

# Distortion Compensation for PIV Systems

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## ABSTRACT

PIV images can be distorted for many reasons, including oblique viewing and window aberrations. Calibration methods are now available for a rigorous description of distortions and extraction of all the information available in PIV images even under difficult measurement conditions.

The concept of the *measurement plane* is introduced. This plane is tangential to the distorted surface being imaged. Orientation of the measurement plane, specified by the *Measurement Plane Pointer* (MPP), may vary from point to point within the region being viewed (the object space).

Generalized relationships are provided for computing MPPs over the entire object space using 3D calibration data. Also, a  $2 \times 3$  matrix is derived for converting displacements from the image plane into the measurement plane. This matrix is a generalization of the  $2 \times 2$  matrix conventionally used for distortion correction in standard (single-camera) PIV.

When the above scheme is applied to a dual-camera stereoscopic system, the two maps of MPPs can be combined to compute the sensitivity of the out-of-plane component and its distribution over the entire object space.

In order to specify MPPs, one needs to obtain 3-dimensional calibration data from the object space (in the same manner as the calibration of a stereoscopic PIV system). If a planar target with markers is used for calibration, it must be imaged at two or more locations along the principal axis of the camera. Traversing of the target may be cumbersome and a potential source of errors. A solution to this impediment was first sought by imposing a physical constraint, called *isotropic magnification*, on the imaging process.

Isotropic magnification assumes that magnification is independent of the direction within the measurement plane. Using experimental data, it is shown that isotropic magnification works well for a typical stereoscopic PIV system, provided that the Scheimpflug condition is met and that the measurements are taken in air. In a water flow, where there is a change of medium between the camera and the light sheet, the experimental results show significant deviations from isotropic magnification, indicating that it is not always a reliable means for a complete calibration.

Finally, a bi-planar target (included in Stereoscopic PIV Accessories of TSI, Model 610050) is introduced that provides complete 3D calibration data while obviating the need to traverse the target. This new calibration target includes a reflector on one side that helps align the device precisely with the laser light sheet.

## 1. INTRODUCTION

A scheme for distortion correction in PIV, using the calibration data is given by Soloff et al. (1997). This scheme is further extended to obtain additional information from the distorted images. The transfer function between the object space and the image plane is used to locally define the plane in which measurements are taken. In the case of a single-camera system, this allows to quantify the error introduced by the assumption that the 2D measurement is always taken in the light sheet. In the case of stereoscopic system, this work allows to quantify the local sensitivity of the system to the out-of-plane velocity component.

This work underlines the importance of 3D calibration for all PIV systems. An easy-to-use 3D calibration device is introduced.

## 2. RELATIONSHIP BETWEEN MEASUREMENT VOLUMES AND INTERROGATION SPOTS

One of the purposes of distortion compensation is to correctly associate three-dimensional measurement volumes in an object space (a portion of a fluid flow) with the two-dimensional interrogation spots on an image plane (sensor array in a camera). An imaging function is used to this end.

As elaborated by Soloff et al. (1997), three-dimensional position vectors  $\vec{x}$  in the object space can be related to the two-dimensional position vectors in the image plane using a non-linear relation

$$\vec{X} = \vec{F}(\vec{x}) \quad . \quad (1)$$

The pertinent coordinate systems are illustrated in Fig. 1. It may be noticed that the object space may consist of a thin planar slice, defined by the light sheet used for illumination or the focal depth of the camera. However, the object space is not being approximated by a two-dimensional plane, as the particle motion perpendicular to the light sheet may have a significant contribution to the measured displacement in the image plane, especially in the case of oblique viewing.

