals, and the concentrations inversely determined from R by Eqs. (15) and (6) averaged for ten scans are shown in Figs. 4 (a) and (b).

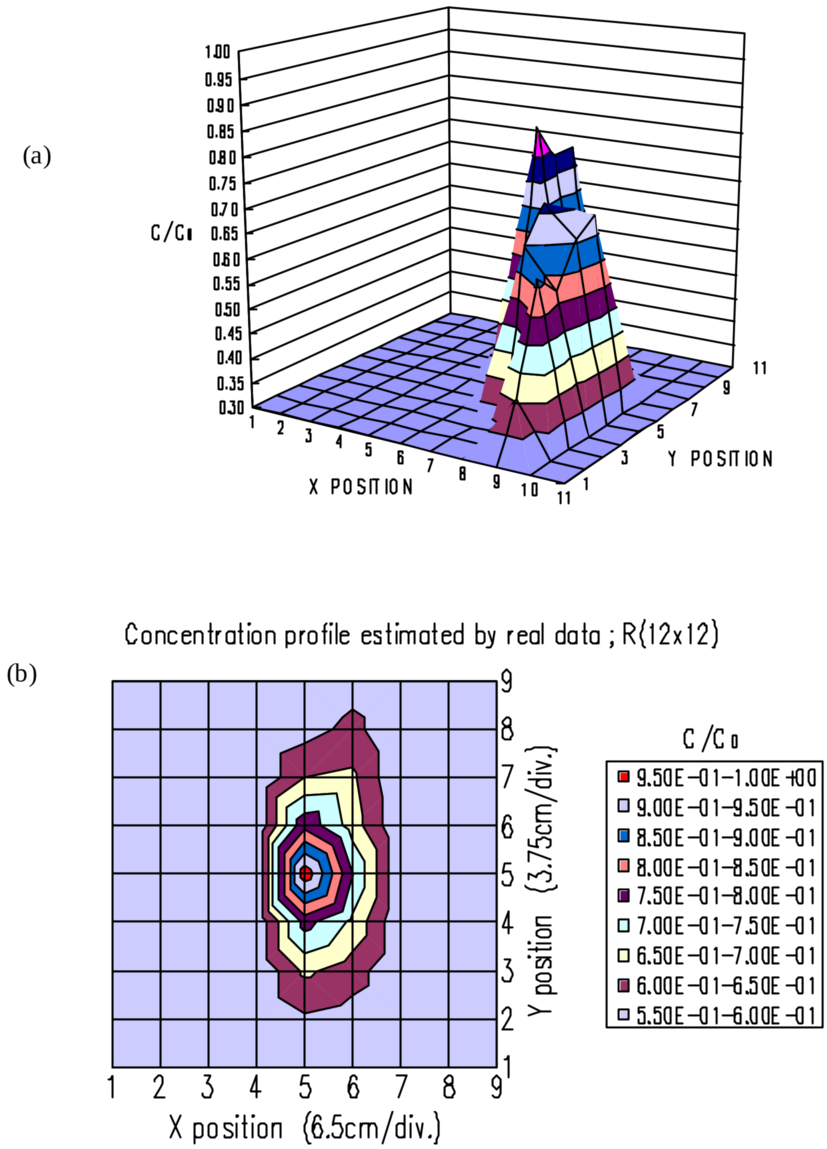


Fig. 4. Examples of 2-D concentration profile measured in a jet spray mist flow.

5.2 Numerical Simulation Experiments on the Effectiveness of Wavelet Transform

The effectiveness of the wavelet transform technique has been examined by the numerical simulation data since the augmentation of the number of emitters and receivers for application of wavelet technique was not yet completed. Figures 5 (a) and (b) show the numerically simulated pure signals received by the array of receivers together with those contaminated by spike-like noise and random noise (error level being 50 %), respectively. In the same graphs, the signals with error suppressed by the wavelet method are also given.
Tests on the effectiveness have further been performed for a concentration field of Gaussian profile. The test results are shown in Figures 6 (a), (b) and (c); (a) no erratic signals, (b) erratic signals of both spike-type (error level being 5 %) and (c) for erratic signals of random noise type (error level 10 %) are superimposed on it. The

inversely estimated (reconstructed) concentration fields (on the left side (i)) are given in Figures 6 (a), (b) and (c), together with the pattern of estimation error distribution. The estimation error level, whose distributions are shown in the right (ii) of Figures 6 (a), (b), (c) are suppressed as low as below appreciable level.