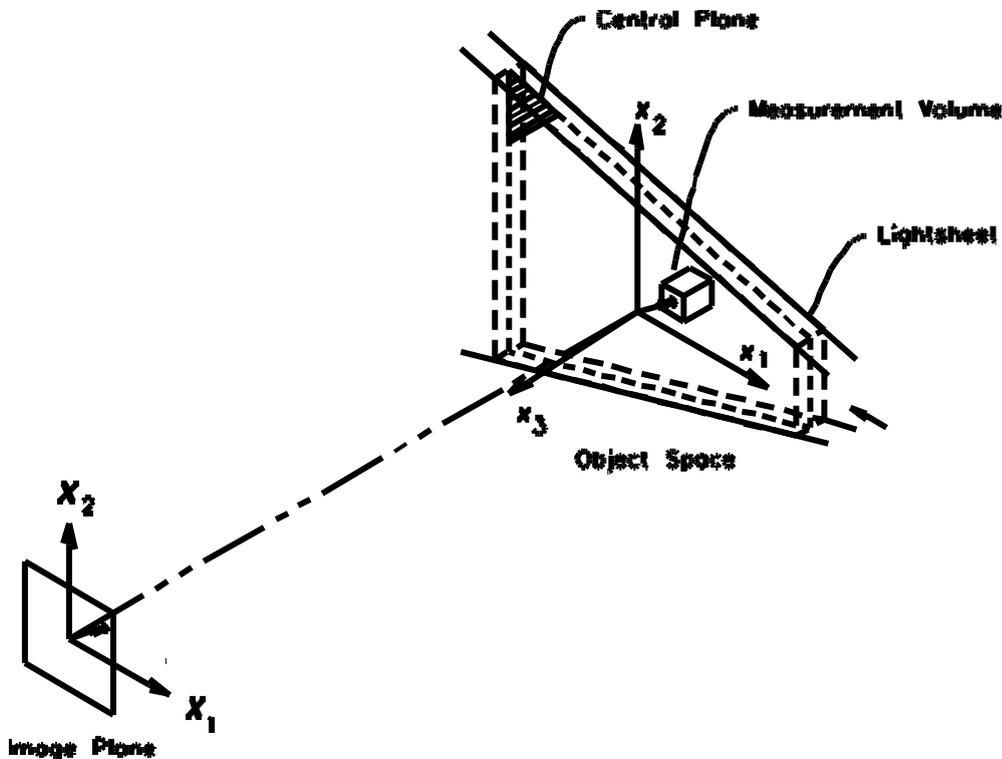


Figure 1: Imaging system: coordinates and other definitions

It has been demonstrated that function  $\vec{F}$  can be represented satisfactorily using polynomials in the components of  $\vec{x}$ . Soloff et al. (1997) have given an example of a polynomial representation that involves cubic dependence in  $x_1$  and  $x_2$  and quadratic in  $x_3$ . This function involves 18 two-dimensional coefficients that can be evaluated using the calibration data, which consists of multiple images of a planar target with markers at known locations. The target is placed parallel to  $x_1x_2$ -plane and multiple images correspond to various positions of the target along  $x_3$ -axis; i.e. the planar target is traversed along  $x_3$ -axis between image captures. Alternatively, a three-dimensional target can be used to capture all the calibration data in a single image.

Knowing that the function  $\vec{F}$  can be specified in practical situations, it is assumed hereafter that this function is known. Furthermore, it is assumed to be a smooth and differentiable function.

A very obvious and simple use of the imaging function is to determine where to read the data from the image plane, in order to compute the velocity vector in a given cell or *measurement volume* in the object space.

The centroidal location  $\vec{x}$  of a three-dimensional measurement volume can be entered into  $\vec{F}$  to compute the corresponding value of image plane location  $\vec{X}$ . The needed information (i.e. particle displacements between two PIV images) may not be available at this particular location, but can be obtained by interpolation with the known values at the neighboring points. The image plane is typically divided into square interrogation spots and average particle displacements are computed at the center of each spot. Hence, imaging function relates the measurement volumes with the interrogation spots.

The object space coordinates are usually selected in a manner that the measurement volumes are centered on  $x_1x_2$ -plane and the position vectors representing their centroids have a vanishing value of  $x_3$  coordinate. This has led some investigators to assume that particle displacements along  $x_3$ -axis can be legitimately set to zero in a single camera application. As shown later, this is not a good choice if the light sheet is viewed obliquely or imaging distortions are large otherwise.

Normally,  $x_1x_2$ -plane would contain the centroids of the measurement volumes and hence may be referred to as the *central plane*. This plane is a convenient choice for presenting the data taken over various measurement volumes.

In the next section, imaging function is utilized to relate the small displacements in a measurement volume with those in the imaging plane.

### **3. RELATIONSHIP BETWEEN MEASUREMENT PLANE AND IMAGE PLANE**

The objective of the PIV measurements, of course, is to estimate displacement of particles in a measurement volume using the measured displacements in an image plane. Quite clearly, a two-dimensional measure of displacement cannot fully unravel three-dimensional displacements in the measurement volume. In fact, the image displacements represent particle displacements along a surface within the measurement volume. For small measurement volumes, a measurement plane (see Fig. 2) approximates this surface, so that particle displacements perpendicular to the measurement plane do not cause any displacements in the image plane.

The orientation of measurement plane may vary from point to point. A unit vector perpendicular to this plane may conveniently define measurement plane orientation. The imaging function needs to be differentiated in order to generate a map of three-dimensional unit vectors (*measurement plane pointers or MPP*) representing the local normal to the plane of measurement.

The above approximation amounts to a Taylor series expansion of the imaging function, in which only first order displacements are retained.

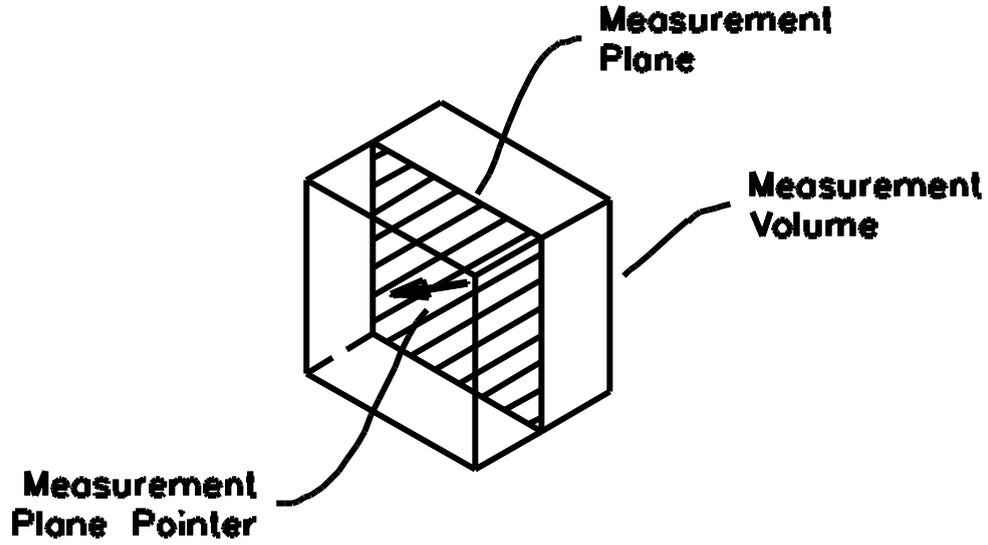


Figure 2: Measurement plane and MPP

As shown by Soloff et al. (1997), these first order displacements in the two coordinate systems can be related through

$$\Delta \vec{X} = [\nabla F] \Delta \vec{x}, \quad (2)$$

where  $\nabla F$  is the *magnification matrix*, whose elements are given as

$$(\nabla F)_{ij} = \frac{\partial F_i}{\partial x_j} = F_{i,j} \quad i = 1,2 \quad \text{and} \quad j = 1,2,3. \quad (3)$$

Local values of six partial derivatives of the imaging function are needed for computing MPP. These can be easily obtained if  $\vec{F}$  is represented as a smooth differentiable function as discussed in the previous section.

An equation for MPPs can be obtained by setting  $\Delta \vec{X} = 0$  in Eq. (2). Hence, MPP equations are expressed as

$$\begin{aligned} F_{1,1} \Delta x_1 + F_{1,2} \Delta x_2 + F_{1,3} \Delta x_3 &= 0 \\ F_{2,1} \Delta x_1 + F_{2,2} \Delta x_2 + F_{2,3} \Delta x_3 &= 0 \end{aligned} \quad (4)$$

After some algebraic manipulations,

$$\frac{\Delta x_1}{G_1} = \frac{\Delta x_2}{G_2} = \frac{\Delta x_3}{G_3}, \quad (5)$$

where

$$\begin{aligned}
G_1 &= F_{1,2}F_{2,3} - F_{2,2}F_{1,3} \\
G_2 &= F_{2,1}F_{1,3} - F_{1,1}F_{2,3} \\
G_3 &= F_{2,2}F_{1,1} - F_{1,2}F_{2,1}
\end{aligned} \tag{6}$$

Hence, a vector along MPP is given by

$$G_1\vec{i} + G_2\vec{j} + G_3\vec{k}, \tag{7}$$

where  $\vec{i}$ ,  $\vec{j}$  and  $\vec{k}$  denote unit vectors along  $x_1$ ,  $x_2$  and  $x_3$  axes.

In general, this is not a unit vector and needs to be divided by its magnitude in order to formally represent the MPP; i.e.

$$\vec{p} = (G_1\vec{i} + G_2\vec{j} + G_3\vec{k}) / \sqrt{H}, \tag{8}$$

where

$$H = G_1^2 + G_2^2 + G_3^2. \tag{9}$$

The vector  $\vec{p}$  as defined above could be pointing into or out of the measurement plane. In other words, Eq. (8) can be multiplied by -1 and it will still represent a unit vector perpendicular to the measurement plane.