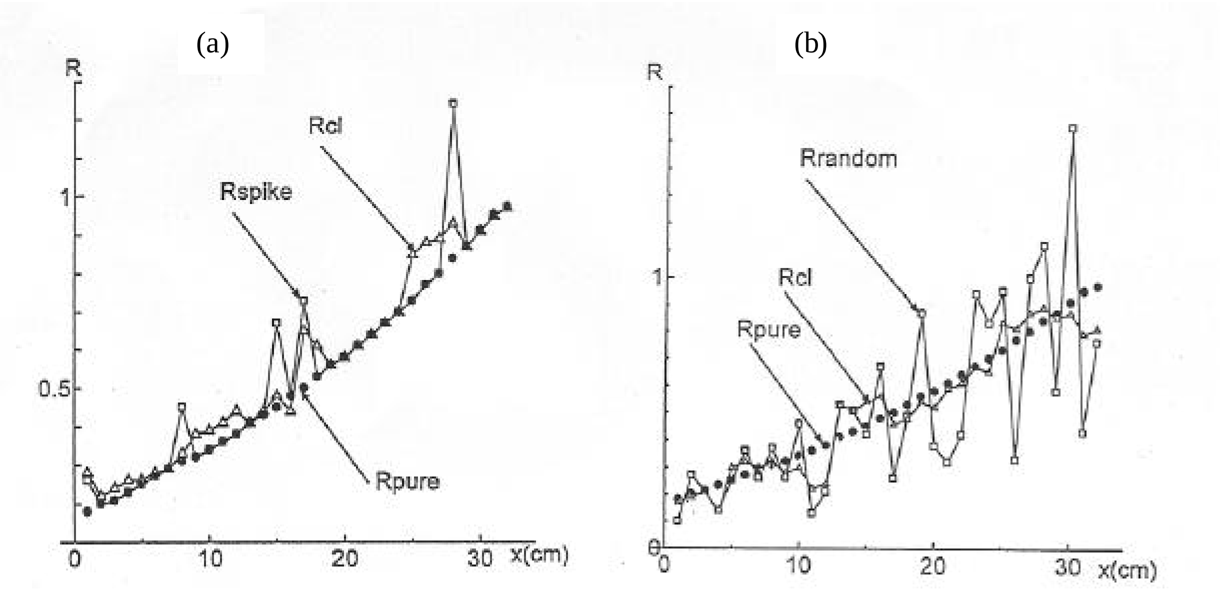
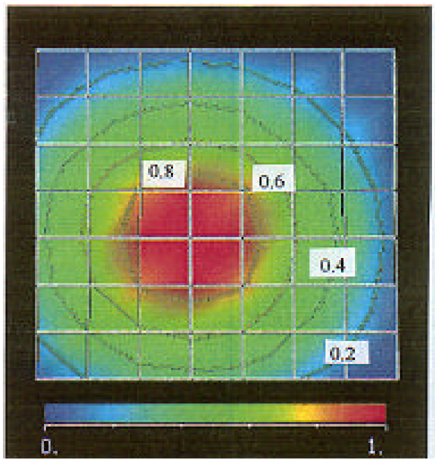


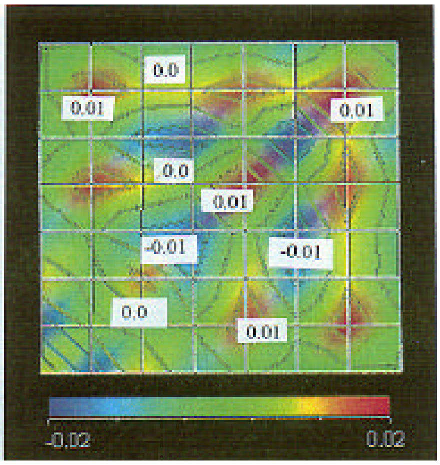
Fig. 5. Simulation results on the suppression/removal of noises for R signals by the wavelet transform method. (a): in the case of spike-type noises; (b): in the case of random noises

(i) Estimated concentration contour

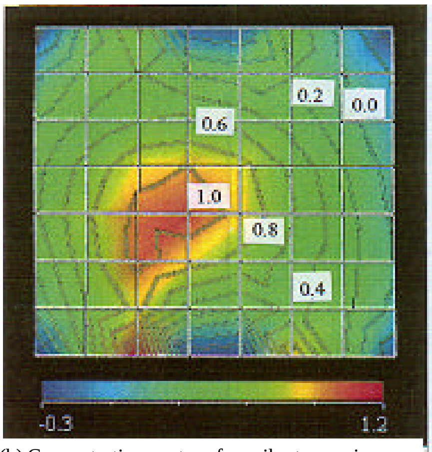
(ii) Estimated error contour



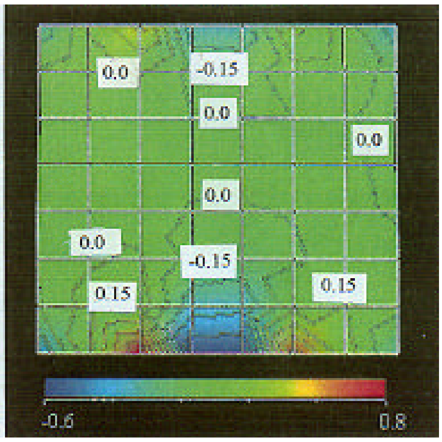
(a) Concentration contour for no noise



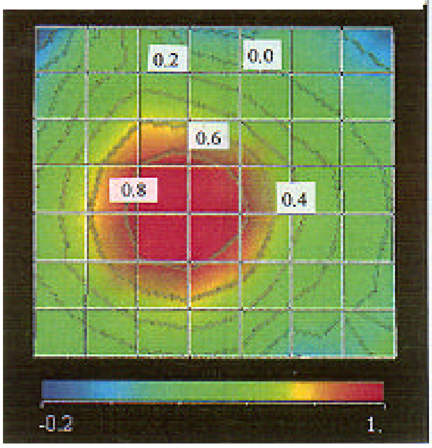
(a) Error contour for no noise



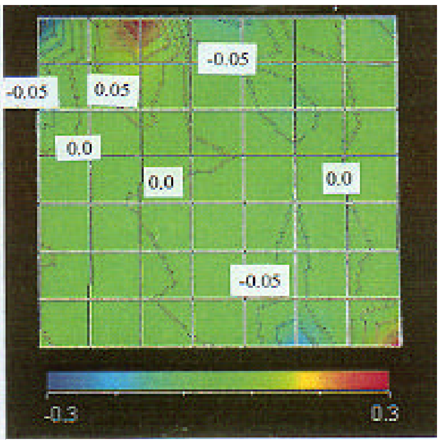
(b) Concentration contour for spike-type noise(5%)



(b) Error contour for spike-type noise(5%)



(c) Concentration contour for random noise(10%)



(c) Error contour for random noise(10%)

Fig. 6. Simulation results on inversely estimated concentration fields (Gaussian profile), with the pattern of estimation error distributions, using the wavelet error suppression method.

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