The above expression will give a vector pointing out of the measurement plane, provided that  $x_3$  is pointing outwards and  $G_3$  is positive. Under normal conditions  $X_1$  and  $X_2$  would be aligned with  $x_1$  and  $x_2$  respectively, so the product  $F_{1,1}F_{2,2}$  will be positive and much larger than  $F_{1,2}F_{2,1}$ . Hence,  $G_3$  as given by Eq. (6) will indeed be positive.

In the case of orthogonal viewing and distortion-free imaging, MPPs are perpendicular to the plane of illumination throughout the field of view. For distortion-free oblique viewing, they are still parallel to the optical axis of the camera but oblique to the plane of illumination. In the presence of distortions, MPPs are not necessarily parallel to the optical axis of the camera and may lie anywhere within the viewing cone of the camera.

The above discussion has shown how to identify the velocity component that is not measured. The next task is to find a means to compute the two velocity components that *are* measured. This involves inverting Eq. (2), such that  $\Delta\vec{x}_m$  (projection of  $\Delta\vec{x}$  on the measurement plane) is perpendicular to MPP, or

$$\vec{p} \cdot \Delta\vec{x}_m = 0. \tag{10}$$

The above statement is a linear equation in  $\Delta\vec{x}_m$  and can be appended to the set of two equations in (2), so that

$$\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ 0 \end{bmatrix} = \begin{bmatrix} F_{1,1} & F_{1,2} & F_{1,3} \\ F_{2,1} & F_{2,2} & F_{2,3} \\ G_1 & G_2 & G_3 \end{bmatrix} \begin{bmatrix} \Delta x_{m1} \\ \Delta x_{m2} \\ \Delta x_{m3} \end{bmatrix}. \tag{11}$$

The solution for  $\Delta\vec{x}_m$  can be obtained by inverting the 3x3 matrix on the right-hand side of Eq. (11). The determinant of this matrix is  $H$  as given by Eq. (9), which is positive definite; i.e. this matrix is non-singular, as long as MPP exists.

Efficient numerical schemes are available to invert the above matrix. The explicit solution of Eq. (11) is also fairly simple and can be expressed as below:

$$\Delta \vec{x}_m = [T] \Delta \vec{X}, \quad (12)$$

where

$$[T] = \frac{1}{H} \begin{bmatrix} F_{2,2}G_3 - F_{2,3}G_2 & F_{1,3}G_2 - F_{1,2}G_3 \\ F_{2,3}G_1 - F_{2,1}G_3 & F_{1,1}G_3 - F_{1,3}G_1 \\ F_{2,1}G_2 - F_{2,2}G_1 & F_{1,2}G_1 - F_{1,1}G_2 \end{bmatrix}. \quad (13)$$

Since  $\Delta \vec{X}$  has only two non-zero components, only six elements of the inverse matrix are needed to express a complete solution.

The transfer function in Eq. (13) is a generalization of the following matrix that is conventionally used for correcting single camera images:

$$[T_{2D}] = \frac{1}{(F_{1,1}F_{2,2} - F_{1,2}F_{2,1})} \begin{bmatrix} F_{2,2} & -F_{1,2} \\ -F_{2,1} & F_{1,1} \end{bmatrix}. \quad (14)$$

Equation (13) reduces to (14) if the derivatives with respect to  $x_3$  ( $F_{1,3}$  and  $F_{2,3}$ ) are neglected.

The equations derived in this section provide a definition of the measurement plane. Also a  $2 \times 3$  matrix is provided to convert the displacements in the image plane into displacements in the measurement plane, so the velocity components in the measurement plane can be computed.

#### 4. APPLICATIONS OF MPP

A range of simple PIV applications—standard and stereoscopic—do not need an extensive distortion analysis using MPPs. The applications that may need such an analysis include wide-angle measurements, measurements through curved windows and single-camera measurements at oblique angles. For these complex applications, MPP provides a quantitative measure of the contribution of out-of-plane motion to the image plane displacements. An example is given in Fig. 3, where tilt angles of MPPs are plotted for a wide-angle system.

The MPPs were measured for a standard single-camera PIV system. The camera had a  $10 \times 10 \text{ mm}^2$  array that was imaged to about  $55 \times 55 \text{ mm}^2$  area in the object space at about 200 mm stand-off. Calibration images were taken using a planar target with markers. Five images of the target were taken with a  $x_3$  spacing of 0.25 mm between the images.

Calibration data was processed using the Insight<sup>TM</sup> software of TSI that generated a gradient file (with extension .grad) containing the values of  $F_{i,j}$  as a function of  $(x_1, x_2)$  at  $x_3 = 0$ . The MPPs were calculated using Eqs. (6—9). MPP tilt angles, as shown in Fig. 3, represent the inverse sine of the corresponding component of MPP. The lower edge of the measurement region (see Fig. 1) is considered in Fig. 3. As expected, MPPs are tilted up along the entire lower edge. They are tilted along positive  $x_1$  axis for the negative values of  $x_1$  and vice versa.

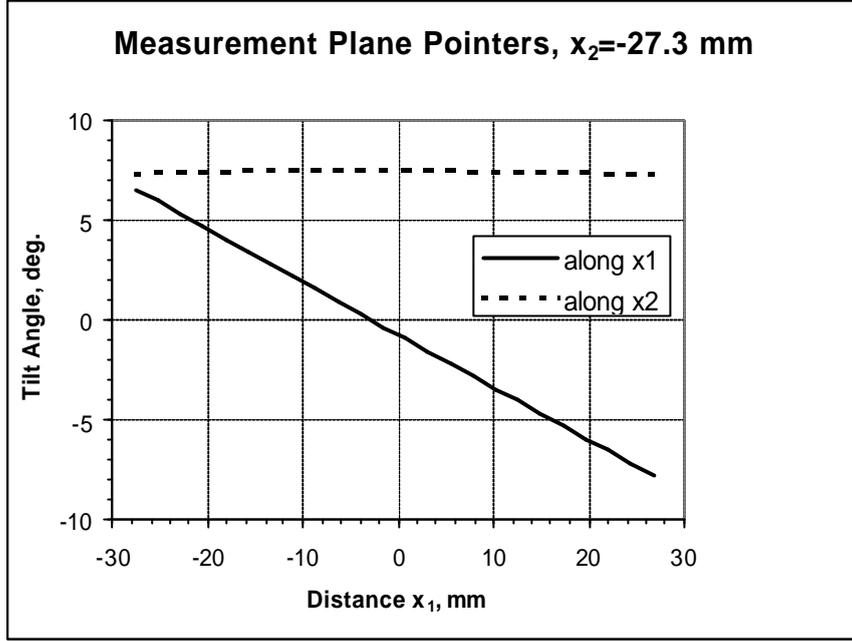


Figure 3: Orientation of MPPs near the lower horizontal edge of the measurement region

Having obtained quantitative information about MPPs, some of the choices the user has are as follows:

1. Determine a level of MPP tilt angles that may be acceptable (e.g. if the out-of-plane velocity component is less than 10% of in-plane components and MPP tilt is less than  $5^\circ$ , the error introduced by ignoring MPP tilt will be less than 1%, which may be acceptable). Continue using Eq. (14) for inversion of 2D data, but reject the data from the regions where the tilt angle exceeds the acceptable limit.
2. Express results in the local measurement planes (instead of the light-sheet plane), i.e. use Eq. (13) for inversion.
3. Extend the measurement system to stereoscopic configuration. This will not only eliminate the complexities with the measurement of in-plane motion, but will also provide a quantitative measure of the out-of-plane motion.

In a stereoscopic system, out-of-plane sensitivity is determined by the angle between the MPPs of the left and right camera, denoted by  $\vec{p}_\ell$  and  $\vec{p}_r$  respectively. The out-of-plane sensitivity relative to the in-plane sensitivity may be expressed as

$$S = \frac{|\vec{p}_\ell - \vec{p}_r|}{|\vec{p}_\ell + \vec{p}_r|}. \quad (15)$$

Alternatively,

$$S = 1 - (\vec{p}_\ell \bullet \vec{p}_r)^2. \quad (16)$$

For either definition, this parameter varies from 0 to 1 as the angle between the MPPs of the two cameras is increased from  $0^\circ$  to  $90^\circ$ . The present formalism allows to generate a map of  $S$  for the entire field of view, in a given measurement setup.

## 5. ISOTROPIC MAGNIFICATION

Some investigators have reported using a planar target, without traversing, for 3D PIV calibration. Details of these methods are not known. However, it is obvious that the pure mathematical transformations discussed so far do not allow 3D calibration with a planar target. To accomplish this, some physical constraints need to be included in the above model.

In physical terms, the measurement plane may be considered as the plane that the camera ‘looks at’ perpendicularly. It is then reasonable to expect that magnification could be isotropic between the measurement plane and the image plane. That is, a plus (+) sign in the image plane is transformed into a plus sign in the measurement plane, so that the two lines of the plus sign are magnified by the same amount and remain perpendicular to each other.

Two columns of the matrix in Eq. (13) represent transformation of unit displacements along  $X_1$  and  $X_2$  - axis. These may be represented by the vectors  $\vec{A}$  and  $\vec{B}$ . If magnification is isotropic, these column vectors have equal magnitude and are perpendicular to each other, i.e.

$$|\vec{A}|^2 = |\vec{B}|^2; \quad (17)$$

and

$$\vec{A} \bullet \vec{B} = 0. \quad (18)$$

Using Eq. (13),

$$|\vec{A}|^2 = (F_{2,2}G_3 - F_{2,3}G_2)^2 + (F_{2,3}G_1 - F_{2,1}G_3)^2 + (F_{2,1}G_2 - F_{2,2}G_1)^2, \quad (19)$$

$$|\vec{B}|^2 = (F_{1,3}G_2 - F_{1,2}G_3)^2 + (F_{1,1}G_3 - F_{1,3}G_1)^2 + (F_{1,2}G_1 - F_{1,1}G_2)^2, \quad (20)$$

and

$$\begin{aligned} \vec{A} \bullet \vec{B} = & (F_{2,2}G_3 - F_{2,3}G_2)(F_{1,3}G_2 - F_{1,2}G_3) + (F_{2,3}G_1 - F_{2,1}G_3)(F_{1,1}G_3 - F_{1,3}G_1) \\ & + (F_{2,1}G_2 - F_{2,2}G_1)(F_{1,2}G_1 - F_{1,1}G_2) \end{aligned} \quad (21)$$

Equations (17) and (18) impose two independent constraints on the values of six elements,  $F_{i,j}$ , of the magnification matrix. Hence, only four elements need to be calibrated. The other two elements can be computed using Eqs. (17) and (18).

A two-dimensional calibration target, placed in the central plane, provides sufficient data to determine  $F_{1,1}$ ,  $F_{1,2}$ ,  $F_{2,1}$  and  $F_{2,2}$ . These data can be inserted in Eqs. (17) and (18), combined with Eqs. (6), in order to obtain two equations representing  $F_{1,3}$  and  $F_{2,3}$ . Hence, the assumption of isotropic magnification should enable one to perform a complete calibration using a simple planar target, without having to capture calibration images at multiple locations along  $x_3$ -axis.

The equations representing  $F_{1,3}$  and  $F_{2,3}$  are 4<sup>th</sup> order polynomials, so multiple solutions are expected. In fact, there is a physical basis for multiple solutions. This can be understood by considering two markers on a planar target with a known horizontal spacing. If the horizontal spacing is reduced in the image plane, it can be concluded that the camera is looking obliquely at the target. However, the image alone does not suggest whether the camera is looking at the markers from the left or the right side. Similarly, for a pair of vertically spaced markers, there will be two solutions pertaining to the camera looking up or down the markers.

Additional information is needed in order to resolve the above ambiguities. This information is usually available. In a stereoscopic system, cameras are typically placed on the left and right side and they view the entire measurement zone from one side only. Also, cameras may look up when viewing the upper edge of the measurement zone and the viewing direction may tilt downward gradually. This information may also help to select the correct solutions of the 4<sup>th</sup> order polynomial equations.

Validity of isotropic magnification was tested for two experimental arrangements of stereoscopic PIV systems. The following parameters were computed, using three-dimensional calibration data:

$$\mathbf{d}_1 = 2 \left[ \begin{array}{c} \left| \vec{A} \right| - \left| \vec{B} \right| \\ \left| \vec{A} \right| + \left| \vec{B} \right| \end{array} \right]; \quad (22)$$

$$\mathbf{d}_2 = \frac{\vec{A} \cdot \vec{B}}{\left| \vec{A} \right| \left| \vec{B} \right|}. \quad (23)$$

The parameters  $\mathbf{d}_1$  and  $\mathbf{d}_2$  represent the amplitude and angular distortion of a plus sign respectively. Both are zero in the case of isotropic magnification.

The first test was conducted with the calibration target in air and without a window between the target and the camera, which was positioned at 15° from the side-scatter. The Scheimpflug condition for blur-free image was met. The results are plotted in Figure 4. An area of 100×100 mm<sup>2</sup> in the central plane is covered. The distortions are small and generally within the limits of experimental uncertainty. Hence, the data in Fig. 4 show that isotropic magnification is applicable to an external airflow.

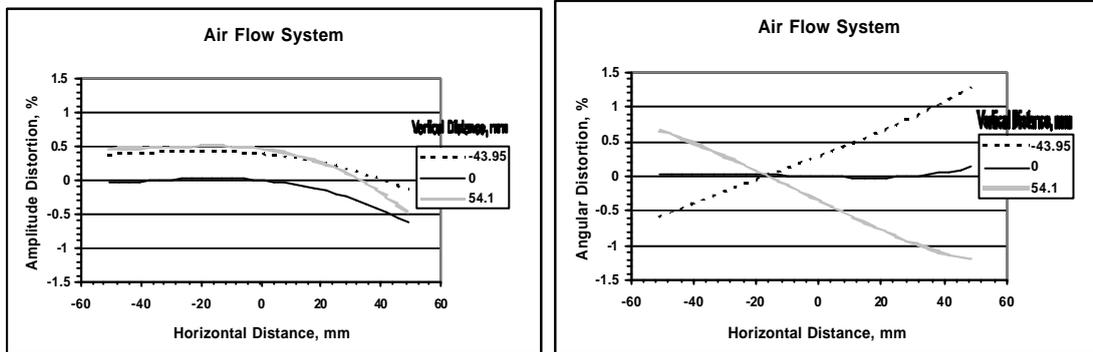


Figure 4: Deviations from isotropic magnification for an external air flow

The second test was conducted with the target in a plexi-glass tank filled with water. The results are shown in Figs. 5 and 6. Results from the left and the right cameras, positioned at  $\pm 20^\circ$ , are plotted. The symbol  $y$  in Fig. 6 represents the vertical distance. In this case distortions are as large as 4%. An expected symmetry between the results of the left and the right camera is seen.

These results show that the assumption of isotropic magnification is not always reliable, especially if a complex geometry is involved. Direct three-dimensional calibration is still the most reliable means to calibrate the PIV systems for stereoscopic recombination as well as computation of MPP. The need to make 3D calibration easier is addressed in the following section.

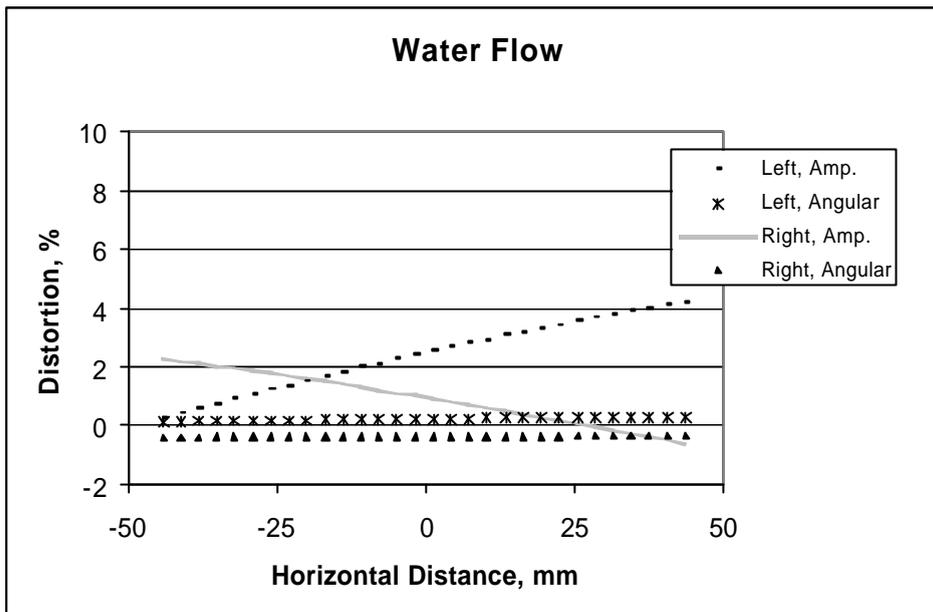


Figure 5: Deviations from isotropy for an internal water flow, along a horizontal line near the center of the measurement region

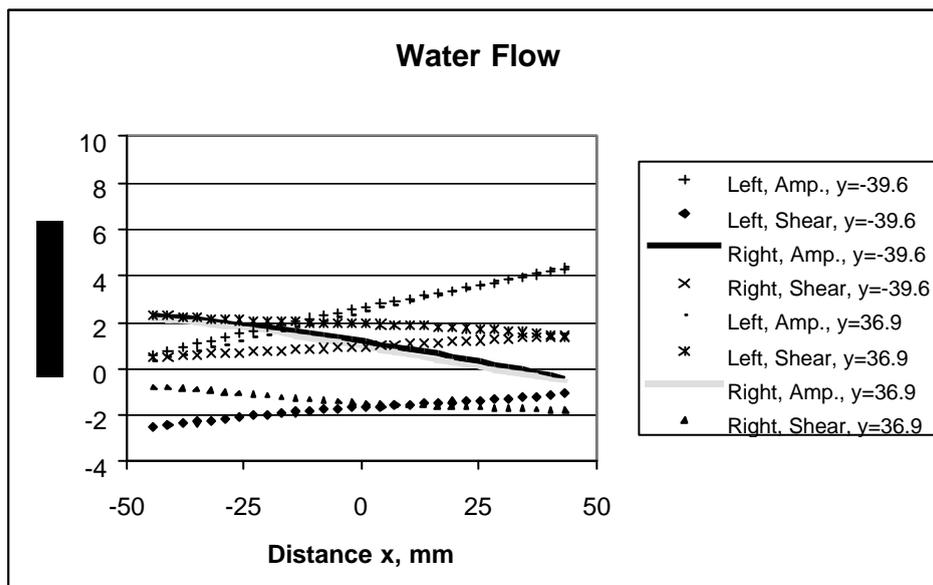


Figure 6: Deviations from isotropy for an internal water flow, along horizontal lines near the top and bottom of the measurement region

## 6. BI-PLANAR TARGET FOR 3D CALIBRATION

A convenient means to obtain three-dimensional calibration is provided by a target that has markers spread out in all three dimensions, instead of just one plane. Such a target (included in Stereoscopic PIV Accessories of TSI, Model 610050) is shown in Fig. 7. It consists of an anodized aluminum plate with diagonal grooves of known depth. The top surfaces of the ridges and the bottom surfaces of the grooves define two distinct planes with a known spacing. White marker dots are painted in such a manner that alternate dots are in the outer and inner planes along a horizontal line as well as a vertical line.

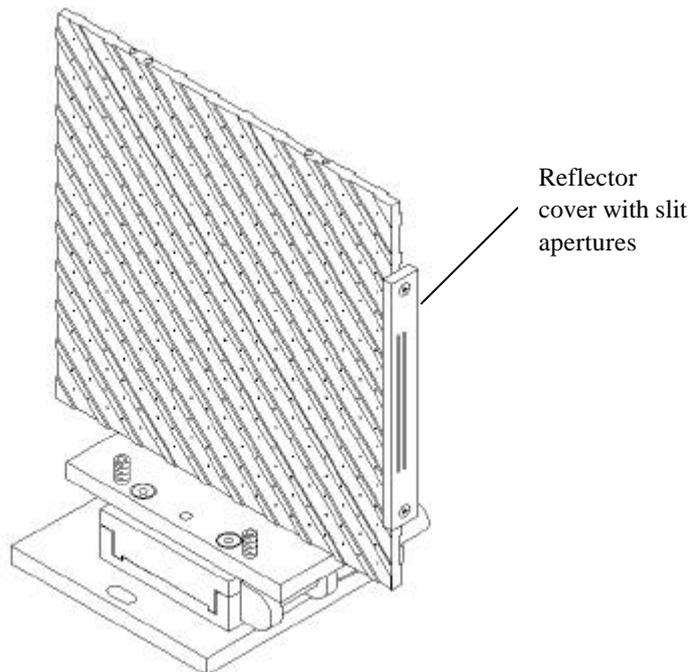


Figure 7: Bi-planar target for PIV calibration

The bi-planar target is positioned in the laser light sheet, so that the two target planes are aligned with  $x_1x_2$  -plane and are separated along  $x_3$  -axis.

In order to facilitate the alignment, a reflector is provided on the side of the target that faces the light sheet. A cover is placed on the reflector allowing reflection of the light sheet from two slit apertures only. These reflections allow the user to align the target planes with the light sheet.

Once properly aligned, this target provides a complete 3D calibration of the imaging function without any traversing of the target.

## 7. CONCLUSIONS

Compensation of imaging distortions using target-based calibration is discussed for PIV measurements. It was assumed in some of the previous works that the out-of-plane motion must be neglected in order to interpret the measurements taken with a single camera. This approximation is shown to be unnecessary. Three-dimensional calibration allows determining the orientation of the measurement plane at each location. The measured displacements can be expressed as two-dimensional vectors in the measurement plane.

In the case of single-camera systems with distortions, the measurement is shown to truly represent two components of the velocity vectors over the entire field. However, the information is available in a complex form as the orientation of the measurement field may vary from point to point.

In the case of stereoscopic measurements, the above scheme allows to generate a map for sensitivity of the out-of-plane component.

Finally, the concept of isotropic magnification is presented that enables one to obtain all the calibration data using a planar target at a fixed location. This concept is evaluated experimentally and found to be valid only for airflow. In a water flow, where a change of medium occurs between the camera and the target, isotropic magnification is accurate only to a few percent.

This reinforces the need to have a convenient means to obtain 3D calibration in a complex geometry. A bi-planar target is shown to satisfy this need.

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